Forward Vs. Spot Buying of Information Goods on Web: Analyzing the Consumer Decision Process

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Abstract

Several information goods, such as movie distribution rights or newspapers, are sold either at spot prices, or through forward subscription buying. Our paper considers a firm that offers an information good through spot buying, or through forward buying at a reduced price, or a combination of the two. We propose a consumer decision-making model that captures the market reaction to such an offering and provide interesting insights into the key elements of the consumer behavior. We show how firms selling information goods can increase their revenues by using a mixed offering strategy using both spot and forward offerings. This strategy may provide an effective price discrimination setting when there are potentially heterogeneous groups of customers in terms of their reservation prices. We also formulate the firm’s problem when it is either a price taker or a price setter.

1. Introduction

In several industries, a price discount is given for advanced purchase of goods. One example is airline tickets where advance purchase, typically before two weeks prior to start of travel, qualifies to a significant reduction in price. In this way, airlines are able to price differentiate between passengers who value price over flexibility while still extracting a high price from business travelers requiring high flexibility on short notice. Maxim’s bakery in Hong Kong offers a 25% price discount to customers ordering cakes for the mid-Autumn festival at least one month prior to delivery (Tang, Rajaram and Ou, 1999). American Adventures offered discounts up to 30% for early booking of travels for the 1995 season. Tickets to sporting events can be purchased in advance at a lower price than at the day of the event. We call this advanced purchase as forward buying and purchase close to delivery of a good as spot buying.

In this paper, we study this phenomenon for information goods on the Web. Digitized information goods deviate from physical goods mainly by having close to zero marginal cost and no supply limit. The latter implies that while sales for a physical good is limited by both demand and supply, the sales of an information good is only limited by the demand.

A specific case of forward buying is the subscription of a bundle of information goods. Newspapers and magazines typically offer significant savings for subscriptions, typically for periods between 3 months and 2 years. This fits within the framework of forward buying with the consumer committing to several issues of an information good at a lower price per issue vs. having the flexibility to buy any issue on spot. The Wall Street Journal offers both one-year and daily subscriptions to its online edition.

A consumer offered the choice between forward buying and waiting for buying the good (possibly) on spot at a higher price faces the tradeoff between a lower unit price and the value of updated preferences. We propose a consumer model that takes this into account, and outlines how the consumer model can be incorporated into the firm’s decision in its offering. For the case of the firm being a price taker, the choice is whether to offer the good as forward buying, spot buying, or a combination of the two. For the case of the firm being a price setter, the choice is how to price the two offerings.

2. Literature Review
Yield management literature incorporates the idea of forward buying at a discounted price but does not explicitly model consumer behavior as we do here. It is commonly assumed that the aggregate demand at each price level is a Poisson process whose intensity is constant.

While several studies have looked at the issues of information goods subscription and bundling, the issues of spot versus forward buying on information goods as such has not been addressed to the best of our knowledge. Dudley (1993) addresses the question of how magazines decide on the level of discount to offer to customers for a second year subscription in a two-year setting. Few other articles in the IS economics literature address some sorts of bundling where complementary goods were sold together as away to decrease the variation across customers in their willingness to pay. These include Bakos and Brynolfsson (1997), Chuang and Sirbu, (1997), and Varian (1995).

3. Modeling Consumer Behavior

For simplicity of presentation, we model forward and spot buying in a single period framework. We consider a single seller of an information good. The seller offers the spot good at unit price \( p \). At the time of offering of the spot good, each consumer \( i \) will realize her reservation price, \( r_i \). If and only if her reservation price exceeds the unit price of the information good, i.e. \( r_i > p \), this consumer will decide to buy the good on spot. The surplus of consumer \( i \) will then be \( r_i - p \) in the case of buying the spot good and 0 otherwise. Define

\[
x^+ = \begin{cases} x & \text{if } x \geq 0, \\ 0 & \text{otherwise.} \end{cases}
\]

Consumer \( i \)'s surplus for spot buying can then be expressed as \( (r_i - p)^+ \).

