Optimal Electric Power Capacity Expansion in the Presence of Options

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Abstract

This paper studies optimal long-term electric power capacity strategies with capacity options. Gencos can sign contracts with Discos, where such contracts take the form of capacity options that may or may not be executed by Discos at some pre-specified maturation date. Capacity not offered in the options market, or for which options by Discos are not executed, can then be offered in the spot market. The purpose of this paper is to derive the optimal capacities for Gencos in the long run, given full knowledge of the short-term equilibria in previous literature. We determine the best response strategies for each Genco in the game derived from the short-term outcome resulting from capacity decisions. We then characterize the long-run equilibrium and derive an efficient algorithm to compute it, when it exists. This allows us also important insights into the nature of technologies that can survive in the long run.

1. Introduction

The papers by Wu, Kleindorfer and Zhang (20001a,b,c), hereafter cited as WKZ have characterized the necessary and sufficient conditions of the short-term equilibrium, i.e. with a fixed capacity for every Genco, for a market in which Discos can reserve capacity through options obtained from individual Gencos. Output on the day can be either obtained through executing such options or in a spot market. In this paper, we extend the WKZ short-term equilibrium results to determine optimal capacity strategies and study the equilibrium issues related to these strategies. To make this paper manageable in size, we will rely entirely on the framework and notation of WKZ (2001c).

Linking capacity expansion games with short-term pricing has been an important area of study in industrial organization, with a major contribution coming in the Kreps-Scheinkman (1983) paper, showing that precommitments of capacity, followed by Bertrand competition, gives rise to Cournot outcomes. This paper enriches these same results in the context of the more complicated arena of interest here in which competition occurs in an integrated contract-spot market setting. The results obtained reflect the interaction of these two markets. Indeed, the characterizing condition for long-run equilibrium is that Tobin’s marginal $q$ be equal to unity for all Gencos, with the definition of Tobin’s marginal $q$ (Tobin, 1969; Abel, 1983, 1990; Abel et. al, 1996) modified to reflect the presence of two markets (the contract and the spot) for procurement. This brings new insights in putting together the traditional conditions for efficient equilibrium based on Tobin’s marginal $q$ with options markets. (See Dixit and Pindyck, 1994, pp. 4-7 for a discussion.)

A further issue of some interest in the present context is the characterization of efficient technology mixes in long-run equilibrium. This has been discussed by Crew and Kleindorfer (1976), Allaz (1992), Allaz and Vila (1993), and Gardner and Rogers (1999). The conditions characterizing the efficient mix are extended here to account for the integration of the two markets of interest. The usual cost conditions (indicating tradeoffs between unit capacity costs and unit variable costs across different technologies) need to be extended in the present context to account for the interaction of each technology with the characteristics (especially the volatility) of the spot market.

We proceed as follows. In Section 2, we first define some necessary notation and summarize assumptions and conditions needed for our model. In Section 3, building on the short-term results of WKZ (2001b, c), we structure the long-term capacity game among Gencos. This game is determined by the expected profits for each Genco in the short-term game of participating in the combined contract-spot markets. These profits, of course, depend on the capacity decisions made by Gencos prior to the play of this game. We determine best response and equilibrium strategies for this game and show some properties of the price and capacities that result in equilibrium. We then consider the characteristics of efficient technology mixes in the long-run equilibrium. In section 4, we give some numerical examples to illustrate key insights derived in this paper. In
section 5, we further characterize the long-run market segmentation for Gencos. Section 6 concludes the paper with some extensions and directions for future research.

2. Preliminaries

We assume a set of $I$ Gencos, $\Xi$, and any number of Discos. Following WKZ (2001c), we use the following notation.

$P_s$: spot market price. Its cumulative distribution function $F(P_s)$ is assumed to be common knowledge

$\beta_i$: Genco $i$’s unit capacity cost per period

$b_i$: Genco $i$’s short-run marginal cost of providing a unit $K_i$: Genco $i$’s total capacity. Let $K = (K_1, \ldots, K_I)$

$s_i$: Genco $i$’s reservation cost per unit of capacity if the contract is signed

$g_i$: Genco $i$’s execution cost per unit of output actually used from the contract. Recall from WKZ (2001b,c) that the (optimal) price of $g_i = b_i$

$Q_i$: Contract market demand for Genco $i$’s output. Recall that in WKZ (2001c), we have shown when there are multiple Gencos, Greedy Contracting in order of the index $s_1 + G(g_1) \leq s_2 + G(g_2) \leq \ldots \leq s_I + G(g_I)$ is optimal for the Discos

$U(z)$: Discos’ aggregate Willingness-To-Pay for output level $z$

Define $p = s_i + G(g_i)$ as the contract market price, symmetric for all Gencos at equilibrium, with the effective price function $G(p)$ defined as

$$G(p) = \int_0^p (1 - F(y))dy = E\{\min(P_s, p)\}$$

and $G^{-1}$ as the inverse function of $G$

$D_s(x)$: Disco’s normal demand function when there exists only the spot market, so $D_s(x) = U'^{-1}(x)$. Let $D(p) = D_s(G^{-1}(p))$

Define $c_i = s_i + G(b_i)$, in which $s_i = E\{m(P_s - b_i)^+\}$ is Genco $i$’s unit opportunity cost on the spot market if the Disco chooses to exercise his contract

$m$: the probability that the Genco can find a last-minute Disco on the spot market\(^1\)

Define $X(M) = \sum_{i \in M} K_i$ as the total capacity of all Gencos in set $M$.

