An Optimization Model for Tactical Planning of Wood Procurement

Anders Skjäl  
IAMSR / Åbo Akademi University  
Joukahainen 3-5A, FIN-20520 ÅBO, Finland  
anders.skjal@abo.fi

Kaj-Mikael Björk  
IAMSR / Åbo Akademi University  
Joukahainen, 3-5A, FIN-20520 ÅBO, Finland  
kaj-mikael.bjork@abo.fi

Jenny Östman  
IAMSR / Åbo Akademi University  
Joukahainen, 3-5A, FIN-20520 ÅBO, Finland  
jenny.ostman@abo.fi

Christer Carlsson  
IAMSR / Åbo Akademi University  
Joukahainen, 3-5A, FIN-20520 ÅBO, Finland  
christer.carlsson@abo.fi

Abstract

This paper presents a new LP (Linear Programming) model to solve a tactical wood procurement and harvesting problem. This optimization problem occurs in several wood supply chains today. The goal with the optimization is to find a 12 month procurement, harvesting and transportation plan that minimizes total costs. The case study is found in one of the Finnish wood supply chains and the model is implemented to aid the decision making process. The focus of the paper is on the modeling of some phenomena peculiar to wood procurement.

1. Introduction

Supply Chain Management (SCM) is growing in importance with the growing unit size in both the production and distribution layers of the logistics networks. The increasing integration of the supply chain actors is a by-product of advances in information technology and large-scale, real-time information systems for the SCM. This has made it important to manage the whole supply-demand chain. SCM has even been considered one of the most important areas of management today. Therefore, there are the metaphors of “corporate wars” amongst the supply chain networks and not between independent enterprises, (Cohen & Roussel [5], Houlihan [6]). The decisions made in SCM concern all levels of the company processes. Strategic decisions on long lasting relationships with the right suppliers as well as production and inventory capacity investments belong to the strategic level of SCM decisions. Tactical decisions (procurement plans for 12 months, for instance) as well as operational decisions need to be optimized in order to achieve excellence in supply chain performance.

Process industry works with quite different principles than assembly-line industry. Production facilities are often not optimized for short production series or for many tailor-made products. They are better suited to make a few products in large quantities. Many solutions in the SCM literature are developed for assembly-line industry systems and operations. These results may not be applicable as such for process industry based supply chains (Björk [2]). Given the feasible decision space of strategic solutions, managers are forced to solve complex problems on a daily basis and these problems need to be addressed properly. The trade-off between production, inventory and demand responsiveness is not easily understood or managed optimally. There has been a decent amount of research activity in this area, however. For instance, production-distribution and production-distribution-production networks are often a central SCM theme in the research conducted today (cf. Simchi-Levi et al. [9] for a overview of some current contributions). Russel et al. [8] present an interesting case study of a multi-product Production Distribution Network (PDN) problem in the newspaper business. They used a Tabu search method for an operational planning problem, where vehicle routing was an important issue. The recent contribution by Rizk et al. [7] solves a problem where a multi-item PDN is optimized (on a tactical/operational level). The model...
is applied with great success to a large pulp and paper company in North America. The MILP-model does not focus on long-term scheduling and it does not incorporate important aspects of the wood procurement (seasonality among others). In [4], five different projects at the same Swedish pulp producing company are reviewed. The methods and targets vary, using MILP-solution techniques, however, but all the projects are concerned with forest industry supply chains. They did not, however, make a tactical harvesting and procurement plan that would incorporate inventory, transportation and harvesting costs on a monthly aggregated level.

In a couple of research projects over the last few years the coordination of production-distribution networks has been tackled. Many companies are forced to operate on a non-optimal level (see Björk & C. Carlsson 2007 [1] and 2006 [3], for instance) for a number of reasons: their market and competitive positions, long-term contracts for key parts of the supply chain, the lack of data and facts on which to build optimal production and distribution plans, etc. In the most recent project we are working with companies which operate in three different supply chains: The plastic raw-material used for wrapping tissue paper (SC1), the supply and procurement of wood to pulp-mills (SC2) and the production-distribution of food-sweeteners (SC3). Common factors in these supply chains are the complexity in coordination of production-distribution networks with inflexible chemical processes. The focus in this paper is set on one of the supply chains (SC2), the supply and procurement of wood to pulp-mills and more precisely on the tactical decisions in a Finnish wood harvesting supply chain.

