Abstract

This paper investigates the time dynamics of online user behavior in a series of overlapping electronic auction mechanisms. An auction is a dynamic game where the valuation and strategic space of bidders determine the final outcomes. When those mechanisms exist concurrently or in a series, multiple sources are visible and accessible, and there is likely to be a form of interdependency. While heterogeneous bidder behavior has been studied in some literature, the focus is mainly on individual auction level and the underlying dynamics regarding the interdependency among the auctions in the market has not been explained. We use two-phased approach. First, we classify user strategy using k-means clustering. Then we characterize the transition pattern of heterogeneous clusters using dynamic systems framework. Long-term behavior of the system is effectively and efficiently predicted using system parameters. Our model can serve as a decision support framework for optimal market design.

1. Introduction

Due to the advancement of Internet Technologies, e-commerce has significantly transformed the fundamentals of business processes and financial transactions in today’s economy. In this paper we consider multiple overlapping online auction market which relies on the current technology advancement and is a major branch of web-based e-commerce system.

Eliminating spatial and temporal constraints to connect consumers who are physically apart, online auction mechanism provides an effective environment for pricing and allocation of goods. An interesting aspect of current online auction is that often multiple mechanisms exist concurrently or in overlapping manner by the same website.

We define overlapping mechanisms as the auctions sharing a common time span. By this we mean an auction starts before the termination of another auction. Figure 1 shows a stylized view of overlapping auctions in a B2C online auction site. Each line segment represents an auction and those auctions are offered by a single seller.

Defining a market as a web-based subspace where multiple overlapping online auctions exist, it is hard to say that those auctions in the market are independent of each other. Given that multiple auctions are visible and accessible by a simple mouse click and the progress information of each auction (i.e., current winner, winning price, and remaining time) propagates quickly through the market, it is natural to expect that the strategic space of bidders should be influenced by the surrounding market forces.

This interesting environment motivates us to look at a series of overlapping auctions at the market level view to analyze bidder strategy. While heterogeneous bidder behavior has been studied in [5] and [16], they mainly focused on individual auction level and didn’t expand their view to the market level.

Figure 1. A series of overlapping auctions in Sam’s club for ‘TiVo Series2 DVR’

We investigate the interdependency of multiple auction mechanisms in the context of time dynamics of user\textsuperscript{1} behavior. Our analysis can be summarized in two phases:

\textsuperscript{1} We use the terms ‘user’ and ‘bidder’ interchangeably.
• At the auction mechanism level, as in [5] we apply $k$-means clustering approach to classify bidders based on their bidding and participation strategy. We found that different clusters reveal different ranges of final bid value.

• We then develop dynamic systems model to understand market level time dynamics. The transition pattern of heterogeneous cluster distribution in a series of multiple overlapping auctions is explicitly characterized in our model. System parameters such as Eigen-values and Eigen-vectors are applied to analyze long-term behavior and equilibrium state of the market.

As Reinartz and Kumar [15] argued, accurate consumer profiling is a crucial factor for a firm’s profitability. In our dataset we observe different clusters reveal significantly different pattern of entry, exit and bidding behavior. Given that an auction mechanism is a dynamic game environment where the bidder-interplay is the determinant of final outcomes, correct identification of heterogeneous user strategy is a key component in designing mechanism formats and bidding systems.

Further, since multiple auctions are visible and accessible by a simple mouse click, there should be a form of interdependency among multiple auctions in the market. From a practitioner’s perspective, it is also crucial to understand the underlying dynamics about how individual mechanism level strategy is influenced by surrounding market forces.

Using our dataset collected from a B2C online auction site, we found that there is a significant transition pattern in cluster distribution across multiple auctions in the market. We demonstrate how our model can be empirically calibrated to serve as a decision support framework for optimal market design.

2. Prior works

A substantial difference between current research in online auctions and prior works on physical domain is the expansion of dimensions in strategic space of bidders [3]. The nature of bidding activities and corresponding auction progress is more complicated than traditional physical setting. While Bayesian-Nash equilibrium with the assumption that bidders are risk-neutral and they have homogenous, symmetric and privately-known valuation explains physical settings well [17], it does not take account of online auction participation cost and potential variation of bidder strategy which depends on the cost. As indicated in [5] and [6], bidding in online auctions involves costs for Internet connection, participation and progress monitoring and this makes the strategic bidding activities more complicated than a simple mapping between bidder valuation and their bid placements.