We make the following assumptions.

A1: The Disco’s WTP $U(z)$ is strictly concave and increasing so that $U''(z) > 0$, $U''(z) < 0$, for $z \geq 0$

A2: $zD''(z) + 2D'(z) \leq 0$, $\forall z \geq 0$

A3: $P_i[D_s(g_i) - \sum_{i=1}^I Q_i] \geq 0$, $i = 1, \ldots, I$, assuming $g_1 \leq g_2 \leq \ldots \leq g_I$

A4: When there is a bid-tie among Gencos, then the Discos’ demand for Genco $i$’s output is proportionally allocated to the Gencos according to their bid capacity, thus $Q_i = D(p)\frac{K_i}{X(M)}$

Concerning A1, these are standard assumptions on the Willingness-to-Pay function. From A1, we can easily know that $D(p)$ is monotonously descending. A2 is equivalent to $R^2 > 0$ and $R < 1$ where $R = -U''(z)z/U'(z)$ is the Arrow-Pratt measure of relative risk aversion. This, too, is standard in the financial economics literature (e.g., Rothschild and Stiglitz, 1970, 1971). A3, noted as the No Excess Capacity Condition in WKZ (2001c), implies that Discos will not contract for more than what they are sure they will use if they buy under contract on the day, i.e., if $Q_i > 0$ then the sum of all contracted capacity with execution fees less than or equal to $g_i$ must not exceed $D_s(g_i)$.

In WKZ (2001c, Theorem 2), we characterized the short-term equilibrium as the following. Let $K, \hat{p}, \hat{M}$ be any short-term equilibrium, where $\hat{M}(K) \subseteq \Xi$ is the equilibrium set of all Gencos having positive capacity contracts, i.e., $Q_i(\hat{p}) > 0, i \in \hat{M}$ and $Q_j(\hat{p}) = 0, j \in \Xi \setminus \hat{M}$. Assume $\hat{M}$ is non-singleton such that $|\hat{M}| > 1$ and $\min\{c_i | i \in \Xi\} < G(U'(0))$. Then the necessary and sufficient conditions for an equilibrium $\hat{p}$ to exist are\(^2\)

SC1: $D(\hat{p}) = \sum_{i \in \hat{M}} K_i = X(M)$

SC2: $\partial f_k(p_k)/\partial p_k < 0$ if $p_k > \hat{p}$, where $f_k(p_k)$ is defined as $f_k(p_k) = (p_k - c_k)(D(p_k) - \sum_{i \in \hat{M}_k} K_i)$, and $\hat{M} = \hat{M} \setminus \{k\}$ and

SC3: $\forall j \in \Xi \setminus \hat{M}, \hat{p} < c_j$.

Condition SC1, noted as the “symmetry condition” in WKZ (2001c), says that in the short-term equilibrium, for any Genco “in the money”, i.e., for any $k \in \hat{M}$, its entire capacity will be contracted in the contract market. Condition SC2 is a special case of the standard economic assumption (see, e.g., Friedman, 1988) for the behavior of the profit function. In Friedman (1988), it is assumed to be strictly concave, here we only require the function to be non-increasing to the right of the equilibrium price $\hat{p}$, where $\hat{p} \geq \arg\max x f_k(p_k)$. Condition SC3 implies that any Genco “out of the money” does not have any incentive to join in the short-term contract market equilibrium, as doing so results a net loss in its profit.

Further discussion of these assumptions and conditions is in WKZ (2001a, b, c).

\(^1\)To minimize notational complexity, we analyze only the case where $m$ is uniform and fixed for all Gencos. It is straightforward to generalize these results to allow $m$ to vary as a function of $P_s$ and to vary across Gencos. See WKZ (2001c) for such an extension.

\(^2\)In the singleton case when $|\hat{M}| = 1$, the only Genco providing positive contract output (which we denote as Genco 1) satisfies $c_1 = \min\{c_i | i \in \Xi\} < G(U'(0))$. The necessary and sufficient conditions for a single-Genco short-term equilibrium $\hat{p}$ to exist are (i) $\hat{p} = \max\max\{p^H, x^H\}$, where $p^H = \arg\max\{p \leq c_1 \} D(p)$, and $x^H = D^{-1}(K_1)$, and (ii) $\hat{p} \leq \min\{c_i | i \in \Xi \setminus \{1\}\}$.\n
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3. Optimal Capacity Expansion

The outcome of the short-term options-pricing game in the integrated contract and spot markets leads to the following profit function for any Genco $k$:

$$E \pi_k(\hat{p}, K) = (\hat{p} - c_k)Q_k + (c_k - \beta_k - G(b_k))K_k$$

Genco $k$'s problem is to choose an optimal capacity $K^*_k$ to maximize $k$’s long-run expected profit, i.e.,

Maximize $K_k E \pi_k(\hat{p}, K)$.