This optimization model is to be used as a decision support system to aid in the tactical planning. A simplified description of the supply chain is seen in Fig. 1. Wood is bought from the suppliers and added to inventory A of standing stock. The wood is harvested and moved to inventory B, stockpiles at roadsides or other transportation routes. Inventory C is the factory stocks which should be kept large enough to secure the production plan.

The rest of the paper is organized as follows: The next section describes some phenomena in the wood market that should be accounted for in a model. Our modeling solutions are described in parallel. Section 3 contains the actual model with parameters, decision variables and constraints. Some formulas are numbered and described further at the end of the section. The fourth section is a brief look at how the model was implemented to serve the business partner. Some example results obtained with semi-fictional data are presented. Section 5 lists some further lines of inquiry and how this optimization model will be used in a risk analysis setting. At the end of the paper are conclusions and references.

2. Dynamics on the wood market

Most supply chain models share some similarities. The availability of material and some bottlenecks are usually found as constraints and costs are often present in the objective function. Most lines of business also contain their own distinct features, and wood procurement is no exception. The unwritten “rules of the trade” must be pinpointed and then translated to a mathematical framework. This section contains some special features which were included in the model dynamics.

2.1. Accessibility

Many Finnish forests are not accessible for harvesting throughout the year. A harvester can get stuck in a swampy forest. The damage done to the forest in summer time would also be significant, which will not be allowed.

Another reason for avoiding some areas in parts of the year is the condition of roads. The forest industry is more dependent on a widespread infrastructure compared to other industries, e.g. the metal industry. With diminished maintenance of public roads some are now avoided, especially in the wet periods of spring and fall. Also, roads have previously been designed to withstand 40 tons, whereas today full loaded log trucks can weigh 60 tons.

Accessibility is strongly present for the purchasers as there should be enough accessible wood to follow the procurement schedules. The Finnish wood market is limited and the areas available throughout the year are scarce. To ignore accessibility in the modeling would be to risk getting solutions which are either not implementable or useless.
To model this aspect of procurement we divided stumpage sales wood into three ‘accessibility classes’ in accordance with the actual practice of our project partner. The classes are ‘always’, ‘winter’ and ‘summer’. ‘Always’ means that the wood is accessible throughout the year; ‘Winter’ is accessible only when the ground is frozen solid. ‘Summer’ is accessible in both winter and summer, but not necessarily in Spring or Fall. It should be noted that the last two winters, 2006–2008, have been too warm to allow harvesting of all areas classified as ‘winter’ or ‘summer’.

<table>
<thead>
<tr>
<th>Table 1. Accessibility settings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accessible for harvesting (x)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>always summer winter</td>
</tr>
<tr>
<td>January x x x</td>
</tr>
<tr>
<td>February x x x</td>
</tr>
<tr>
<td>March x x x x</td>
</tr>
<tr>
<td>April  x</td>
</tr>
<tr>
<td>May  x</td>
</tr>
<tr>
<td>June  x x</td>
</tr>
<tr>
<td>July  x x</td>
</tr>
<tr>
<td>August x x</td>
</tr>
<tr>
<td>September  x x</td>
</tr>
<tr>
<td>October  x</td>
</tr>
<tr>
<td>November  x</td>
</tr>
<tr>
<td>December  x</td>
</tr>
</tbody>
</table>

The three classes are given their own price, supply and inventory parameters. ‘Always’ is naturally the most expensive class. It is also the smallest, estimated to be roughly 10% of all forests. The accessibility is provided by ticking the appropriate months (Table 1).

### 2.2. Delivery sales

There are two main types of contracts on the wood market. The prevalent one is where the purchasers buy the right to harvest the wood within a specified time. The company will then send their own personnel or contractors to harvest the wood and transport it to roadside. This is commonly called stumpage sales.

The other type of contract is offered to forest owners who want to do the harvesting themselves. They should then deliver the wood to the roadside or a terminal where the forest company receives it. The price paid is naturally higher since it includes costs for harvesting and hauling out of the forest. This type of contract is referred to as delivery sales. Delivery sales account for about 20% of the volumes sold. It is more common for owners to take care of thinning, as opposed to regeneration felling, and for this reason delivery sales cover about 40% of pulpwood volumes.