Further, the bidder behavior and resulting price formation of a given auction may be influenced by surrounding market forces. While the vast majority of papers consider the auction mechanism as an independent and isolated environment, there are several works that consider the inter-dependencies of multiple auctions [9, 13]. Concerning the effect of concurrent or sequential markets, Engelbrecht-Wiggans and Weber [7] pointed out that when there are multiple simultaneous auctions, the situation couldn’t be treated as multiple independent auctions. Since bidders may have non-linear utility functions for multiple items, if the value of the items is not just simple sum of multiple objects, the independence assumption might not be appropriate. McAfee [12] proposed a multiple auction problem in which auctioneers compete with arbitrary auctions. In equilibrium, the auctioneers would use identical auctions with reserve price equal to the cost. Peters and Severinov [14] analyzed a similar situation. They argued that, when there are multiple competing auctions, the demand and the number of bidders are endogenously determined by the mechanism factors offered by auctioneers. Our dataset consists of overlapping auctions for the same items offered by auctioneers. Using the data of eBay coin auctions, Bajari and Hortacsu [1] examined the determinants of bidder and seller behavior. They showed a low minimum bid, a high book value, low negative ratings, and high overall ratings increase the entry of bidders to an auction. Bapna, Goes and Gupta [4] used a simulation approach to analyze the effect of changing control factors such as bid-increment on the seller’s revenue.
and the bidders’ welfare. Our research provides new explanation on how the endogeneity, which is driven by the market format, can affect the bidders’ entry behavior and bidding pattern.

3. Problem domain, market and data

We collected data from the online auction site of Sam’s club. In contrast to other online auction sites such as eBay, there is only one seller in Sam’s club auctions. The bias related with seller reputation, shipping method, and buy-it-now option, which are commonly occurring in multi-seller auctions does not exist in Sam’s Club auction. Since we are considering the environment in which multiple bidders can find multiple sources of identical goods, the lack of seller heterogeneity makes it an ideal place to study the characteristics of bidders’ participation and bidding.

Using automated scripts in PHP, we downloaded the auction information and bidding history (opening time, closing time, winner and winning price of every auction as well as each bidder’s bidding time and bid amount) to construct entire bidding history of all the auctions in the market.

<table>
<thead>
<tr>
<th>Item Name</th>
<th>Number of Auctions</th>
<th>Average Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>14” Apex Flat-Screen TV/DVD Combo</td>
<td>44</td>
<td>98.02</td>
</tr>
<tr>
<td>2.3” Casio Portable LCD TV</td>
<td>5</td>
<td>37.60</td>
</tr>
<tr>
<td>20” Toshiba LCD TV</td>
<td>54</td>
<td>387.76</td>
</tr>
<tr>
<td>22” Samsung LCD TV</td>
<td>19</td>
<td>725.89</td>
</tr>
<tr>
<td>23” Philips WideScreen Flat LCD TV</td>
<td>33</td>
<td>777.67</td>
</tr>
<tr>
<td>27” Samsung Flat TV</td>
<td>31</td>
<td>146.74</td>
</tr>
<tr>
<td>30” Panasonic Widescreen HDTV/EDTV Monitor</td>
<td>10</td>
<td>456.00</td>
</tr>
<tr>
<td>Cyberhome 17” HDTV-Ready LCD TV/DVD</td>
<td>27</td>
<td>353.26</td>
</tr>
<tr>
<td>Echostar Dish301 Receiver</td>
<td>24</td>
<td>35.00</td>
</tr>
<tr>
<td>JVC 500 Watt Home Theater System</td>
<td>48</td>
<td>141.46</td>
</tr>
<tr>
<td>JVC DVD Home Theatre System</td>
<td>199</td>
<td>158.44</td>
</tr>
<tr>
<td>JVC Mini DV Camcorder</td>
<td>25</td>
<td>292.80</td>
</tr>
<tr>
<td>Philips mz1000 Micro Stereo System</td>
<td>46</td>
<td>99.11</td>
</tr>
<tr>
<td>Philips Universal Remote - 2 pk.</td>
<td>63</td>
<td>33.46</td>
</tr>
<tr>
<td>Samsung 4.1MP Mini-DV Camcorder DuoCam</td>
<td>54</td>
<td>374.80</td>
</tr>
<tr>
<td>Sony 23” WEGA HDTV Monitor</td>
<td>14</td>
<td>830.29</td>
</tr>
<tr>
<td>Sony 650W 5-Disc DVD Dream System</td>
<td>57</td>
<td>330.14</td>
</tr>
<tr>
<td>Sony Digital Video Camera Recorder</td>
<td>29</td>
<td>539.10</td>
</tr>
<tr>
<td>Sony KLV-23HR1 23” WEGA Flat-Panel LCD TV</td>
<td>11</td>
<td>836.18</td>
</tr>
<tr>
<td>Sony Sports Digital FM/AM &amp; CD Boombox</td>
<td>13</td>
<td>45.85</td>
</tr>
<tr>
<td>Symphonic 13” TV/VCR</td>
<td>16</td>
<td>60.31</td>
</tr>
<tr>
<td>TiVo Series2 DVR - 140 hours</td>
<td>15</td>
<td>168.13</td>
</tr>
<tr>
<td>TiVo Series2 DVR - 80 hours</td>
<td>36</td>
<td>108.83</td>
</tr>
<tr>
<td>VoiceStation 10 Conference Phone</td>
<td>59</td>
<td>107.95</td>
</tr>
</tbody>
</table>