Prior to realizing her reservation price, consumer \( i \) will have some belief on the realization of her reservation price given in terms of a known probability distribution. Hence, prior to observing her reservation price, consumer \( i \)'s expected surplus for spot buying is \( E(r_i - p)^+ \).

We now turn to forward buying of the information good. The seller offers a price discount \( \alpha \) for forward buying. Typically, a consumer also derives value (incurs cost) owing to the convenience (inconvenience) of forward buying over spot buying, expressed as a fraction of the selling price given by \( \beta \) (which in some cases might be negative). The experienced price discount (consisting of both the direct price discount and the convenience benefit) to the consumer from forward buying is then given by \( z \) where \( z = \alpha + \beta \). We assume that \( z \in (0,1) \). The expected surplus of consumer \( i \) from forward buying is then given by \( E(r_i - (1-z)p) \). Note that as a result of the uncertainty in reservation price, the realized (as well as the expected) surplus of the consumer when forward buying might be negative. When choosing between forward and spot buying, a rational consumer is facing the tradeoff between price and information uncertainty. This decision is illustrated in the following figure:

![Decision Tree](image)

At the first epoch, Time 0, the consumer decides whether or not to forward buy the good prior to observing her reservation price. At the second epoch, Time 1, the consumer might spot buy the good if she has not already committed to the good by forward buying.

Note that at Time 0 the choice of waiting and buying the good on spot at Time 1 only if \( r-p \) is positive is like having a call option with strike price \( p \) and stock price at the exercise date \( r \).

Let \( I_i \) be the indicator value for forward buying of consumer \( i \). Formally,

\[
I_i = \begin{cases} 1 & \text{if consumer } i \text{ forward buy,} \\ 0 & \text{otherwise.} \end{cases}
\]

Similarly, let \( J_i \) be the indicator value for spot buying of consumer \( i \). Formally,

\[
J_i = \begin{cases} 1 & \text{if consumer } i \text{ spot buy,} \\ 0 & \text{otherwise.} \end{cases}
\]
3.1. Consumer Decision

When only forward buying is available to the consumers, then \( I_i = 1 \) if and only if \( E(r_i - (1 - z)p) > 0 \). Similarly, when only spot buying is available, then \( J_i = 1 \) if and only if \( r_i > p \). We then turn to the situation when both forward and spot buying are available to the consumers.

Since a consumer will never both buy forward and spot, we have that \( I_i \in \{0, 1\} \) and \( J_i \in \{0, 1\} \setminus (I_i \cap \{1\}) \). The decision problem of a rational consumer maximizing her expected surplus can then be expressed as the following simple two-stage stochastic dynamic program:

\[
\max_{I_i \in \{0, 1\}} E \left[ I_i (r_i - (1 - z)p) + \max_{J_i \in \{0, 1\} \setminus (I_i \cap \{1\})} J_i (r_i - p) \right]
\]

Since the optimal second stage decision is trivially given by

\[
J_i = \begin{cases} 1 & \text{if } r_i \geq p \\ 0 & \text{otherwise} \end{cases}
\]

we can reformulate the above program as the following one stage program:

\[
\max_{I_i \in \{0, 1\}} E \left[ I_i (r_i - (1 - z)p) + (1 - I_i)(r_i - p)^+ \right]
\]

Define \( \Delta_i \) as the difference between forward buying and not forward buying (i.e. having opening to spot buying) in expectation:

\[
\Delta_i = E_{r_i} [(r_i - (1 - z)p) - (r_i - p)^+] .
\]

This can be rewritten as

\[
\Delta_i = E_{r_i} [(r_i - p)^+ - (p - r_i)^+ + zp - (r_i - p)^+] .
\]

Canceling equal terms gives

\[
\Delta_i = zp - E_{r_i} (p - r_i)^+ ,
\]

which completes the proof. QED

4. Modeling Consumer Classes

Consumers within a population will vary in characteristics in terms of the distribution function of \( r_i \) denoting their belief as well as the parameters within the distribution function. In our analysis, we limit these distributions to the ones that can be completely characterized by the two parameters mean \( \mu \) and standard deviation \( \sigma \).

