A Real-time Scheme for Validation of an Auto-regressive Time Series Model for Power System Ambient Inter-area Mode Estimation

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Abstract

A lattice time series model may be used to estimate the inter-area electromechanical modes of a power system from measured synchrophasor data. The accuracy of these estimates is sensitive to the order of the model. This paper describes a methodology for real-time, order-recursive whiteness testing of the prediction errors. This hypothesis testing methodology may be used in conjunction with the lattice mode-meter algorithm to validate the model in real-time. Results are presented on a model of the western North American power system.

1. Introduction

Inter-area electro-mechanical modes of a power system provide indications of the stability of the system [1]. These modes may be estimated from measured power system synchrophasor data, see [2]–[16] and references therein.

Various parameters such as frequency, current and voltage phasor, and other quantities are measured throughout the grid by a network of phasor measurement units (PMU). Under ambient conditions, the power system may be approximated by a linear time invariant (LTI) system. The random load changes at various locations on the grid act as an approximate white noise input to the system [8] in the frequency band of interest. These measured colored noise signals may be regarded as stochastic time series.

The time series data may be used to estimate an autoregressive (AR) model and hence the modes. The modes are related to the roots of the z-transform of the AR model. This general approach was first demonstrated in [8] and has been refined through many other techniques such as least mean square (LMS) [16], recursive least squares (RLS) [13], [14] and others [2].

These algorithms belong to a class of estimation methods that provide near real-time estimates of the power system dominant modes and thereby, provide a measure of wide area situational awareness for the power system operators. But, these approaches require the user to fix the order of the AR model. It is known that the quality of the estimates is sensitive to the choice of model order [17]. This paper extends order-recursive estimation of the modes [18], [19]. In particular, the availability of the prediction errors at all stages of the lattice filter is used in a statistical hypothesis testing framework to validate the time series model. Such validation may help in determining whether the underlying assumptions about the data and the model are satisfied or not.

This paper is organized as follows. In the following section, the lattice prediction filter is introduced and is related to the modes of the system. The modes are estimated via a least squares approach [19]. Section 3 describes the proposed whiteness testing scheme. The results are discussed in section 4. Section 5 concludes the paper with a note on future work.

2. Background

Let \( y(t) \) be measured ambient signal from the power system, \( t \) is the index of the current sample. The traditional method used to estimate an auto-regressive model of \( y(t) \), such as that used in [8], [13], [16] and others, linearly combines the past \( M \) measurements to predict the current measurement, i.e.,

\[
\hat{y}_{f,M}(t) = - \sum_{i=1}^{M} a_{f,M,i} y(t - i).
\] (1)
Then the parameter vector $\mathbf{a}_{f,M}$ of this auto-regressive model of order $M$ (AR($M$)) may be estimated by searching for the global minimum of a weighted squared prediction error cost function using gradient descent method [13]. This is the commonly used recursive least squares (RLS) algorithm.

$$\hat{\mathbf{a}}_{f,M}(t) = \min_{\hat{\mathbf{a}}_{f,M}} \left\{ \sum_{i=0}^{t} \lambda^{t-i} f_{M,i}(i) \right\}, \quad (2)$$

where,

$$f_{M}(t) = y(t) - \hat{y}_{f,M}(t) = y(t) + \sum_{i=1}^{M} a_{f,M,i} y(t - i),$$

is the forward prediction error of $M$-th order.

The lattice filter model, see Figure 1, is an alternate formulation of the linear prediction. Due to the special structure of the filter, both forward and backward linear predictions of all orders $m = 1, 2, ..., M$ are also available. The $m$-step backward prediction error is,

$$b_{m}(t) = y(t - m) - \hat{y}_{b,m}(t) = y(t - m) + \sum_{i=0}^{m-1} a_{b,m,i} y(t - i).$$

The coefficients $k_{f,M}$ and $k_{b,M}$ are called the reflection coefficients. The reflection coefficients are related to the corresponding AR prediction filters of all orders $m = 1, 2, ..., M$ via the Levinson-Durbin algorithm [19].

$$a_{f,m} = \begin{bmatrix} a_{f,m-1} + k_{f,m} & a_{f,m-1} \\ 0 & 1 \end{bmatrix},$$

$$a_{b,m} = \begin{bmatrix} 0 & a_{b,m-1} \\ a_{b,m-1} + k_{b,m} & 1 \end{bmatrix}, \quad (3)$$

where $a_{f,0} = a_{b,0} = [1]$. Therefore one can estimate the AR prediction filters $a_{f,m}$ by estimating the reflection coefficient vector $\hat{k}_{f,M}(t)$ and $\hat{k}_{b,M}(t)$ for all model orders $m = 1, 2, ..., M$. By rooting these polynomials, one may estimate the modes for all the model orders $m = 1, 2, ..., M$.