The dataset includes 932 auctions for various electronic goods during March 23 – May 11, 2005\(^2\). As in Table 1, there are 24 items and each item was sold by multiple overlapping English auctions - an auction with single unit ascending price. Average auction duration is 0.9 days and there are 9 bidders on average in an auction. All the auctions have bid increment of $1 and bids are submitted by proxy bidding system in which bidders can enter their maximum willingness to pay then the automatic bidding agent increases bid according to the auction progress.

4. Model

For phase 1 analysis, we apply \( k \)-means clustering to categorize heterogeneous bidders’ strategic space. In phase 2, we implement the clusters from phase 1 in a dynamic systems model to analyze the time trajectory of transition, long-term behavior of the system and equilibrium states.

4.1. Mechanism level clustering (phase 1)

Since our objective in this section is creating a mapping between user strategy and classified operationalizable measures, we apply \( k \)-means clustering algorithm which is an effective unsupervised learning method. \( k \)-means clustering algorithm partitions the data points into \( k \) disjoint subsets in which the sum of Euclidian distances between data points and the cluster centroid is minimized.

\(^2\) As of 2009, Sam’s Club uses the same price progress, winner determination rules and bidding procedure.
\[ \text{Min} \sum_{i=1}^{k} \sum_{x \in D_i} |d_x - \text{centroid}_i|^2 \]  \hspace{1cm} (1) 

where:  
- \( i \): cluster index  
- \( D_i \): a set of data points in cluster \( i \)  
- \( d_x \): a data point  
- \( \text{centroid}_i \): centroid in cluster \( i \) 

For the clustering, since the timing and frequency of bids play a crucial role in understanding the heterogeneity of real-world bidder strategy [5], we deploy variables which reflect bidders’ entry, exit, and bidding characteristic. The 932 auction instances in our dataset include 8369 bidders’ entry, exit, bid frequency, and bid value information. The variables are:

- **TOE (Time of Entry):** normalized entry time of bidder \( b \) in auction \( i \)
  \[ \frac{t_{ib}^E - t_i^O}{t_i^C - t_i^O} \]

- **TOX (Time of Exit):** normalize exit time of bidder \( b \) in auction \( i \)
  \[ \frac{t_{ib}^X - t_i^O}{t_i^C - t_i^O} \]

- **NOR (Number of bid revision):** actual numeric measure of bid revision of bidder \( b \) in auction \( i \)

where:
- \( i \): auction index  
- \( b \): bidder index  
- \( t_i^O \): opening time of auction \( i \)  
- \( t_i^C \): closing time of auction \( i \)  
- \( t_{ib}^E \): the first bidding time of bidder \( b \) in auction \( i \)  
- \( t_{ib}^X \): the last bidding time of bidder \( b \) in auction \( i \)

As in Table 2 and Table 3, we identify three clusters. Performing the dissimilarity ratio\(^4\) test (see [5]) we find the value of \( k = 3 \) gives the largest inter

- **Cluster 1 (Evaluators):** Bidders in this cluster show an early-stage arrival and exit pattern in a course of an auction. During the short duration of stay, the bidders make a single bid without any bid adjustment at later stage. We categorize this strategy as ‘early stage evaluation’ (see [2]). Nearly half of the bidders (48%) use this strategy.