**Lemma 1:** Let $\hat{p}(K)$ be the short-run equilibrium price and let $\hat{M}(K)$ be the set of Gencos in the contract-market equilibrium. Assume $\hat{M}(K)$ is non-singleton so that $|\hat{M}(K)| > 1$. Then the best response capacity strategy for each Genco $k \in \hat{M}(K)$ is

$$K^*_k = \max \left\{ \frac{\hat{p} - \beta_k - G(b_k)}{\hat{p} - c_k} X(M), 0 \right\}.$$  

(1)

**Proof:** See Appendix.

We note that the proof in the appendix takes the equilibrium set $\hat{M}(K)$ as given, and determines the best response strategy for every Genco in $\hat{M}(K)$, assuming that the set $\hat{M}(K)$ does not change as $K$ is adjusted. In the long-term equilibrium, where $K$ is adjustable, what is required is that all best capacity responses $K^*$, given $\hat{M}(K^*)$, result in a short-run equilibrium $p^*$ with $p^* = \hat{p}(K^*)$ and $M^* = \hat{M}(K^*)$. Thus, the long-run equilibrium we seek to characterize is defined as follows.

**Definition:** A long-run non-singleton contract market equilibrium $K^*, p^*, M^*$ is a vector such that $p^* = \hat{p}(K^*)$ and $M^* = \hat{M}(K^*)$ and such that $K^*_k > 0$ for all $k \in M^*$, where $K^*_k$ satisfies the best-response condition (1), i.e.,

$$K^*_k = \frac{p^* - \beta_k - G(b_k)}{p^* - c_k} X^*(M^*)$$

(2)

where $X^*(M^*) = \sum_{i \in M^*} K^*_i$.

**Definition:** Let $c_1 = \min \{ c_i | i \in \Xi \}$ so that Genco 1 has the lowest $c_i$ index among all Gencos. A long-run singleton contract market equilibrium $K^*_1, p^*, M^*$ is a triple such that the following conditions are satisfied: (i) $p^* = \text{argmax}(p - \beta_1 - G(b_1))D(p)$; (ii) $K^*_1 = D(p^*)$; (iii) $c_1 \leq \beta_1 + G(b_1)$; (iv) $p^* < \min \{ \max \{ c_i, \beta_i + G(b_i) \} | i \in \Xi \}$.

3The case of singleton when $|\hat{M}| = 1$ is dealt with later.

**Definition:** Define $\zeta_k$ - a modified Tobin’s marginal $q$ (Abel 1983; 1990; Abel et al. 1996) for Seller $k$ as:

$$\zeta_k = \frac{\partial((p - c_k)D(p)\frac{K^*_k}{\partial K_k})}{\partial((\beta_k + G(b_k) - c_k)K_k)\partial K_k}$$

(3)

**Corollary 1:** Let $K^*, p^*, M^*$ be a long-run equilibrium solution. Then for any Genco $k \in M^*$, $\zeta_k = 1$ where $\zeta_k$ is Tobin’s marginal $q$ for Genco $k$, whether $M^*$ is singleton or not.

**Proof:** See Appendix.

**Corollary 2:** For any Genco $k \in M^*$, whether $M^*$ is singleton or not, if $\exists K^*_k > 0$, then $p^* > \beta_k + G(b_k) > c_k$.

**Proof:** This is a direct consequence of Lemma 1 and the definition of singleton contract market equilibrium. Q.E.D.

It should be noted that the above lemma and corollaries characterize capacity conditions only for the long-term contract market. It may very well be the case that some Gencos build capacity and only participate in the spot market. Corollary 2 says that those who participate in the contract market in the long run, $\beta_k + G(b_k) > c_k$ or equivalently $\beta_k > \frac{c_k}{b_k}$. This implies that in any long-term contract market equilibrium, every Genco (with positive capacity) satisfies $\beta_k + mG(b_k) > m\mu$, where $\mu$ is the mean of the spot market price. This is very intuitive. As the mean of the spot market price increases, or access conditions improve to the spot market (m increases), Gencos are less interested in participating in the contract market and more interested in participating in the spot market.

