A 1 is present initially at each ULM representing an initial state, all others being zero.

One remarks that there may be no uniform decomposition because one input symbol can enter one state from two or more states. In order to overcome this lack, one can transform the machine into its reverse [8] and define the dual of this reverse. In this case, the number of component machines is equal to the number of states of the nondeterministic automaton, that is, the dual of the reversed machine.

Each component has n external terminals connected to external input lines and 2n internal terminals connected to binary constants or the outputs of other component machines.

Fig. 8 shows the nondeterministic automaton of the machine of Fig. 7(a) and its uniform decomposition using delayed ULM-2's.

IV. CONCLUSION

Interpretation of a ULM-n with 2n control lines as a 2n-in-1 multiplexer allows direct synthesis of sequential machines from its state transition map.

Decomposition is nonuniform in the sense that there may be two or more states entered into one state by the same input symbol.

Decomposition can be made uniform by starting the synthesis not with the state transition map of the initial machine, but with the map of its nondeterministic automaton.

REFERENCES


Comments on “On a New Class of Bounds on Bayes’ Risk in Multihypothesis Pattern Recognition”

GODFRIE T. TOUSSAINT

Recently, Devijver [1] has written an excellent paper on the Bayesian distance and its relation to various measures of distance and information found in the literature.

In Section III, Part E, Devijver discusses a family of measures proposed in [2], Equation (57), which defines this family of measures, is incorrect. Using Devijver’s notation, the correct definition is given by

$$M_k(X/Y) = E_T \left( \sum_{i=1}^{m} \Pr(x_i/y) - \frac{1}{m} \log \frac{1}{X}, \frac{1}{Y} \right), k=0,1,\ldots,\infty.$$  

This does not change any of Devijver’s results since for k = 0, the case of interest in [1], the above expression is the same as (57) in [1].

A second important remark should be made regarding the descriptive title of the measure. In [1] Devijver calls it a separability measure. In [2] and [3] the above measures were proposed as equivocation measures. While there are close similarities between the above measures and separability measures, as shown in [1]–[3], the above measures have the structure of a generalization of the concept of certainty or negative entropy as shown below.

The Shannon entropy of a distribution is given by

$$H(X) = -\sum_{i=1}^{m} \Pr(x_i) \log \Pr(x_i).$$

$$H(X)$$ is a measure of how peaked the distribution is. Shannon’s equivocation is given by

$$H(X/Y) = -E_T \left( \sum_{i=1}^{m} \Pr(x_i/y) \log \Pr(x_i/y) \right).$$

In [2] and [3] the concept of entropy, or peakedness of a distribution, was generalized to include any measure of distance between the given distribution, say \(\Pr(x_i)\), \(i=1,2,\ldots,m\), and the uniform distribution \(\frac{1}{m},\frac{1}{m},\ldots,\frac{1}{m}\). For example, the discrimination information distance measure \(I(1,2)\) becomes

$$I(1,2) = \sum_{i=1}^{m} \Pr(x_i) \log \frac{\Pr(x_i)}{1/m}$$

$$= \log m - H(X).$$

In other words, it corresponds to the Shannon entropy (certainty), except for a constant. Many families of such measures are considered in [3]. If one uses the rth power of the rth order Minkowski metric as distance measures, then one obtains

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as measures of certainty or negative entropy. The corresponding equivocation measures are then given by

\[ M_r(X) = \sum_{i=1}^{m} | \Pr(x_i) - 1/m |^r \]

In the abstract, Devijver [1] states that the Bayesian distance "bears much resemblance to the information theoretic concept of equivocation." From the above one can see that the Bayesian distance is in fact a special case of the generalized equivocation

\[ M_r(X/Y) \]

for \( r = 2 \), except for a constant.

REFERENCES