- **Cluster 2 (Opportunists):** The second cluster bidders tend to be the ‘last minute opportunists’ (see [2]). They join in at near end of an auction, place one bid and leave. 37% of bidders in the market belong to this group.

- **Cluster 3 (Participators):** Bidders in the third group stay active for longer time duration and make multiple bids. We categorize these bidders as ‘active participators’ (see [2]) relying on ratchet bidding. Only about 10% of bidders are in this cluster.

### Table 2. Cluster centers

<table>
<thead>
<tr>
<th>Cluster</th>
<th>TOE</th>
<th>TOX</th>
<th>NOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cluster 1</td>
<td>0.156</td>
<td>0.164</td>
<td>1.093</td>
</tr>
<tr>
<td>Cluster 2</td>
<td>0.828</td>
<td>0.828</td>
<td>1.000</td>
</tr>
<tr>
<td>Cluster 3</td>
<td>0.612</td>
<td>0.840</td>
<td>2.594</td>
</tr>
</tbody>
</table>

### Table 3. Inter cluster distance

<table>
<thead>
<tr>
<th>Cluster 1</th>
<th>Cluster 2</th>
<th>Cluster 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cluster 1</td>
<td>0.000</td>
<td>0.949</td>
</tr>
<tr>
<td>Cluster 2</td>
<td>0.949</td>
<td>0.000</td>
</tr>
<tr>
<td>Cluster 3</td>
<td>1.708</td>
<td>1.608</td>
</tr>
</tbody>
</table>

### Table 4. Average distance in cluster

<table>
<thead>
<tr>
<th>Cluster</th>
<th>Number of observations</th>
<th>Average distance in cluster</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cluster 1</td>
<td>4005</td>
<td>0.575</td>
</tr>
<tr>
<td>Cluster 2</td>
<td>3065</td>
<td>0.497</td>
</tr>
<tr>
<td>Cluster 3</td>
<td>1299</td>
<td>1.501</td>
</tr>
<tr>
<td>Overall</td>
<td>8369</td>
<td>0.690</td>
</tr>
</tbody>
</table>

An auction mechanism is a dynamic pricing environment where the valuation and strategic space of participating bidders are the determinants of final outcome. As in Figure 2 the final bid distribution of three clusters are significantly different. It is an indication that different mix or distribution of the

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\(^3\) In this mechanism level analysis, each bidder in each auction is counted individually.

\(^4\) Dissimilarity ratio is inter-cluster distance (the minimum distance among the different clusters) divided by intra-cluster distance (average distance between each data point and the cluster center). We selected the \( k \) which yields the highest dissimilarity ratio.
clusters can generate different financial outcomes in an auction.

Further, considering the nature of the market where multiple overlapping auctions exist and bidders’ strategic space is likely to be influenced by other overlapping auctions, the distribution of heterogeneous clusters would be influenced by the surrounding market forces.

In next section, we analyze the transition pattern of the clusters distribution associated with the interdependency of multiple overlapping auctions at the market level.

### 4.2. Market level dynamic system (phase 2)

To model the market level dynamics which involves transition of heterogeneous cluster distribution, we use a recursion system as proposed by Luenberger [11]. Let \( t, t \in T \) be the auction index in a series of \( T \) auctions in ascending order of closing time (i.e., auction \( t = 1 \) ends earlier than auction \( t = 2 \)). We define \( N_i(t) \) as the number of users who belong to cluster \( i \) in auction \( t \). The proportional transition of clusters to the following auction and the magnitude of new bidder arrival are represented by \( u_{ij} \) and \( v_i \). The bidder transition dynamics is as follows.

\[
\begin{align*}
N_i(t+1) &= u_{i1} \cdot N_1(t) + u_{i2} \cdot N_2(t) + u_{i3} \cdot N_3(t) + v_i \\
N_i(t+1) &= u_{21} \cdot N_1(t) + u_{22} \cdot N_2(t) + u_{23} \cdot N_3(t) + v_2 \\
N_i(t+1) &= u_{31} \cdot N_1(t) + u_{32} \cdot N_2(t) + u_{33} \cdot N_3(t) + v_3
\end{align*}
\]

Equation (2) can be rephrased using \( N(0), U, \) and \( V \) as follows.