**Corollary 3:** Let $K^*, p^*, M^*$ be a long-run equilibrium solution. Assume $|M^*| > 1$, then $p^*$ must satisfy (in addition to being a short-run equilibrium price corresponding to $K^*$)

$$\sum_{i \in M^*} \frac{p^* - \beta_i - G(b_i)}{p^* - c_i} = 1$$

(4)

or equivalently,

$$\sum_{i \in M^*} \frac{\beta_i + G(b_i) - c_i}{p^* - c_i} = |M^*| - 1.$$  

(5)

**Proof:** Summing over $M^*$ on both sides of (2) results in (4). It is straightforward to get (5) from (4). Q.E.D.

**Lemma 2:** If there exists any equilibrium set $M(p^*) \subseteq \Xi$, it must be unique.
Theorem 1: The long-term equilibrium set \( M^* \subseteq \Xi \), whether it exists or not, is characterized by the following algorithm. Index Gencos in the order of \( c_t \), i.e.,
\[ c_1 \leq c_2 \leq \ldots \leq c_t. \]

(i) \( p^* = \arg \max (p-\beta_t - G(b_t))D(p) \). If \( c_1 > \beta_t + G(b_1) \) then exit else if \( p^* \leq c_2 \), then
\( M^* = \{ 1 \} \) exit. Else \( M^* = \{ 1 \} \) and \( i = 2 \).

(ii) Loop While \( (p^*>\beta_t + G(b_t)) \) and \( (\beta_t + G(b_t) > c_i) \)
begin
\( M^* = M^* \cup \{ i \} \); compute \( p^*(M^*) \) via (5).
if \( i < I \) then \( i = i + 1 \) else exit.
end.

(iii) If \( (p^* > c_i) \) and \( (c_i \geq \beta_t + G(b_i)) \) then \( M^* = \phi \).
(iv) If \( \partial f_i(p_i)/\partial f_t \geq 0 \) and \( p_t > p^* \) then \( M^* = \phi \).

Proof: This is direct consequence of Lemmas 1, 2 and Corollary 3. Q.E.D.

The above algorithm generates the equilibrium essentially by testing, in increasing order of \( c_t \), the compatibility between the short-run pricing equilibrium and the long-run capacity equilibrium at the best-response strategies characterized in Lemma 1. An equilibrium can, of course, fail to exist. As embodied in the above algorithm, this occurs when adding a further Genco \( k \) to the contract market, at the long-term capacities implied by the best-response capacity strategies, the short-term equilibrium price drops below the required feasibility index \( c_t \) for Genco \( k \). Thus, without Genco \( k \) in the contract market, Discos’ demand intensity signals that entry is desirable beyond the current participants in the contract market. But adding \( k \) drops the price below that which is sustainable in this market. Let us consider some examples to illustrate these points.

### 4. Numerical Examples

**Numerical Example 1.** Assume there are five Gencos with technology parameters \( (G(b), \beta, K) \) as shown in Table 1 and the risk factor \( m = 0.5 \). We can compute \( (s, c, \beta + G(b)) \) as shown in Table 1. Suppose the spot market price follows an exponential distribution, \( f(y) = \frac{1}{30}e^{-y/30} \), so the mean of the spot market price is \( \mu = 30 \). Then the effective price function is \( G(x) = -30(e^{-x/30} - 1) \), where \( 0 \leq x < \infty \), and thus we have \( G^{-1}(p) = 30\ln\frac{30}{-30-p} \), where \( 0 \leq p < 30 \). Suppose the WTP function is \( U(z) = 30z(\ln\frac{30}{z} + 1) \), where \( 0 < z \leq 30 \), it is obvious that this function satisfies Assumption 1 as follows: \( U''(z) = 30\ln\frac{30}{z} \geq 0 \) and \( U'''(z) = -\frac{30}{z} < 0 \); thus we have \( D_s(x) = U''^{-1}(x) = 30e^{-x/30} \), where \( 0 \leq x < \infty \). So the contract market demand function is \( D(p) = D_s(G^{-1}(p)) = U''^{-1}(G^{-1}(p)) = 30 - p \), where \( p \in [0, 30] \). It is straightforward to compute that in the short term, four Gencos, namely 1, 2, 3, and 4 achieve an equilibrium at a price \( p = 26 \). Genco 5 is not in the short-term equilibrium even though 5 has strong incentives to participate since 5 can not make any money on the spot market due to very high short-run marginal cost \( G(b_5) = 30 \). Using the above Theorem 1, we can compute that there are only two survivors in the long run, namely 1 and 2, with the equilibrium price, \( p^* = 23.26 \). The optimal capacity investments for these two Gencos are \( K_1^* = 4.180 \) and \( K_2^* = 2.584 \). Genco 3 and Genco 4 find themselves out of the contract market, and both participate only in the spot market. Genco 5 is “out of business” in the long run, and is better off by shutting down all its plants. A more general result is summarized in Corollary 5.

The reader will note that equilibrium does not always exist. Here’s another example.