The vector $\hat{k}_{f,M}(t)$ and $\hat{k}_{b,M}(t)$ may be estimated by simultaneously minimizing weighted sum of squared prediction errors,

$$\hat{k}_{f,M}(t) = \min_{\hat{k}_{f,M}} \left\{ \sum_{i=0}^{t} \lambda^{t-i} f_{M,i}^2(i) \right\}, \quad (4)$$

$$\hat{k}_{b,M}(t) = \min_{\hat{k}_{b,M}} \left\{ \sum_{i=0}^{t} \lambda^{t-i} b_{M,i}^2(i) \right\}. \quad (5)$$

The $m$-th order mode estimates are obtained by rooting the AR ($m$) polynomial,

$$A_{f,m}(z) = 1 + \sum_{i=1}^{m} a_{f,m,i} z^{-i}.$$

Let $z_m = \text{roots}(A_{f,m}(z))$ and $\tilde{z}_m = F_r \log(z_m)$. The frequency $\omega_m$ (Hz) and damping ratio (DR) $\xi_m$ are given by,

$$\omega_m = \frac{f(z_m)}{(2 \pi)} \text{ (Hz)}, \quad (6)$$

$$\xi_m = -\Re(z_m)/|z_m| \times 100 \text{ (%)}. \quad (7)$$

Here $|\cdot|, \Re(\cdot)$ and $f(\cdot)$ are the absolute, real and imaginary values of a complex number and $F_r$ is the sampling rate. The QR Decomposition based Least Squares Lattice (QRDLSL) algorithm may be used for the estimation of reflection coefficients [18], [20]. This paper focuses on the information that may be available in the prediction errors $f_{M}(t)$ regarding the validity of the model.

![Figure 1: Lattice prediction filter model for order-recursive estimation of the power system ambient modes and validation of the time series model.](image)
The conceptual description is provided in Figure 1. The “Estimator” blocks estimate the reflection coefficients according to a pre-selected algorithm such as the QRDLSL algorithm. The “Whiteness Testing Block” collects the samples of all prediction errors and performs a real-time whiteness test on each of the time series. The output $m_h$ is fed to a mode calculator block which runs the Levinson-Durbin algorithm up to $m = m_h$, roots the $A_{f,m_h}(z)$ polynomial and outputs the mode frequency and damping ratio estimates. All this calculation is performed at each time step.

This paper discusses the design of “Whiteness Testing Block”. This block gathers the time series of prediction errors from all the stages and outputs an integer $m_h < M$. The integer $m_h$ is the minimum order for which the prediction errors are white.

### 3. Model Validation via Whiteness Testing

Both the forward and backward prediction errors may be thought of as the portion of the input time series that could not have been predicted with the given model. Therefore, those residual time series contain information about the adequacy of the model [20].

Such residual analysis is common in all system identification and time series analysis [20]. Since the AR model is extracting all the inter-sample correlations, the consecutive samples of, say the forward prediction error $f_m(t)$, should be independent of each other and identically distributed such that the time series $f_m(t)$ may be considered white. This is true provided $m$ is large enough. Let this value of $m$ be denoted as $m_h$.

The traditional model in (1) provides only the time series $f_m(t)$. The lattice filter in Figure 1 provides $f_m(t)$ for all $m = 1, 2, \ldots, M$. Assume that $M > m_h$. Then the prediction errors $f_m(t)$, $m = 1, 2, \ldots, M$ may be used to determine a statistically significant estimate of $m_h$.

Consider a time recursive definition of the sample auto correlation of the prediction error $f_m(t)$,

$$
\hat{r}_{f_m}(t, \tau) = (1 - \lambda) \sum_{k=1}^{t} \lambda^{t-k} f_m(k) f_m(k - \tau), \quad (8)
$$

$$
= \lambda \hat{r}_{f_m}(t, \tau) + (1 - \lambda) f_m(t) f_m(t - \tau), \quad (9)
$$

where $0 < \lambda \leq 1$ is a forgetting factor.