\[
N(t+1) = U \cdot N(t) + V
\]

where

\[
\begin{pmatrix}
N_1(t+1) \\
N_2(t+1) \\
N_3(t+1)
\end{pmatrix} =
\begin{pmatrix}
u_{11} & u_{12} & u_{13} \\
u_{21} & u_{22} & u_{23} \\
u_{31} & u_{32} & u_{33}
\end{pmatrix}
\begin{pmatrix}
N_1(t) \\
N_2(t) \\
N_3(t)
\end{pmatrix} +
\begin{pmatrix}
v_1 \\
v_2 \\
v_3
\end{pmatrix}
\]

The vector components are

- **State vector**: the number of users in cluster \( i \) in each auction.

\[
N(t) = \begin{pmatrix}
N_1(t) \\
N_2(t) \\
N_3(t)
\end{pmatrix}
\]

- **Transition matrices**: Matrix \( U \) captures the dynamics of how the proportion of a particular cluster transition from auction \( t \) to the next auction \( t+1 \).

\[
U = \begin{pmatrix}
u_{11} & u_{12} & u_{13} \\
u_{21} & u_{22} & u_{23} \\
u_{31} & u_{32} & u_{33}
\end{pmatrix}
\]

\( u_{ij} \): proportional transition of cluster \( j \) in auction \( t \) to cluster \( i \) in auction \( t+1 \).

- **Vector \( V \)** captures periodic arrival of new users.

\[
V = \begin{pmatrix}
v_1 \\
v_2 \\
v_3
\end{pmatrix}
\]

\( v_i \): number of new users who join in cluster \( i \) in auction \( t \)

Equation (3) can be rephrased using \( N(0), U, \) and \( V \) as follows.

when \( t = 1 \)

\[
N(1) = U \cdot N(0) + V
\]

when \( t = 2 \)

\[
N(2) = U \cdot N(1) + V
\]

\[
= U^2 \cdot N(0) + U \cdot V + V
\]

when \( t = 3 \)

\[
N(3) = U \cdot N(2) + V
\]

\[
= U^3 \cdot N(0) + U^2 \cdot V + U \cdot V + V
\]
and

\[ N(t) = U' \cdot N(0) + \sum_{l=0}^{t-1} U'^{t-l-1} \cdot V \quad t, l = 0, 1, 2 \quad (4) \]

From equation (4),

\[ \sum_{l=0}^{t-1} U'^{t-l-1} = U'^{t-1} + U'^{t-2} + \cdots + I \quad (5) \]

Equation (5) is equivalent to

\[ U \cdot \sum_{l=0}^{t-1} U'^{t-l-1} = U' + U'^{t-1} + \cdots + U \quad (6) \]

Subtracting equation (6) from equation (5),

\[ (I - U) \cdot \sum_{l=0}^{t-1} U'^{t-l-1} = I - U' \quad (7) \]

Therefore,

\[ \sum_{l=0}^{t-1} U'^{t-l-1} = (I - U)^{-1} \cdot (I - U') \quad (8) \]

Now equation (4) is

\[ N(t) = U' \cdot N(0) + (I - U)^{-1} \cdot (I - U') \cdot V \quad (9) \]

Using the above equation (9) with Eigen-values and Eigen-vectors, we analyze the time trajectory, long term behavior of the system and equilibrium states.

For a 3-by-3 matrix, we can find three scalar Eigen-values \( \lambda_1, \lambda_2, \) and \( \lambda_3 \) if there are nonzero 3-by-1 Eigen-vectors \( e_1, e_2, \) and \( e_3 \) satisfying \( U \cdot e_i = \lambda_i \cdot e_i \) where \( i = 1, 2, 3. \) The Eigen-values and Eigen-vectors are obtained using the characteristic equation, \( \text{det} [U - \lambda \cdot I] = 0. \)

From equation (9),

\[ N(t) = U' \cdot N(0) + (I - U)^{-1} \cdot V - (I - U)^{-1} \cdot U' \cdot V \quad (10) \]

The first term and the third term of equation (10) can be rewritten using Eigen-values and Eigen-vectors of \( U. \)

\[ U' \cdot N(0) = \lambda_1 \cdot c_1 \cdot e_1 + \lambda_2 \cdot c_2 \cdot e_2 + \lambda_3 \cdot c_3 \cdot e_3 \quad (11) \]

\[ U' \cdot V = \lambda_1 \cdot c_4 \cdot e_1 + \lambda_2 \cdot c_5 \cdot e_2 + \lambda_3 \cdot c_6 \cdot e_3 \quad (12) \]

where \( c_1, \ldots, c_6 \) are constant coefficients.