**Numerical Example 2.** Assume there are three Gencos in the contract market. Genco 1’s and 2’s technology parameters and all the other market conditions are the same as in Example 1, except that Genco 3’s \( G(b_3) = 16 \) and \( \beta_3 = 6 \) as in Table 2. The short-term equilibrium price is 27. However, there is no long-term equilibrium in this example, because the long-term contract price formed by Genco 1 and 2, 23.2, is higher than Genco 3’s index \( c_3 = 23 \), at this contract market price \( p = 23.2 \). Genco 3 finds participation in the contract market is more profitable than staying in the spot market per unit capacity, since \( p-(\beta_3 + G(b_3)) = 23.2 - 22 = 1.2 > s_3 - \beta_3 = 7 - 6 = 1 \). However, if Genco 3 does participate in the contract market, the contract market price drops to 22.2, this
Table 2. Summary of Parameters and Results for Numerical Example 2. \( m = 0.5, \mu = 30, \) 
\( D(p) = 30 - p. \)

<table>
<thead>
<tr>
<th>( i )</th>
<th>( G(b_i) )</th>
<th>( \beta_i )</th>
<th>( K_i )</th>
<th>( s_i )</th>
<th>( c_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>14</td>
<td>1</td>
<td>12</td>
<td>18</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>12</td>
<td>1</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>16</td>
<td>6</td>
<td>1</td>
<td>7</td>
<td>23</td>
</tr>
</tbody>
</table>

Table 3. A Two-Genco Investment Game

<table>
<thead>
<tr>
<th>( K_1(\text{Don't Invest}) )</th>
<th>( K_2(\text{Don't Invest}) )</th>
<th>( K_1(\text{Invest}) )</th>
<th>( K_2(\text{Invest}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>6, 4</td>
<td>5, 9</td>
<td>16, 2</td>
<td>13.4, 3.1</td>
</tr>
</tbody>
</table>

makes participation undesirable, since net profit per unit capacity is less than staying in the spot market, i.e., \( p - (\beta_i + G(b_i)) = 22.2 - 22 = 0.2 < \frac{m}{2} - \beta_i = 7 - 6 = 1. \) This is an example that shows there need be no long-term equilibrium in the contract market.

**Numerical Example 3.** It is interesting now to conduct a game-theoretical analysis of the investment game in Numerical Example 1. Assume Gencos 3, 4, 5 would not invest, and their capacity will be fixed throughout, i.e., \( K_3 = K_4 = K_5 = 1. \) Genco 1 and Genco 2 each has to decide whether to invest or not. Genco 3, 4, and 5 decide whether or not to participate in the contract market based on the resulting contract equilibrium price due to the capacity adjustment of Gencos 1 and 2.

In the above numerical example 1, we computed the profits for both parties in short-term equilibrium \((K_1, K_2)\) and in long-term equilibrium \((K_1^*, K_2^*)\). Now we compute the profits of both parties when only one Genco is using the best response strategy, \((K_1^*, K_2)\) and \((K_1, K_2^*)\). When Genco 1 expands its capacity to \( K_1^* = 4 \) but Genco 2 does not, \( p = 24 \), so Genco 4 is out, Genco 1’s profit increases while Genco 2’s profit decreases. Genco 3 is indifferent between participating in the contract market or in the spot market. On the other hand, when Genco 2 raises its capacity level to \( K_2^* = 3 \) while Genco 1 does not, \( p = 25 \), Genco 4 is indifferent between participating in the contract market and in the spot market. However, Genco 3 now finds the contract market more profitable than the spot market alone. Genco 2’s profit increases while Genco 1’s profit decreases due to Genco 2’s capacity expansion. Table 3 contains the payoff matrix for Gencos 1 and 2. Clearly, the Nash equilibrium of this investment game is that both Genco 1 and 2 choose to invest, as characterized in our Theorem 1.

Our analysis leads to a somewhat different conclusion than that advanced by Allaz (1992), Allaz and Vila (1993) based on a simpler model in which capacity is assumed to be completely scalable within the timeframe of the contract market. They show that a prisoner’s dilemma results in the contract-spot market with Oligopoly. They claim that Gencos always prefer to stay in the spot-market, but are forced to enter into the contract market because of competition, even though doing so results in a net loss of profit for both parties. The above example clearly shows such a dilemma is not a general result in the richer model studied here in which capacity precommitments (at a cost) are present! The long-run equilibrium we obtained in this example is indeed Pareto efficient.

5. Long-Run Market Segmentation

**Corollary 5:** Assume a long-term equilibrium \( K^*, p^*, M^* \) exists. The market segmentation of Gencos in the long-run is the following. (i) For any Genco \( k \) “in the money”, then \( s_k < \beta_k \), i.e., \( \forall k \in M^* \), \( k \) participates both in the contract and in the spot market. (ii) For any Genco \( k \) “out of the money”, i.e., \( \forall k \in \Xi \setminus M^* \), the necessary and sufficient condition for \( k \) to participate in the spot market is \( s_k > \beta_k \).

(iii) For any Genco \( k \in \Xi \setminus M^* \) but \( s_k \leq \beta_k \), \( k \) will be “out of business” in the long-run.

**Proof:** See Appendix.

Note that if \( k, k + 1 \in \Xi \setminus M^* \) and if \( s_{k+1} > \beta_k \), then \( s_{k+1} > \beta_{k+1} \) implies \( s_k > \beta_k \)\(^4\). This means that if \( k \) is “out of the money” w.r.t. the contract market, but still competes in the spot market, then every other Genco not in the money with lower capacity costs than \( k \) will also participate in the spot market alone.