Forest owners with harvesting capacity have traditionally been farmers. In the growth season they were occupied with agriculture and forestry was mainly done in the winter. This rhythm is still present in the delivery sales (see [4] p. 595). The model should try to reflect this and not use delivery sales to fill out the gaps caused by accessibility problems.

In the input parameters we stipulate a distribution for the delivery sales throughout the year, see Table 2. The shape of the delivery sales procurements is then decided and the only decision we can optimize is the total volume we buy.

<table>
<thead>
<tr>
<th>Table 2. Delivery sales distributions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delivery sales distribution (%)</td>
</tr>
<tr>
<td>Pine sawlogs</td>
</tr>
<tr>
<td>January 15 15</td>
</tr>
<tr>
<td>February 15 15</td>
</tr>
<tr>
<td>March 15 15</td>
</tr>
<tr>
<td>April 3 3</td>
</tr>
<tr>
<td>May 3 3</td>
</tr>
<tr>
<td>June 3 3</td>
</tr>
<tr>
<td>July 3 3</td>
</tr>
<tr>
<td>August 3 3</td>
</tr>
<tr>
<td>September 3 3</td>
</tr>
<tr>
<td>October 7 7</td>
</tr>
<tr>
<td>November 15 15</td>
</tr>
<tr>
<td>December 15 15</td>
</tr>
<tr>
<td>Total 100 100</td>
</tr>
</tbody>
</table>

An average delivery time, m, is provided in the inputs and the volumes turn up in the B inventory (roadside) m months after the contract is made. Rather than asking for the expected deliveries during the first months we divide the pending contracts evenly between the first m months in the model.
2.3. Target levels

To be able to meet customer demand at all times some stock must be kept in the inventories. The size of the needed reserves vary with seasons and demand. Winter is the easiest time to acquire wood and in the summer there will be holidays to take into account. The pulp mill customers on the other hand strive for continuous production all year round. Therefore stocks are usually large in spring.

Experience and business planning suggests some target levels which the procurement personnel tries to maintain. Stock levels a bit below targets may not be hurtful, but too low levels will cause problems with logistics, and possibly shortages for the customers. Too large stocks should also be avoided since they tie up capital.

The target levels were implemented in the model. A crisp constraint setting definite lower bounds on the stocks is possible but not very natural. Crisp limits would also cause infeasibility if there are not sufficient amounts of wood available. Instead we use the stock level variables and some slack variables to shape the objective function so that the solution will tend towards the target levels. The contribution of a stock level to the objective function is as seen in Fig. 2. The dotted line represents inventory costs without penalty costs.

Above the target levels there are only inventory costs, calculated as interest on the invested capital. Below the target level there are “risk penalties”. The penalties reflect the risk taken with low stocks. The aggregation and averaging in our model means that the situation might be okay in some locations and bad somewhere else. The critical level describes where the risk is becoming imminent. Below this level the penalties grow faster with decreasing inventories. The reason for having two different breakpoints are found in the fact that if the inventory level will fall below the target level, some extra costs will probably occur due the some additional logistics solution necessary (re-routing decisions or similar). The extra costs are not huge, however. If the average inventory level will fall below the critical level, it is very likely that some pulp mills need to reduce their production. This is much more costly than if only the low inventory can be solved with additional logistics solutions.

2.4. Price-supply dynamics

Wood is different from many other raw materials as it is a slowly degrading asset. As long as the wood is still growing in the forest the owner can postpone selling for years, waiting for a better price. It has also been claimed that for some owners the forest has a sentimental value.

The prices do reflect in the volumes sold on the market. Especially the active owners might try to sell at the best possible price and at the most opportune time. Forest companies can try to stabilize the market and secure their procurement by offering a price guarantee for price changes the following months.

The expert opinion here states that the activity on the market is either low or high. It is hard to jumpstart the supply but a higher price might work. Saw log prices matter the most to forest owners because they make up most of the profit. A rough estimate was that +5 €/m³ on saw log prices could increase the monthly supply by 50%.