Substituting equations (11) and (12) into equation (10), we get

\[ N(t) = (\lambda_1 \cdot c_4 \cdot e_1 + \lambda_2 \cdot c_5 \cdot e_2 + \lambda_3 \cdot c_6 \cdot e_3) + (I - U)^{-1} \cdot V \]

\[ - (I - U)^{-1} \cdot (\lambda_1 \cdot c_4 \cdot e_1 + \lambda_2 \cdot c_5 \cdot e_2 + \lambda_3 \cdot c_6 \cdot e_3) \quad (13) \]

This equation effectively predicts the long-term behavior of the system. Note that when the absolute value of eigen-values \( |\lambda_i| < 1, \) the first and third term of equation (13) approach zero as \( t \) increases. Thus only the second term becomes dominant. The vector \( N(t) \) eventually reaches the equilibrium state.

\[ \lim_{t \to \infty} N(t) = (I - U)^{-1} \cdot V \quad (14) \]

The values in Table 5 are given by the least square estimates from our dataset. We find statistically significant transition pattern in both \( U \) and \( V. \)

### Table 5. Least square estimates, \( u_{ij} \) and \( v_i \)

<table>
<thead>
<tr>
<th>( u_{ij} )</th>
<th>( v_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.410*</td>
<td>-0.044*</td>
</tr>
<tr>
<td>-0.097***</td>
<td>0.394*</td>
</tr>
<tr>
<td>0.095*</td>
<td>0.053*</td>
</tr>
</tbody>
</table>

* Significance in 0.01 level  
** Significance in 0.05 level  
*** Significance in 0.1 level

### Table 6. Eigen-values and Eigen-vectors

| Characteristic polynomial : \( x^3 - 1.063x^2 + 0.338x -0.028 \) |
|---------------------|---------------------|---------------------|
| Eigen value | 0.129 | 0.438 | 0.496 |
| Eigen vector | -0.585 | -0.240 | 0.821 |

Three Eigen-values and corresponding Eigen-vectors are presented in Table 6. All the Eigen-values are less than one in absolute value.

Without loss of generality, we assume \( N(0) = 0. \) Therefore, equation (11) is
\[ \mathbf{U}' \cdot \mathbf{N}(0) = 0.129' \cdot 0 \cdot \begin{pmatrix} -0.585 \\ 0.632 \\ \end{pmatrix} + 0.438' \cdot 0 \cdot \begin{pmatrix} -0.240 \\ 0.958 \\ 0.157 \\ \end{pmatrix} + 0.496' \cdot 0 \cdot \begin{pmatrix} 0.821 \\ -0.531 \\ -0.209 \\ \end{pmatrix} \]

(15)

\[ \mathbf{U}' \cdot \mathbf{V} = 0.129' \cdot (2.330) \cdot \begin{pmatrix} -0.585 \\ 0.632 \\ \end{pmatrix} + 0.438' \cdot (2.250) \cdot \begin{pmatrix} -0.240 \\ 0.958 \\ 0.157 \\ \end{pmatrix} + 0.496' \cdot (0.443) \cdot \begin{pmatrix} 0.821 \\ -0.531 \\ -0.209 \\ \end{pmatrix} \]

(16)

As time goes infinity, the two terms in equation (15) and (16) approach zero. The system converges to

\[ \lim_{t \to \infty} \mathbf{N}(t) = (I - \mathbf{U})^{-1} \cdot \mathbf{V} = \begin{pmatrix} 4.23 \\ 3.31 \\ 1.38 \end{pmatrix} \]

(17)

Figure 3 shows a stylized view of the system growth direction and magnitude, which is consistent with equation (17). Due to the term in equation (16) which diminishes over time, the number of cluster 1, 2, and 3 gradually approaches the equilibrium point.

The time trajectory of user transition in a finite time horizon as well as the long-term equilibrium over infinite time horizon are explicitly captured in our dynamic systems model.