Corollary 5 implies that the index line of \( c_1 < c_2 < \ldots < c_T \) can be used to identify the unique group of Gencos who participate both in the contract and in the spot market, and a further disjoint and unique group of Gencos who only participate in the spot market, and lastly, the remaining unique group of Gencos who will be “out of business” in the long-run.

Unsurprisingly, the nature of the spot market (volatility and price level) as well as both variable and capital costs

\(^4\)Assuming the standard conditions in the public economics literature hold, i.e., \( b_1 < b_2 < \ldots < b_T \) and \( \beta_1 > \beta_2 > \ldots > \beta_T \), see, e.g., Crew and Kleindorfer (1976).
and the access parameter \((m)\) are factors affecting which technologies survive in the long run. The above results provide the key insights on how these cost and market factors interact strategically to determine which markets will exist in the long run and which Gencos will be able to survive in each respective market.

6. Some Extensions and Future Research

6.1 Efficiency of the Two-Part Options Contract

It is well known (e.g., Allaz and Vila 1993) that forward markets are generally inefficient under Cournot competition among suppliers/Gencos (though these results typically ignore capacity constraints), unless there are many suppliers and many trading/contracting periods prior to the spot market. Our results provide a richer framework for analyzing the efficiency of forward contracts for the case of Bertrand-Nash competition with capacity constraints both in the short run and in the long run (arguably the natural form of competition for electronic markets for electricity). The reader should note that our two-part options \([s^*, g]\) are equivalent to forward contracts when \(g = 0\); if \(g = 0\), then clearly the Discos will always exercise the contracts on the day (since \(U'(z) > 0, \forall z\)), and Gencos will therefore be forced to deliver the full amount of any option committed with \(g = 0\). Such a contract is therefore a “must-produce, must-take” contract, i.e., a forward. However, as shown in Wu et al. (2001c), this contract is strictly dominated by an appropriately designed options contract from the Genco’s perspective without changing the Disco’s utility. Thus, any such forward contract is Pareto dominated by some options contract when both contract and spot markets are active. Naturally, if custom features of a product make spot markets infeasible, then forward contracts can still be efficient, especially if they allow better, cheaper planning of production through advance reservation\(^5\). In a similar fashion, one should note the more intuitively obvious fact that contracts that pre-commit capacity without a reservation fee, of the form \([s, g] = [0, g]\) are also Pareto dominated.

A related interesting question would be whether generalizations to allow for state-dependent options contracts would perform better than the simpler contracts studied here. Such a contract would take the form \([s, g(\omega), Q(\omega)]\) where \(g\) and \(Q\) both may depend explicitly on the state of the world \(\omega\). However, such contracts are easily shown to be dominated by the two-part options contract studied here. But such contracts might be of interest if either Gencos’ costs \(b\) depend on \(\omega\) or if Discos’ demands depend on \(\omega\), for example, if the strength of Disco demands depends on the “weather”. However, the characterization of the optimal Disco’s choices are considerably more complicated when costs or demands are state-dependent, as worked out in detail in Spinler, Huchzermeier and Kleindorfer (2000) generalizing the single-Genco results of Wu, Kleindorfer and Zhang (2001b) to the state-dependent case.

6.2 Continuous Time

We define \(f_t(P_T)\) as the probability density function of the spot market price at time \(T\), given the information available until time \(t < T\). As in the above analysis, we assume the density function \(f_t(P_T)\) is common knowledge. We impose some structure on the evolution of this density function as \(t\) progresses. As in standard options theory, we assume that the logarithm of the commodity spot market price \(X_t = \ln P_t\) is normally distributed and follows the Ornstein-Uhlenbeck (mean-reverting) process (Dixit and Pindyck 1994) [Schwartz (1997) proposes that the commodity spot price \(P_t\) follows a stochastic process given by

\[
dP_t = \kappa(\mu - \ln P_t)dt + \sigma P_t dz_t.
\]

Applying Ito’s lemma to this process, the logarithm of the price, \(X_t = \ln P_t\), follows the process (6):\]

\[
dx_t = \kappa(\alpha - X_t)dt + \sigma dz_t
\]

where \(\kappa\) is the speed of reversion \((\kappa > 0)\) measuring the degree of mean reversion to the long-run mean log price, \(\alpha\); the second term in equation (6) characterizes the volatility of the process, with \(dz_t\) being an increment to a standard Brownian motion. The following single-Genco result is generalized in Wu et al. (2001) from the results of this paper.