Let the population consist of \( C \) classes of consumers indexed by \( c=1,...,C \) where each class specifies a probability distribution and standard deviation \( \sigma_c \). Consumers within each class might have different expectations \( \mu_i \) following some distribution function. Suppose that any such arbitrary distribution \( f \) has support in the interval \([l, h]\). We make a non-restrictive assumption that for any \( a \in (l, h) \), \((a - l)f(t) \leq \int_{l}^{h} f(x)dx \), which, in words, means that the support at the lowest possible realization is no more than the average support up to an arbitrary realization. Distributions that satisfy this assumption include the family of symmetric unimodal distributions, e.g. the Normal and the Uniform distributions.

To motivate consumer behavior under this classification, consider the following simplified example for consumer demand of tickets for the Rhinos soccer team. One class consists of consumers who are ardent fans but do not have much spending power. Their reservation price is $10 if the weather is not nice and $12 if the weather is nice. The other class consists of consumers who are not ardent fans but have larger spending power. Their reservation price is $0 if the weather is not nice and $20 if the weather is nice. If the probability of nice weather...
is ½, then the first class of consumers can be captured with any price of forward buying of less than $11, while the second class can be captured with probability ½ with spot buying price of less than $20. Note that this would be a very efficient way to price differentiate between the two consumer groups – one which emphasizes on expectation, the other having emphasis on information uncertainty.

### 4.1. Attractiveness of Forward Buying

We now formally analyze behavior of the consumer population, since the aggregate behavior is what is relevant for the firm when choosing its sales policy (see section 5). We first establish some very useful properties of consumer behavior with respect to the distribution parameters. Later, we do a comparative analysis of the expected consumer surplus from alternate sales policies.

**Proposition 1**

For a consumer $i$ in an arbitrary class, we have that

$$\frac{\partial \Delta_i}{\partial \mu_i} \geq 0$$

which means that the attractiveness of forward buying is increasing in $\mu_i$.

**Proof:** Let $Y_i = E_i \left( p - r_i \right)^+$. It is sufficient to show that

$$\frac{\partial Y_i}{\partial \mu_i} \geq 0$$

Let $f(r)$ be the density function and $g(r, \mu) = f(r)$. Then,

$$Y = \int_{I} (p - r) f(r) dr$$

$$= \int_{I} (p - r) g(r, \mu) dr = \int_{I} (p - r - \mu) g(z) dz.$$

Differentiation wrt $\mu$ gives

$$\frac{\partial Y}{\partial \mu} = - \int_{I} g(z) dz + (p - l) g(l - \mu)$$

$$= \int_{I} (g(l - \mu) - g(z)) dz$$

By the assumption on the probability distribution, this completes the proof. QED

**Proposition 2**

For a consumer $i$ in an arbitrary class $c$ there exists a unique $\bar{\mu}_c$ such that the following consumer behavior is optimal in the first epoch:

$$I_i = \begin{cases} 
1 & \text{if } \mu_i \geq \bar{\mu}_c \\
0 & \text{otherwise}
\end{cases}$$

Then, $\bar{\mu}_c$ is the solution to $zp = E_i \left( p - r_i \right)^+$ with respect to $\mu_c$.

**Proof:** Existence follows from continuity and the facts that $\lim_{\mu_i \to -\infty} \Delta_i = -\infty$ and $\lim_{\mu_i \to -\infty} \Delta_i = zp > 0$, and uniqueness follows from Proposition 1. The solution for critical mean value is then immediate. QED

**Proposition 3**

For a consumer $i$ in an arbitrary class that follows either a normal or uniform distribution, we have that

$$\frac{\partial \Delta_i}{\partial \sigma_c} \leq 0$$

which means that the attractiveness of forward buying is decreasing in $\sigma_c$.