<table>
<thead>
<tr>
<th>Total overlapped time with preceding auctions</th>
<th>Total overlapped time with following auctions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pearson correlation</td>
<td>P-value</td>
</tr>
<tr>
<td>Proportion of Cluster 1</td>
<td>-0.324</td>
</tr>
<tr>
<td>Proportion of Cluster 2</td>
<td>0.473</td>
</tr>
<tr>
<td>Proportion of Cluster 3</td>
<td>-0.017</td>
</tr>
</tbody>
</table>

Table 8. Correlation between the proportion of three clusters and price

<table>
<thead>
<tr>
<th>Proportion of Cluster 1 in a given auction</th>
<th>Pearson correlation</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportion of Cluster 2 in a given auction</td>
<td>-0.262</td>
<td>0.000</td>
</tr>
<tr>
<td>Proportion of Cluster 3 in a given auction</td>
<td>0.068</td>
<td>&lt; 0.05</td>
</tr>
</tbody>
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The model delivers useful managerial insights for multiple online auction market design. Using a similar definition as in [2], we define “total overlapped time with preceding auctions” as the sum of time-intervals which are overlapped by other preceding auctions (i.e., auctions which have lower time-index value than the focal auction). Similarly, the “total overlapped time with following auctions” is the sum of time-intervals overlapped by the auctions which have higher time-index than the focal auction. We find that as total overlapped time increases the proportion of opportunistic strategy (Cluster 2) increases (see Table 7). These bidders tend to monitor multiple auctions in the market to find auctions which potentially will maximize their surplus. Given that this cluster is one of the primary price determinants in a given auction, the multi-auction-monitoring and last-minute-bidding strategy decreases the price (see Table 8). However, participatory bidders (Cluster 3) focus on the current auction without paying attention to the other auctions (see Table 7). They are likely to reveal their true-
valuation and this strategy increases the price of a given auction (see Table 8).

We examined the impact of total overlapped time with other auctions on the price. We find strong negative correlations between them. As Table 9 indicates, preceding overlapping auctions and following overlapping auctions decrease the current auction price and these decreases are proportional to the total overlapped time.

<table>
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<th>Table 9. Correlation between total overlapped time and price</th>
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<tr>
<td>Pearson correlation</td>
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<td>Total overlapped time with preceding auctions</td>
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<td>Total overlapped time with following auctions</td>
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It implies that the mixture of three strategies, which has a significant impact on the market revenue, is influenced by the shape of the market. Different overlapping pattern can generate different proportional distribution of three clusters and eventually yield different outcome. Practitioners can benefit from this prior knowledge to optimally schedule the multiple auctions in the market to control the pattern of overlap. The optimal pattern will alter the transition matrices $U$ and $V$ to perturb the cluster distribution and, in turn, achieve a different equilibrium state which has a low proportion of opportunists (Cluster 2) and a high proportion of participators (Cluster 3). In this case, the overall market revenue will become higher.

The associated time-dynamics regarding long-term behavior of the system and the equilibrium state, which can be analyzed using Eigen-values and Eigen-vectors, will provide increased precision in allocating operational and marketing resources.

5. Conclusion

This paper shed new lights on time dynamics of online user behavior in the market which comprise a series of multiple overlapping auctions. We present a two-phased research which includes $k$-means clustering analysis and dynamic systems model at auction mechanism level and overall market level.

While some research such as [5] and [16] examined heterogeneous user behavior at the mechanism level, their focus has not been extended to the underlying dynamic across multiple auctions in the market. Since multiple sources are easily visible and accessible by online users in a series of overlapping auction mechanisms, there is likely to be a form of interdependency across multiple auctions in a time horizon. This interesting environment motivates us to investigate the impact of the market forces on the strategic space of users and associated time dynamics in the market.

At auction mechanism level, we found different clusters show different range of willingness to pay distribution which may alter the final outcome of individual auctions. We then develop dynamic systems model to understand the time dynamics of heterogeneous cluster distributions in a series of multiple overlapping auctions. The transition of cluster distributions is explicitly characterized in our model. System parameters such as Eigen-values and Eigen-vectors are applied to analyze long-term behavior and equilibrium state of the system.

Our data set validates there exists significant pattern of user transition in matrices $U$ and $V$. We demonstrated how our decision support framework can be empirically calibrated using the data to deliver managerial insights.

Based on the results, our future research will include multiple overlapping online auction market design problem that optimizes the duration of individual auctions and degree of overlap of multiple auctions which together form the overall market shape. Given that user transition pattern is closely tied with the overlapping pattern of multiple auctions, the optimal strategy which controls the mechanism and market factors along with operating costs will be investigated.

Our research also suggests the examination of user behavior and underlying time dynamics in different auction venues with newer datasets. How do different auction format, winner determination and price formation rule work? How do other features such as seller reputation and buy-it-now option influence bidder strategy and market equilibrium?

6. References