This definition of the auto correlation leads to a small bias for finite time. Taking expected value of (8),

$$
E [\hat{r}_{f_m}(t, \tau)] = E \left[ (1 - \lambda) \sum_{k=1}^{t} \lambda^{t-k} f_m(k) f_m(k - \tau) \right] = (1 - \lambda^{t-\tau}) r_{f_m}(\tau), \quad (10)
$$

$$
r_{f_m}(\tau), \quad \text{for } t \gg \tau, t \to \infty, \quad (11)
$$

where $r_{f_m}(\tau)$ is the true auto-correlation of the sequence $f_m(t)$ and $E[.]$ is the statistical expectation operator. The simplification in (10) is obtained by summing the geometric series. Now consider,

$$
E [\hat{r}_{f_m}^2(t, \tau)] = E \left[ \sum_{k=1}^{t} \lambda^{t-k} f_m(k) f_m(k - \tau) \right] \left(1 - \lambda^2 \sum_{l=1}^{t} \lambda^{t-l} f_m(l) f_m(l - \tau) \right)
$$

$$
= (1 - \lambda)^2 \lambda^{2t} \sum_{k=1}^{t} \sum_{l=1}^{t} \lambda^{-k-l} E[f_m(k) f_m(k - \tau) f_m(l) f_m(l - \tau)].
$$

This equation can be further simplified under the assumption that $f_m(t)$ is distributed normally i.e. $f_m(t) \sim N(0, F_m(t))$.


$$
E [\hat{r}_{f_m}^2(t, \tau)] = (1 - \lambda^{t-\tau})^2 r_{f_m}(\tau) + (1 - \lambda)^2 \lambda^{2t} \sum_{k=1}^{t} \sum_{l=1}^{t} \left[ r_{f_m}^2(k - l) + r_{f_m}(k - l + \tau) \right] r_{f_m}(k - l - \tau).
$$

The auto-correlation sequence of a white-noise process is an impulse function in the delay parameter [17]. Therefore, under the null hypothesis that $f_m(t)$ is white, for $\tau = 0$,

$$
\text{var} [\hat{r}_{f_m}(t, 0)] = E [\hat{r}_{f_m}^2(t, 0)] - E [\hat{r}_{f_m}(t, 0)]
$$

$$
= 2(1 - \lambda)^2 \lambda^{2t} r_{f_m}^2(0) \sum_{k=1}^{t} \lambda^{-2k} = 2 \frac{(1 - \lambda)}{1 + \lambda} (1 - \lambda^{2t}) r_{f_m}^2(0), \quad (12)
$$

where (12) is obtained by summing the finite geometric series. For $\tau \neq 0$, using a similar reasoning,
\[ \text{var}[\hat{r}_{f,m}(t,\tau)] = (1 - \lambda)^2 \lambda^{2t} \sum_{k=t+1}^{\tau} \lambda^{-2k}, \]
\[ = \left( \frac{1 - \lambda}{1 + \lambda} \right) (1 - \lambda^2(t-\tau)) r_{f,m}^2(0). \]

For large \( t \),
\[ \text{var}[\hat{r}_{f,m}(t,\tau)] = \begin{cases} 
(1 - \lambda)^2 \lambda^{2t} r_{f,m}^2(0), \quad \tau = 0 \\
\left( \frac{1 - \lambda}{1 + \lambda} \right) r_{f,m}^2(0), \quad \tau \neq 0 
\end{cases}. \] (14)

Under the null-hypothesis, for large \( t \) the sample autocorrelation is normally distributed with zero-mean and variance given in (14) [19], [20], i.e.,
\[ \hat{r}_{f,m}(t,\tau) \sim \mathcal{N} \left( 0, \left( \frac{1 - \lambda}{1 + \lambda} \right) r_{f,m}^2(0) \right), \tau \neq 0. \] (15)

This distribution may be recast as a zero-mean unit variance distribution by scaling with the inverse of the standard deviation (for \( \tau \neq 0 \)),
\[ \frac{\hat{r}_{f,m}(t,\tau)}{\text{std}(\hat{r}_{f,m}(t,\tau))} \sim i d \, \mathcal{N}(0,1). \] (16)

Based on the above distribution, a chi-squared (\( \chi^2 \)) test may be formed to test for the whiteness of the sequence \( f_m(t) \).

Define a test statistic,
\[ \rho_m(\tau_{\text{max}}, t) = \sum_{\tau=1}^{\tau_{\text{max}}} \hat{r}_{f,m}^2(t,\tau). \] (17)

The statistic \( \rho_m(\tau_{\text{max}}, t) \) is a \( \chi^2 \) random variable with \( \tau_{\text{max}} \) degrees of freedom, i.e.
\[ \rho_m(\tau_{\text{max}}, t) \sim \chi^2(\tau_{\text{max}}). \]

A level \( \alpha \) hypothesis test for determining if there is statistically significant evidence to suggest if the sequence \( f_m(t) \) is white, is given by,
\[ \text{fail to reject } \mathcal{H}_0(m, t) \text{ if } \rho_m(\tau_{\text{max}}, t) \leq T \]
\[ \mathcal{H}_0(m, t) \text{ is rejected if } \rho_m(\tau_{\text{max}}, t) > T \] (18)

where, \( \mathcal{H}_0(m, t) \) is the hypothesis that \( f_m(t) \) is white and \( T = \chi^2(\tau_{\text{max}}) \) is the value of the argument of the cumulative distribution function of the chi-squared random variable of \( \tau_{\text{max}} \) degrees of freedom corresponding to the level \( \alpha \).