**Proof:** It is sufficient to show that

$$\frac{\partial Y_i}{\partial \sigma_c} \geq 0.$$

**Normal**

We have the following expression:

$$Y_i = \int_{-\infty}^{p} (p - r_i) f(r_i) dr_i = \sigma_c \int_{-\infty}^{p} \left( \frac{p - H_i - x}{\sigma_c} \right) \phi(x) dx$$

By Leibnitz’s rule, we have that
\[
\frac{\partial Y_i}{\partial \sigma_c} = \int_{-\infty}^{\infty} \left( \frac{p - \mu_i}{\sigma_c} - x \right) \phi(x) \, dx
\]
\[
\quad - \frac{p - \mu_i}{\sigma_c^2} \left( \frac{p - \mu_i}{\sigma_c} - x \right) \phi \left( \frac{p - \mu_i}{\sigma_c} - x \right)
\]
\[
+ \sigma_c \left[ -0 \cdot \lim_{x \to -\infty} \left( \frac{p - \mu_i}{\sigma_c} - x \right) \phi \left( \frac{p - \mu_i}{\sigma_c} - x \right) \right]
\]
\[
= - \int_{-\infty}^{\infty} \phi(x) \, dx \geq 0,
\]

which completes the proof for the case of normal distribution.

**Uniform**

Let the reservation price for consumer \( i \) be uniformly distributed \( r_i \sim U(a_i, b_i) \). Then,

\[
Y_i = \frac{(p - a_i)^2}{2(b_i - a_i)}
\]

Denote the mean as \( \mu_i \) and the standard deviation as \( \sigma_i \), so that \( b_i = \mu_i + 3^{1/2} \sigma_i \) and \( a_i = \mu_i - 3^{1/2} \sigma_i \).

To see how \( Y_i \) varies with \( \sigma_i \), consider the following expression:

\[
Y_i = \frac{(p - \mu_i + \sqrt{3} \sigma_i)^2}{2 \sigma_i}
\]

Differentiation wrt \( \sigma_i \) yields

\[
\frac{\partial Y_i}{\partial \sigma_i} = \frac{(p - \mu_i + \sqrt{3} \sigma_i)(\mu_i + \sqrt{3} \sigma_i - p)}{2 \sigma_i^2}
\]
\[
= \frac{(p - a_i)(b_i - p)}{2 \sigma_i^2}
\]

Since \( a < p < b \) (price has to be greater than the lowest reservation price and lesser than the highest reservation price), this expression is positive, which completes the proof for the uniform case. QED

Given our earlier comparison that at Time 0 the choice of waiting and buying the good on spot only if \( r - p \) is positive is like having a call option, the above result can also be explained as follows. With increase in \( \sigma \), option theory suggests that the expected value of this option may be expected to increase. But the expected value of forward buy does not change with \( \sigma \). Thus, with increase in \( \sigma \), the attractiveness of forward buy decreases.

### 4.2. Comparative Analysis of Consumer Surplus from Alternate Sales Policies

We then move to making a comparative analysis of the expected surplus of consumers within an arbitrary class. Define \( S_c^* \) as the surplus of consumers within class \( c \) when both forward and spot buying are available. We have

\[
S_c^* = \begin{cases} 
    r - (1 - z) p & \text{if } \mu \geq \mu_c, \\
    (r - p)^+ & \text{otherwise.}
\end{cases}
\]

Similarly, define \( S_c^f \) as the surplus of consumers within class \( c \) when only forward buying is offered, and \( S_c^s \) as the surplus of consumers within class \( c \) when only spot buying is offered. We have

\[
S_c^f = \begin{cases} 
    r - (1 - z) p & \text{if } \mu \geq (1 - z) p, \\
    0 & \text{otherwise,}
\end{cases}
\]

and

\[
S_c^s = (r - p)^+.
\]

The expected surplus of a consumer in class \( c \) taken over the possible \( \mu \)'s is then

\[
E_{\mu} S_c^* = \Pr(\mu < \mu_c) E_{\mu|\mu < \mu_c} [(r - p)^+] + \Pr(\mu > \mu_c) E_{\mu|\mu > \mu_c} [r - (1 - z) p]
\]

By manipulation of this expression, we can isolate the expected consumer surplus over consumers within class \( c \) for the cases when either only forward buying or only spot buying is offered. For instance, we can rewrite...
The first term in this expression is the expected surplus of consumers in class $c$ when offered only the forward buying while the second term is the additional surplus for consumers in class $c$ due to the offering of spot buying in addition to forward buying.