After some trials we decided on a 50% increase in supply on any lumber type when the marginal price is raised by 8% (0.08*60 € = 4.8 €). This can be done two times. A part of the supply will be available to the prevailing market price and the actual volume will depend on if the saw log market price is high or low. The actual evaluation and decision if the prices are to be judged “low” or “high” is decided by comparing saw log prices to some given upper and lower limits, see Fig. 3.

3. The model

In this section we present the parameters supplied by the user, the decision variables and the model. Throughout the document the index i will denote the time step (month). The index k denotes to the type of lumber. The numbers 1, 2, 3 or the letter n is used as superscript indices to denote the accessibility classes,
(always \(= 1\), summer \(= 2\), winter \(= 3\)). Other superscript indices used are: \(A\), \(B\) and \(C\) for the different inventories; \(\alpha\) and \(\beta\) for stumpage and delivery sales; \(T\) and \(Q\) for target and critical levels, minus and plus signs for slack variables; \(U\) and \(L\) for upper and lower limits.

The model is of course independent of units. In our project we used \(\text{€}\), \(\text{m}^3\) and \(\text{€}/\text{m}^3\) for amounts of money, volumes and prices respectively. Interest rates and percentages are unit-less.

### 3.1. Parameters

Some of the needed parameters are individual numbers; some are vectors with values for every time step or lumber type, and some are matrices with indices for both time step and lumber type. Some parameters are subject to computations before they enter the linear programming (LP) model at the core of the inventory model.

- **\(r\)**: annual interest rate
- **\(a^\alpha\), \(a^\beta\)**: advance percentages
- **\(M\)**: procurer’s profit percentage
- **\(d_{i,k}\)**: forecast demand
- **\(u_k\)**: harvesting costs
- **\(t_k\)**: transport costs
- **\(h_i\)**: upper monthly limit to harvesting
- **\(g_i\)**: upper monthly limit to transportation
- **\(D_{i,k}\)**: distribution of delivery sales contracts
- **\(m\)**: average delivery time for delivery sales, an integer
- **\(s^1_k, s^2_k, s^3_k, s^\beta_k\)**: upper limit to the supply in the modeled year
- **\(p^1_{i,k}, p^2_{i,k}, p^3_{i,k}, p^\beta_{i,k}\)**: forecast prices
- **\(L^{A,1}_{i,k}, L^{A,2}_{i,k}, L^{A,3}_{i,k}, L^{A,\beta}_{i,k}, L^{B}_{i,k}, L^{C}_{i,k}\)**: initial stock levels
- **\(L^{A,T}_{i,k}, L^{B,T}_{i,k}, L^{C,T}_{i,k}\)**: target stock levels
- **\(L^{A,Q}_{i,k}, L^{B,Q}_{i,k}, L^{C,Q}_{i,k}\)**: critical stock levels
- **\(p^1_{i,k}, p^2_{i,k}, p^3_{i,k}\)**: price limits on pine and spruce saw logs
- **\(f^u_{i,k}, f^l_{i,k}\)**: extra cost limits for high procurements
- **\(V^1_i, V^2_i, V^3_i\)**: accessibility settings (0 or 1)

### 3.2. Variables

- **\(L^{A,1}_{i,k}, L^{A,2}_{i,k}, L^{A,3}_{i,k}, L^{B}_{i,k}, L^{C}_{i,k}\)**: stock levels
- **\(y^1_{i,k}, y^2_{i,k}, y^3_{i,k}, y^\beta_{i,k}\)**: volumes in contracts made
- **\(x^B_{i,k}, x^A_{i,k}, x^A_{i,k}, x^B_{i,k}, x^C_{i,k}\)**: volumes moved to inventory B
- **\(x^C_{i,k}\)**: volumes transported to inventory C
- **\(x^C_{i,k}\)**: volumes taken from inventory C

The following decision variables are slack variables used for penalties in the objective function. They measure volumes below ideal and critical stock levels and pushed supplies.
3.3. Objective function

The objective function to be minimized consists of costs for procurement, logging, transportation and storage. Additionally there are penalties for exceeded supply levels and stock levels below the desired.