Theorem 2 (Disco’s) Optimal Contracting Policy at time \(t\); Wu et al. 2001: Assume \(\lim_{t \to \infty} U'(z) = \infty\). If \(s(t) + G_t(g(t)) > G_t(U(0))\), then \(Q(s(t), g(t), t) = 0\) is optimal; otherwise, if the Disco purchases a capacity contract at time \(t\), then the optimal \(Q(s(t), g(t), t)\) will satisfy the following identity:

\[
s(t) = G_t(U'(Q(t))) - G_t(g(t))
\]

\[
= E[(P_{t,t} - g(t))^{+}] - E[(P_{t,t} - U'(Q(t))^{+}].
\]

The Genco’s optimal contract at time \(t\) will be of the form \([s(t), b]\) where \(s(t)\) solves the following profit maximization problem with the spot price distribution given by \(f_t(P_T)\):

\[
s(t) = \arg\max_{t} E[H(s(t), b, t)]
\]

\[
= \max_{t} (s - E[m(P_{t,t})(P_{t,t} - b)^{+}]Q + E[m(P_{t,t})(P_{t,t} - b)^{+}] - \beta )K.
\]

\(^5\)This problem is analyzed in detail in Levi, Kleindorfer and Wu (2001), and includes an extension of the WKZ framework to allow different production costs for the same Genco producing for the contract market and the spot market.
The bidding price $s(t)$ in equation (7) has two terms, in which the first term $E[(P_{T,t} - g(t))^+]$ is essentially the Black-Scholes-Merton (BSM, Black and Scholes 1973, Merton 1973) option value and the second term $E[(P_{T,t} - U(Q(t)))^+]$ is related to the willingness-to-pay of the Disco. The second term is the new feature of the real option we are studying. It shows that the price of the real option here is always smaller than the BSM option price $E[(P_{T,t} - g(t))^+]$. Note that this is the case notwithstanding the fact that in the classical BSM model the contract market is competitive, whereas here the Gencos have some market power. The reason that the reservation fee here is bounded above by the BSM option price is directly related to the fact that Buyers can also purchase from the spot market and this constrains the reservation price that can be supported in the market. It is straightforward to show that, as $t \to T$, the difference between the BSM and the option value here ($s(t)$) gets smaller and smaller, and eventually disappears at option maturity ($s(t) \to 0$).

A key companion result to the above Theorem 2 is that the standard non-arbitrage condition holds for Discos at the equilibrium contract $[s(t), b]$ so that Discos do not have any incentive to wait to purchase options. This continuous-time result extends previous results of Wu, Kleindorfer and Zhang (2001a, forthcoming), in the context of this paper, neither the Gencos have an incentive to buy excess options and sell these on the spot market on the day, nor do Discos have an incentive to sell short and cover their positions on the day through spot purchases. These results show that in our framework, at least for the Single Genco, the dynamic or continuous time case can be reduced to the static (two period) case. If this is true for the multiple Gencos case, then the significance of the results reported in this paper would pave the way for a general solution to the capacity options pricing problem for non-storable goods.

6.3 Future Research

The above characterizes long-run equilibrium in the usual “putty-putty” world of completely flexible capacity investments, or investments at least which could be evaluated and changed to any level before the fact. In many markets, capacity is a “putty-clay” investment, i.e., irreversible. In such markets, it would be interesting to characterize the long-run equilibria that would result if the only capacity choice options were complete withdrawal from the market or expansion of capacity.

An open question is the efficiency of a variation of the two-part tariff contract studied here, where the Gencos have to pay pre-specified fixed cost to reserve the transmission rights so that the contracted amount can be guaranteed to be delivered to the Discos on the day if the contract option is exercised. Along the same line, also interesting is what if the Gencos, like the Gencos, incur risk access factors $m_i(P_x)$ due to transmission congestions.

Finally, another interesting matter to study is the dynamics of commitment and profitability in those cases in which contract market equilibria do not exist because of the cycling phenomena illustrated in the above numerical example 2. For some initial results on this matter, see Wu and Sun (2002).

We are actively pursuing these research lines.

7. Appendix

Proof of Lemma 1

Proof: Take any capacity vector $K$ and let $\hat{M}(K)$ be the short-term equilibrium set (assuming it exists, and is non-singleton). Substituting $Q_k = D(\hat{p})K_k / X(M)$ into the profit function, we obtain the following expression for the profit function for Genco $k \in \hat{M}(K)$:

$$E \pi_k(\hat{p}, K) = (\hat{p} - c_k)D(\hat{p})K_k / X(M) + (c_k - \beta_k - G(b_k))K_k$$

The FOC condition for maximizing $E \pi_k(\hat{p}, K)$ gives

$$K_k^* = X(M) - (X(M))^2 \beta_k + G(b_k) - c_k$$

$$\frac{\hat{p} - c_k}{(\hat{p} - c_k)D(\hat{p})\frac{X(M)}{X(M)^3}} < 0.$$

Since $K_k^* = 0$. Hence the above solution is indeed optimal. Q.E.D.

Proof of Corollary 1

Proof: (a) First we show the claim is true in the singleton case when $|M^*| = 1$. Since $p^* = \arg\max p - \beta_1 - G(b_1)D(p)$, we have the FOC

$$(p^* - \beta_1 - G(b_1)) \frac{\partial D}{\partial K_1} + D = 0,$$

since $p^* = D^{-1}(K_1)$, we have

$$\frac{D}{\partial D/\partial K_1} = \beta_1 + G(b_1) - p^* = \beta_1 + G(b_1) - D^{-1}.$$ (8)
Proof: It is trivial for the case when \( |M^*| = 1 \). Suppose \( |M^*| > 1 \). We prove this in two steps: (a) Given any set \( M^* \subseteq \Xi \), \( p^* \) is unique; then we show (b) that \( M^*(p^*) \subseteq \Xi \), is unique.