Therefore, “the sufficient model” case is the minimum value of \( m \) for which the null-hypothesis evaluates to unity, i.e.
\[ m_b = \min_{m \in \{0, M\}} \{ \mathcal{H}_0(m, t) \, \sim \, 1 \} \] (19)

The value of \( \alpha \) provides the trade-off in the rate of false negatives and positives. Generally, \( \alpha \in [0.9, 0.99] \) is selected. The weighing factor \( \lambda \in [0.950, 0.999] \). In equation (16), the true auto-correlation \( r_{f,m}^2(0) \) is replaced by the estimate \( \hat{r}_{f,m}(t,0) \).

The test statistic \( \rho_m(\tau_{\text{max}}, t) \) is defined in terms of the autocorrelation estimates. The \( \chi^2 \) distribution is applicable only under the null-hypothesis and hence it is difficult to ascertain the “power of the hypothesis test”. The simulation evidence suggests that the power of the hypothesis test may be dependent on the choice of \( \tau_{\text{max}} \), the number of delay terms.

The limiting value of the function is \( \chi^2(\tau_{\text{max}}) \rightarrow \tau_{\text{max}} \) for large \( \tau_{\text{max}} \). Experience suggests that small values of \( \tau_{\text{max}} \) degrade the performance of the test. Larger values of \( \tau_{\text{max}} \) seemed to work better in the absence of any transients or non-linear events.

Qualitatively speaking, the transient events typically last only for a short duration. Hence these events affect the auto correlation estimates of smaller lags to a larger extent. This implies that to be able to capture the transient events, one must use smaller value of \( \tau_{\text{max}} \).
It seems that there exists a middle ground on the selection of $\tau_{\text{max}}$. Simulation evidence suggests that $\tau_{\text{max}}$ should be of the order of the settling time of the system, so that any event may be captured by a good portion of the considered auto correlation sequence.

4. Results

The proposed whiteness testing scheme is validated using simulated data. A seventeen machine model of the western North American power system is used. The details of the model may be referred in [13]. Figure 2 shows the one-line diagram of the model and the inter-area modes of the model used are listed in Table 1. A first derivative of the phase of the voltage measurements at bus 17 and 45 are used. These measurements are linearly combined such that an estimate of the power spectrum of the resulting time series has the tallest peak at 0.42Hz. Thirty minute of data is generated (filtered with an anti-aliasing filter and down-sampled to 5 samples per second) and used to estimate the 0.42Hz mode using the QRDLSL algorithm [18]. This experiment is repeated in a 400 trial Monte Carlo analysis.

Table 1: Inter area modes of the seventeen machine model.

<table>
<thead>
<tr>
<th>Freq (Hz)</th>
<th>Damping (%)</th>
<th>Participation areas/buses</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.318</td>
<td>10.74</td>
<td>North vs. South</td>
</tr>
<tr>
<td>0.422</td>
<td>3.63</td>
<td>North vs. South + bus 45</td>
</tr>
<tr>
<td>0.635</td>
<td>3.94</td>
<td>bus 18 vs. Rest of system</td>
</tr>
<tr>
<td>0.673</td>
<td>7.63</td>
<td>buses 20, 21 vs. bus 24</td>
</tr>
</tbody>
</table>

Figure 3: Root mean squared error in the estimation of the 0.42Hz mode of the seventeen machine model as a function of the model order. The estimates are obtained using the QRDLSL algorithm. $\lambda = 0.999$, $M = 60$. RMSE is calculated using 400 independent Monte Carlo trials.

At the end of 30th minute, sample statistics is calculated over the 400 trials. In Figure 3 root mean squared error (RMSE) of the mode frequency and damping is shown as function of the model order. RMSE is the average distance of the estimated mode from the true mode,

$$RMSE[\omega_m(t)] = \left[ \frac{1}{l} \sum_{i=1}^{l} \left( \omega_m^{(i)}(t) - \omega_0 \right)^2 \right]^{\frac{1}{2}},$$

where, $l$ is the total number of trials (400 in this case) and $\omega_m^{(i)}(t)$ is the estimate of frequency of a mode of order $m$, at time $t$ corresponding to the $i^{th}$ Monte Carlo iteration, and $\omega_0$ is the true frequency of the mode. The RMSE of damping ratio is also similarly defined.