Minimize $z$

$$z = \left[ \text{sales prices and interests for delivery sales} \right]$$

$$+ \sum_{i} \sum_{k} \left( p_{i,k} \cdot y_{i,k}^{1} + p_{i,k}^{2} \cdot y_{i,k}^{2} + p_{i,k}^{3} \cdot y_{i,k}^{3} \right)$$

$$+ \sum_{i} \sum_{k} \left( 1 + m \cdot a^\beta \cdot \frac{r}{12} \right) \cdot p_{i,k}^{\beta} \cdot y_{i,k}^\beta$$

$$+ \sum_{i} \sum_{k} 0.08 \cdot p_{i,k}^{1} \cdot \left( y_{i,k}^{2} + y_{i,k}^{2\prime} \right)$$

$$+ \sum_{i} \sum_{k} 0.08 \cdot p_{i,k}^{\beta} \cdot \left( y_{i,k}^{2\prime} + y_{i,k}^{2\prime\prime} \right)$$

$$+ \sum_{i} \sum_{k} u_{k} \cdot \left( x_{i,k}^{1} + x_{i,k}^{2} + x_{i,k}^{3} \right)$$

$$+ \sum_{i} \sum_{k} t_{k} \cdot x_{i,k}^{B}$$

$$+ \sum_{i} \sum_{k} M \cdot p_{i,k} \cdot x_{i,k}^{C}$$

$$+ \sum_{i} \sum_{k} a^\alpha \cdot \frac{r}{12} \cdot \left( p_{i,k} \cdot L_{i,k}^{A^1} + p_{i,k}^{2} \cdot L_{i,k}^{A^2} + p_{i,k}^{3} \cdot L_{i,k}^{A^3} \right)$$

$$+ \sum_{i} \sum_{k} \frac{r}{12} \cdot \left( p_{i,k}^{1} + u_{k} \right) \cdot L_{i,k}^{B}$$

$$+ \sum_{i} \sum_{k} \left( \frac{r}{12} + M \right) \cdot \left( p_{i,k}^{1} + u_{k} + t_{k} \right) \cdot L_{i,k}^{C}$$

3.4. Constraints

The optimization is subject to the following constraints. There is one constraint for every i and k unless they are used as summation indices or something else is specified. The volume balances for $i = 1$ will include initial parameters (index $i-1=0$).

[x demand is always met]

$$x_{i,k}^{C} = d_{i,k}$$

[volume balances]

$$L_{i,k}^{C} = L_{i,k-1}^{C} - x_{i,k}^{C} + x_{i,k}^{B}$$

$$L_{i,k}^{B} = L_{i,k-1}^{B} - x_{i,k}^{B} + x_{i,k}^{A^1} + x_{i,k}^{A^2} + x_{i,k}^{A^3} + x_{i,k}^{A^\beta}$$

$$L_{i,k}^{A^1} = L_{i,k-1}^{A^1} - x_{i,k}^{A^1} + y_{i,k}^{1}$$

$$L_{i,k}^{A^2} = L_{i,k-1}^{A^2} - x_{i,k}^{A^2} + y_{i,k}^{2}$$

$$L_{i,k}^{A^3} = L_{i,k-1}^{A^3} - x_{i,k}^{A^3} + y_{i,k}^{3}$$

[harvesting capacity is limited]

$$\sum_{k} \left( x_{i,k}^{A^1} + x_{i,k}^{A^2} + x_{i,k}^{A^3} \right) \leq h_{i}$$

[transport capacity is limited]

$$\sum_{k} x_{i,k}^{B} \leq g_{i}$$

[annual supply is limited]

$$\sum_{i} y_{i,k}^{\beta} \leq s_{k}^{\beta}$$

$$\sum_{i} y_{i,k}^{1} \leq s_{k}^{1}$$

$$\sum_{i} y_{i,k}^{2} \leq s_{k}^{2}$$

$$\sum_{i} y_{i,k}^{3} \leq s_{k}^{3}$$

[slack variables measuring stock levels]
\[ L_{i,k}^{A-} \geq L_{i,k}^{A,T} - L_{i,k}^{A,1} - L_{i,k}^{A,2} - L_{i,k}^{A,3} \]
\[ L_{i,k}^{B-} \geq L_{i,k}^{B,T} - L_{i,k}^{B} \]
\[ L_{i,k}^{C-} \geq L_{i,k}^{C,T} - L_{i,k}^{C} \]

The slack variables are found in the objective function and will be zero if the stock level is high enough. But if for instance \( L_{i,k}^{B} < L_{i,k}^{B,T} \), then the solution will be \( L_{i,k}^{B-,i} = L_{i,k}^{B,T} - L_{i,k}^{B} \).