First, we prove (a) is true. Take any subset of \( M^* \subseteq \Xi \). Assume (a) is not true, i.e., there are at least two pricing equilibria \( p_1^* \) and \( p_2^* \) corresponding to \( M^* \) satisfying (4) or (5). W.l.o.g. assume that \( p_2^* > p_1^* \geq \max\{c_i \mid i \in M^*\} \).

From (5) of Corollary 3, we know that

\[
\sum_{i \in M^*} \beta_i + G(b_i) - c_i = |M^*| - 1.
\]

However, since \( \forall i \in M^*, p_2^* > p_1^* > \max\{c_i \mid i \in M^*\} \) by assumption and \( \beta_i + G(b_i) - c_i > 0 \) by Corollary 2, we have

\[
\sum_{i \in M^*} \beta_i + G(b_i) - c_i > \sum_{i \in M^*} p_1^* - c_i = |M^*| - 1.
\]

This contradicts the assumption that \( p_1^* \) is an equilibrium since (11) is violated. Thus, we must have \( p_1^* = p_2^* \), as asserted in claim (a).

Second, we show (b) is true. First we note that from WKZ (2001c), for any equilibrium set (short-run or long-run) \( M^* \), if \( j \in M^* \) and \( c_i \leq c_j \), then \( i \in M^* \), so that any equilibrium set for the long-term contract market consists of Gencos with contiguous indices \( c_i \). Now assume there are two equilibrium sets \( M_1^* = \{1, \ldots, l\} \) and \( M_2^* = \{l + 1, \ldots, n\} \) with respective equilibrium prices \( p_1^*, p_2^* \). Moreover, \( p_1^* \leq c_{l+1} \) because otherwise \( l + 1 \) would have an incentive to participate in the contract market and \( M_1^* \) would not be an equilibrium set. From (5) of Corollary 3,

\[
\sum_{i \in M_1^*} \beta_i + G(b_i) - c_i = |M_1^*| - 1; \quad \sum_{k \in M_2^*} \beta_k + G(b_k) - c_k = |M_2^*| - 1.
\]

Subtract (13) from (12), and rearrange terms, we get

\[
\sum_{j \in M_1^*} \left( \beta_j + G(b_j) - c_j \right) \frac{p_1^* - p_2^*}{p_1^* - c_i} = \frac{\beta_k + G(b_k) - c_k}{p_2^* - c_k}.
\]

Since from Corollary 1, we know that \( \forall k \in M_2^* \setminus M_1^* \), \( p_2^* > \beta_k + G(b_k) > c_k \), we have

\[
\sum_{k \in M_2^* \setminus M_1^*} \frac{\beta_k + G(b_k) - c_k}{p_2^* - c_k} < |M_2^*| - |M_1^*|.
\]

or

\[
|M_2^*| - |M_1^*| - \sum_{k \in M_2^* \setminus M_1^*} \frac{\beta_k + G(b_k) - c_k}{p_2^* - c_k} > 0.
\]
Thus the LHS of (14) must be positive, i.e.,
\[
\sum_{i \in M^*_1} \left( \beta_i + G(b_i) - c_i \right) \frac{p_1^i - p_2^i}{(p_1^i - c_i)(p_2^i - c_i)} = (p_1^1 - p_2^1) \sum_{i \in M^*_1} \frac{\beta_i + G(b_i) - c_i}{(p_1^i - c_i)(p_2^i - c_i)} > 0
\]

Since \( \forall i \in M^*_1, \beta_i + G(b_i) - c_i > 0 \) and \( p_2^i - c_i > 0 \), also \( p_1^i + \beta_i < c_i \), the above inequality implies \( p_2^i < p_1^i \). Thus contradicts the fact that Genco \( l + 1 \) is a member of \( M_2^* \), so that claim (b) holds. Coupling (a) and (b), we have the uniqueness of \( M^*(p^*) \). Q.E.D.

**Proof of Corollary 5**

**Proof:** From Corollary 2, we know that for any Genco \( k \in M^* \), \( \beta_k + G(b_k) > c_k \), since by definition, \( c_k = s_k + G(b_k) \), we have \( \beta_k > s_k \), thus claim (i) holds.

Claim (ii) holds, since for any Genco \( k \in \Xi \setminus M^* \), the necessary and sufficient condition for \( k \) to participate in the spot market is \( p^* < c_k \), \( \beta_k + G(b_k) \). Claim (iii) holds since for any Genco \( k \in \Xi \setminus M^* \), if \( p^* < c_k \) and \( s_k \leq \beta_k \), thus \( p^* < c_k = s_k + G(b_k) \leq \beta_k + G(b_k) \), then \( k \) can make money neither in the contract market nor in the spot market; \( k \) is better off by closing its business. Q.E.D.

**References**


