The assertion proved below on the run time required by certain programs is a generalized version of the halting problem, and shares with it simplicity of proof and intuitiveness of the result.

Suppose \( T(P) \) is a timing program which takes an arbitrary program \( P \) with no free variables as its argument, and that for all such \( P \), \( T(P) = \text{true} \) if \( P \) terminates execution in time less than \( t \) when run, and \( T(P) = \text{false} \) if \( P \) runs for at least time \( t \). Suppose further that all programs \( P \) as well as the timing program \( T \) run on the same computer \( C \), and that identical programs will always have equal run times on \( C \).

Then for some argument program \( Q \), \( T(Q) \) must run at least for time \( t - \varepsilon \) where \( \varepsilon \) is some ("small") positive constant dependent on \( C \) but independent of \( t \).

Liberally interpreted, this result says that the best one can do in general to estimate the run time of programs is to execute them. The \( \varepsilon \) in the assertion above is vexing, since one intuitively expects the stronger statement that \( T(Q) \) must run at least for time \( t \), or even \( t + \varepsilon \). However, it does not seem that the proof below can be sharpened to yield a stronger result. The proof is as follows.

Consider a program \( Q \) which, in a free notation, is expressed by \( Q \); recursive procedure; if \( T(Q) \) then run for time \( t \); return; end. Notice that \( t \) is some fixed number, hence \( Q \) has no free variables and qualifies as an argument for \( T \). Notice also that \( T(Q) \) cannot have the value \text{true} because if it had, \( Q \) would run for at least time \( t \), contradicting our assumption on the behavior of \( T \). Hence \( T(Q) \) must be \text{false}, and hence the run time \( r(Q) \) of \( Q \) must be at least \( t \). According to the definition of \( Q \), its run time \( r(Q) \) consists of two parts: 1) the time \( r(T(Q)) \) required to compute the value of \( T(Q) \), plus 2) the time to enter the procedure \( Q \), to branch on \text{false}, and to return. The time required for the latter control operation clearly does not depend on \( T \) (and hence not on the time limit \( t \)), and we denote it by \( \varepsilon \).

Hence we have
\[
r(Q) = r(T(Q)) + \varepsilon \geq t,
\]
and
\[
r(T(Q)) \geq t - \varepsilon,
\]
which was to be demonstrated.

Authors' Reply

Having gone through the paper by Marcus [1], we agree that our method [2] of finding the maximum compatibles is essentially similar to his approach. However, our method was developed independently. The different issues of the IBM journals are not available to us.

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