In this paper, we study the bit allocation problem under multiple rate constraints. This problem has been arisen in many practical situations, such as optimal image and video quantization, buffer-constrained multimedia transmission, joint source channel coding and unequal error protection. One example is bit allocation for image compression, where the goal is to minimize the total distortion by properly allocating quantization levels for the various blocks with a specified total rate budget constraint. Another example is that buffer constraint is introduced in the bit allocation problem, with one specific situation that a source encoded using variable rate coding is to be transmitted through a constant bit rate channel, and the source bits are buffer prior to transmission. To solve the bit allocation problem under both total rate budget and maximum buffer size constraints, Lagrangian-based iterative technique can be used. Since the bit allocation problems under rate multiple rate constraints are usually equivalent to Knapsack problems, which is NP-hard in general, without having good properties, the optimal solution can not be guaranteed by any polynomial time algorithms.

In this paper, we re-examined the bit allocation problem under multiple rate constraints, and propose an efficient algorithm based on marginal analysis to solve the rate-distortion optimization problem under both rate and buffer constraints. Compared with the Lagrangian-based iterative method proposed by Ortega, which can only eliminate buffer overflow, the proposed marginal analysis based method can eliminate both buffer overflow and underflow. In general situations, the proposed algorithm can give good approximation. Furthermore, by assuming that the rate increase between different schemes is unitary, such as in the scalable compression scenario, the solution obtained by the proposed bit allocation algorithm can be proved to be optimal. In this paper, we have also addressed efficient implementation issues of the MA-based method. Based on the heap data structure and priority queue, a low time complexity implementation has been achieved. Let \( N \) be the number of blocks to be optimized independently, and \( M \) the number of available schemes for each block, then the overall time complexity is up-bounded by \( O(NM\log N) \). The paper has used optimal quantization and joint source channel coding based rate allocation as examples, but extension to other scenarios is straight-forward.