When comparing the standing stock to the critical level we count only wood that is accessible in the present month or the next. (The binary operators \( \land \) and \( \lor \) denote the min and max functions, respectively.) The accessibility for \( i = 13 \) is the same as for \( i = 1 \).

\[ L_{i,k}^{A-} \geq L_{i,k}^{B,T} - (V_{i}^{1} \lor V_{i}^{1}) \cdot L_{i,k}^{A,1} - (V_{i}^{2} \lor V_{i}^{2}) \cdot L_{i,k}^{A,2} - (V_{i}^{3} \lor V_{i}^{3}) \cdot L_{i,k}^{A,3} \]
\[ L_{i,k}^{B-} \geq L_{i,k}^{B,T} - L_{i,k}^{B} \]
\[ L_{i,k}^{C-} \geq L_{i,k}^{C,T} - L_{i,k}^{C} \]

[slack variables measuring high monthly procurements]
\[ y_{i,k}^{a+} \geq y_{i,k}^{1} + y_{i,k}^{2} + y_{i,k}^{3} - f_{i,k}^{a,1} \]
\[ y_{i,k}^{b+} \geq y_{i,k}^{b} - f_{i,k}^{b,1} \]
\[ y_{i,k}^{a++} \geq y_{i,k}^{1} + y_{i,k}^{2} + y_{i,k}^{3} - f_{i,k}^{a,2} \]
\[ y_{i,k}^{b++} \geq y_{i,k}^{b} - f_{i,k}^{b,2} \]

[procurement limits]
\[ y_{i,k}^{1} + y_{i,k}^{2} + y_{i,k}^{3} \leq (f_{i,k}^{a,U} \lor 2 \cdot f_{i,k}^{a,L}) \lor f_{i,k}^{a,L} \]
\[ f_{i,k}^{a,L} \leq y_{i,k}^{1} + y_{i,k}^{2} + y_{i,k}^{3} \]

[delivery sales dynamics]
\[ y_{i,k}^{b} = D_{i,k} \cdot \sum_{i} y_{i,k}^{b} \] (3)
\[ x_{i,k}^{A,b} = \frac{L_{i,k}^{A,b}}{m} \quad \text{when } i \leq m \] (4)
\[ x_{i,k}^{A,b} = y_{i-m,k}^{b} \quad \text{when } i > m \] (5)

Equation (3) forces the delivery sales contracts to follow the given distribution. In the first \( m \) months (4) portions out \( 1/m \) of the volume from contracts made earlier. Later on (5) delivers the contracts made \( m \) months earlier.

### 3.5. Variable bounds

Most variables are positive in the model. The exception is \( L_{i,k}^{C} \) which is free in order to guarantee feasibility. Note that a negative \( C \) inventory is severely punished with penalty costs.

The upper and lower monthly limits for delivery sales are given as variable bounds, as they are not compounded constraints like for stumpage sales.

\[ f_{i,k}^{b,L} \leq y_{i,k}^{b} \leq f_{i,k}^{b,U} \]

The accessibility stipulations are enforced by setting harvest volumes to zero in the month when the class is not available.

\[ \forall i, k, n \quad V_{i}^{n} = 0 \Rightarrow x_{i,k}^{A,n} = 0 \]

### 3.6. Pre-calculations

To determine the volumes offered at market prices we use some upper and lower bounds for saw log prices. The sub indices 1 and 3 are pine and spruce saw log respectively.

\[ W_{i} = \frac{1}{2} \left( \frac{p_{1}^{1} - p_{1}^{\min}}{p_{1}^{\max} - p_{1}^{\min}} + \frac{p_{1}^{3} - p_{3}^{\min}}{p_{3}^{\max} - p_{3}^{\min}} \right) \]
\[ f_{i,k}^{a,1} = f_{i,k}^{a,L} + W_{i} \cdot (f_{i,k}^{a,U} - f_{i,k}^{a,L}) \]
\[ f_{i,k}^{b,L} = f_{i,k}^{b,L} + W_{i} \cdot (f_{i,k}^{b,U} - f_{i,k}^{b,L}) \]
\[ f_{i,k}^{a,2} = 1.5 \cdot f_{i,k}^{a,1} \]
\[ f_{i,k}^{b,2} = 1.5 \cdot f_{i,k}^{b,1} \]