The figure suggests that for model order $m = 28$ and beyond, the time series model does as good a job of estimating the modes as most other choices. This may suggest that $m = 28$ is a minimum model order that extracts all the information from the time series data.

The forward prediction errors are tested for whiteness using the proposed scheme. Set confidence level $\alpha = 0.95$, $\lambda = 0.999$, $\tau_{\text{max}} = 35$, $M = 60$. The decision probabilities may be approximated in a Monte Carlo simulation setup,

$$P(m, t) = \frac{1}{l} \sum_{i=1}^{l} H_0^{(i)}(m, t),$$

where, $H_0^{(i)}(m, t)$ is the hypothesis testing decision (19) corresponding to the $i^{th}$ Monte Carlo iteration.

The whiteness testing results of the 400 Monte Carlo trials are shown in Figure 4. The probabilities are approximated as described in (21). A zero probability
is encoded as blue, unit probability is encoded as red and others are encoded as described by the color-bar above Figure 4 and Figure 7.

Figure 4: Results of real-time whiteness testing of the prediction errors of the lattice filter model of the seventeen machine model time series data. Probabilities are approximated using 400 Monte Carlo trials. $\lambda = 0.999, \tau_{\text{max}} = 35, \alpha = 0.95$.

For small $t$, the whiteness test fails to provide any confidence that the time series model is adequate. This is to be expected since both the estimation algorithm and the calculation of the sample auto-correlation would need time to converge to workable estimates.

For time greater than about 8 minutes, the algorithm shows good probability (about 0.95%, the $\alpha$ level of the test) that $m = 29$ resulted in white prediction errors. Therefore, $m_h = 29$ represents a sufficient model order case. Figure 5 shows the results of the real-time estimation of the modes. The mean frequency and mode damping estimates are shown with one standard deviation interval around the mean value. The standard deviations are calculated over the Monte Carlo trials. The mode estimates for model order $m = 30$ are shown.

Each evaluation of $m_h$ required an average of 0.14ms. At 5 samples per second this is a very small fraction of the computation time available between samples. Therefore, addition of this scheme does not result in any computation burden. The algorithm was implemented using MATLAB 7.12 (R2011a) platform on a 32-bit Windows XP SP3 computer with 4GB of RAM and Intel Core 2 Duo 3.2GHz processor.

For a second experiment, a 1400MW brake, similar to the Chief Joseph dynamic brake on the western North American power system, insertion is simulated on one of the lines at the 15th minute mark (lasting 0.5 seconds). This event introduces non-linearities and transient ring-downs in the data.

The time series of the data is shown in Figure 6. It is to be noted that the QRDSL algorithm used in this paper is an ambient time series modeling algorithm and is not designed to handle non-linearities, transients or outliers.

The whiteness testing results are shown in Figure 7. The figure suggests that the time series model fails to extract all the information from the ring-down event. It may also be the case that the signal might cause the algorithm to lose track of the estimates. Note that in this case, it may be inaccurate to say that the model order has changed. It is more likely that the AR time series model is not an appropriate choice for the transient ringdown data. Even though the power system changes very slowly, small and medium sized events (line/load/generation trips) are fairly common.
Therefore, it is important to continuously evaluate the validity of the data model and therefore the validity of the estimates. For this particular example, Prony’s and other related methods may be better suited to work with such ringdown signals rather than AR, ARMA or other ambient time series models, see [4], [21] - [23] among others.

5. Conclusion and Future Work

This paper demonstrated that the prediction errors generated while estimating the electro-mechanical modes of the power system using the lattice filter model contain information about the quality of the model. The order recursive prediction errors were used in a real-time simultaneous whiteness testing scheme. This scheme gives indications of adequacy of the order of the AR model. This information was used to adjust the model order at which modes are calculated.

The choice of confidence level \( \alpha \) and forgetting factor \( \lambda \) provide trade-off in the algorithm performance. The choice of number of delay terms \( \tau_{\text{max}} \) also influences the test results. These influences need to be studied in finer detail.

It was also demonstrated that in the presence of transient signals the testing scheme is more likely to indicate that the prediction error is not white for many model orders. Such decisions are indications that either the order of the model or the choice of the model itself or both may be inadequate. By extension, these decisions are also indicative of the lower reliability of the mode estimates.

6. References