Similarly, we can rewrite the expression as

$$E\mu S^*_c = Pr(\mu < \bar{\mu}_c)E_{\mu|\mu < \bar{\mu}_c} E_{r|\mu}[r-p^+ - (r-(1-\zeta)p)]$$

$$+ E\mu E_{r|\mu}[r-(1-\zeta)p]$$

$$= Pr(\mu > \bar{\mu}_c)E_{\mu|\mu > \bar{\mu}_c} E_{r|\mu}[(p-r)^+ - \zeta p]$$

$$+ E\mu E_{r|\mu}[r-(1-\zeta)p]$$

$$= E\mu E_{r|\mu} S^*_c + Pr(\mu > \bar{\mu}_c)E_{\mu|\mu > \bar{\mu}_c} E_{r|\mu} [\Delta]$$

The first term is the expected surplus of consumers in class $c$ when offered only the spot buying while the second term is the additional surplus for consumers in class $c$ due to the offering of forward buying in addition to spot buying.

The expected surplus per consumer over all consumer classes in the population is given by

$$ES^* = \sum_{x=1}^C Pr(c=x)E\mu S^*_x.$$

This completes the modeling and analysis of consumer behavior. We believe that it has the potential of several interesting applications.

5. The Seller’s Problem

We will here formulate a firm’s problem involving policy decisions for selling information goods, the analysis of which is work in progress. We consider two scenarios. In the first scenario, the firm is a price taker and has to choose whether to offer the consumers only forward buying, only spot buying, or a combination of the two. In the second scenario, the firm is a price setter and sets the unit revenue for spot buying as well as the discount for forward buying in order to maximize its expected profit.

We will here use the expected profit function for the firm. When the firm only offers forward buying, its expected profit is given by

$$\Pi^f = n \sum_{x=1}^C Pr(c=x) (1-\zeta)p Pr(\mu_c > (1-\zeta)p)$$

where $n$ is the total number of consumers in the population.

When the firm only offers spot buying, its expected profit is given by

$$\Pi^s = n \sum_{x=1}^C Pr(c=x) pE_{\mu} Pr(\mu_c < \bar{\mu}_c, r_x > p).$$

When the firm offers both forward and spot buying, Proposition 2 gives the expected profit by

$$\Pi^c = n \sum_{x=1}^C Pr(c=x) \left[(1-\zeta)p Pr(\mu_c > \bar{\mu}_c) + pE_{\mu} Pr(\mu_c < \bar{\mu}_c, r_x > p)\right].$$

5.1. The Firm is a Price Taker

Here the firm has three binary decisions, denoted by the decision vector $K=(k_1, k_2, k_3)$, where the restrictions $k_j \in \{0,1\}$ for $j=1,2,3$ and $k_1+k_2+k_3=1$ applies. The three possible decision are then: offer both forward and spot buying ($k_1=1$), offer only forward buying ($k_2=1$), and offer only spot buying ($k_3=1$). The decision problem then becomes

$$\max_k k_1\Pi^c + k_2\Pi^f + k_3\Pi^s.$$
6. Contribution

We develop a model of consumer behavior towards forward and spot buying and the combination of the two. This model allows several insights into the behavior of rational consumers and forms the potential of interesting applications. We formulate the problem of optimal selling and offering of information goods, both for the firm as price taker and price setter.

Future work includes analyzing the firm’s decision making for the two cases presented. We are also looking at extending the analysis to incorporate bundling of several information goods. It would also be interesting to reflect supply constraints in the same framework, and in this way develop a common platform to analyze forward and spot buying of both information and physical goods.

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7. References