The weight \( W \) is a mean indicating if saw log prices are high or low compared to the given price limits. The first volume limits \( f_{i,k}^{a,1} \) and \( f_{i,k}^{b,1} \) indicate how much wood can be bought at market prices. The limits are used to let 50% more wood become available with an 8% price increase. Note that all lumber types are dependent on the saw logs.

### 4. Implementation and example

The resulting optimization problem is purely continuous and linear. As far as LP problems go, it is of a modest size: around 1500 constraints, 1800 variables and 5000 non-zero elements in the
The user interface is an excel workbook (Fig. 4). Input parameters are given and tweaked and a macro is started. The macro reads the inputs, does the pre-calculations and sends the problem to lp_solve through an application programming interface (API). The solution is returned; the macro analyzes it and presents the optimal strategy with graphs and numbers. The model has not yet been extensively evaluated. The dynamics it shows have improved since the first versions which were inclined to do business in an on/off fashion buying maximum volumes one month and nothing the next. In Fig. 4 are the results of a constructed example. The prices and logistics costs used are close to reality but all the company specific data is fictional.

The shape of delivery sales are decided by the distribution parameters as described earlier. In the example all six lumber types have the same distribution and therefore the same relative shape. Stumpage sales are not governed by any distribution but they happen to be quite low in months three to six. This happens when the standing stocks are large enough to cover the target levels a couple of months forward and no other constraints come into play.

The inventory graphs show how the inventory levels evolve in the planned year. Stocks increase and decrease with the target levels but diverge from these when it benefits the overall objective function. The factory stocks are kept quite steady to secure the production. In fact, the factory stock levels are

![Figure 4. Graphs showing an example run of the optimization model](image-url)
usually kept very close to the target level, since it is most costly to deviate from the target value at this last inventory. The inventory costs are the highest and the shortage penalty costs are also very high. Also the stumpage sales exhibit a quite expected pattern; buy less in the late spring and early summer.

5. Future research

The research projects continue and also within the wood procurement chain there will be further investigations. The optimization model will undergo an evaluation to check its performance with real data. A sensitivity analysis of the model will also be carried out to detect if the model is highly sensitive to some small parameter changes.

The optimization model will also be used as an integral part of a risk analysis. Prognoses are given for a long time ahead. The optimization is run once to optimize one year forward and the decisions for the first month are implemented. After that some simulated outcomes of supply, demand and prices are used to decide the discrepancy between forecasts and “reality”. The prognoses are then adjusted with this knowledge. This process is repeated to emulate the rolling-horizon planning employed in most businesses. The simulation is performed many times to get a statistical material and the stock levels and costs in the simulations are analyzed to give a measure of risk. Two questions concerning risks are of great importance. “What is the risk (probabilities and costs) with some given stock target levels?” and “What target levels could be chosen if the risk should be below a given level?” The second question is in some sense an inverse of the first and the answers would give valuable aid in tactical planning.

6. Conclusions

While this project is carried out the Finnish forest industry is experiencing turbulent times. Stora Enso decided to close down one of their pulp mills, production in Kemijärvi ended in April 2008. Russia has an agenda of raising customs duties on wood exports. If the next rise is implemented in January 2009 it will drain much of the wood import to Finland. Temperatures above zero in the winter have caused accessibility issues and this might become a recurring theme if global warming makes mild winters common.

In the light of these changes it is of utmost importance that the industry is developing new and better tools to aid in the difficult decision situations. This paper gives an example of how mathematical modeling can be employed. The paper introduces a new model for the optimization of tactical forest harvesting, transportation and inventory management in a large forest industry supply chain in Finland.

The behavior of the current model is considered close to implementable. The next step is a more thorough evaluation of the model with real data. If successful, the model can be used as a stand-alone decision support system. It will also be integrated in the simulation-based risk analysis of the same project.

7. References


