precise time on it. That was our first venture. I would say that we were probably very, very slow. We were feeling our way along and any figure I give to you now would perhaps have no meaning a year from today.

William Michle (Burroughs Corporation): In the abstract of the paper in the program, it says, "Analysis . . . illustrates the percentage use of various basic functions of arithmetic and logical decisions." Please explain this.

J. M. Boermeester: Dr. Petrie, maybe you know something about this.

G. W. Petrie, III: Originally it was hoped that in the presentation we would have a very comprehensive system of evaluation as to the percentage breakdowns on the arithmetic instructions and logical decisions and more statistical data. This paper is in the process but has not been completed. Instead of presenting partial results we felt that all of you would be much happier to hear of one actual case in complete detail such as the one that has just been presented to you.

M. Saslow (Airborne Instruments): What are the estimated dollar savings to be gained by this installation of the 650?

J. M. Boermeester: This is a question on which of course nobody expects a precise answer from me. However, I would say that there are other elements in here, questions of administration which have not been analyzed and the question of speedup in time on which we have not made any precise estimate.

L. Flynn (Curtis Publishing Company): The calculation time of 1 second per dividend, does this mean 1,000,000 seconds for 1,000,000 policyholders?

J. M. Boermeester: When I went to school I X 10^6 = 10^6, yes. This time is apart from emergency breakdowns; it does not include time for downtime.

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Small Digital Computers and Automatic Optical Design

N. A. FINKELSTEIN

THE photograph reproduced in Fig. 1 is an example of good optical imagery. The picture is crisp and considerable detail is resolved all over the area, even at the extreme edges and corners. Fig. 2 is a poor picture. The reasons are obvious. While the central region is still sharp and full of detail, the rest of the area is fuzzy and ill-defined. The quality definitely deteriorates as we move further and further out from the center. These two photographs were taken with the same camera, at the same exposure, under the same subject conditions with two different lenses of the same focal length. Fig. 3 shows the lenses in schematic form, essentially a section through the lens along the axis. The poor picture was taken with the upper lens, a simple biconvex element. The good picture was taken with the lower lens, a very-well-known design called a Tessar, containing four elements, two of which are cemented together. The difference between the two pictures shown obviously is related to the difference between the two lenses which took them. The techniques which lead us from poor picture to good picture, from simple biconvex lens to multielement Tessar, form the province of optical lens design.

The problem of the lens designer is to combine elements of different curvature, thickness, and refractive index in such a manner as to approach perfect imagery of the class of objects to be placed before the lens. In perfect imagery each point in the object is transformed into a corresponding point in the image without distortion or blurring; of course this condition can only be approached.

Probably the most important tool in lens design is geometrical ray tracing, a technique in which the paths of light rays emanating from a point in the object are traced through the several lens elements following the laws of geometrical optics to ascertain the manner in which these rays recombine in the image. In the ideal lens all the rays from each point of the object would recombine at corresponding points in the image as shown in Fig. 4. In general, the rays will not recombine as

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shown in this illustration and the resultant image is then said to contain aberrations. One type of aberration, called spherical aberration, is illustrated in Fig. 5; it is that characteristic of a spherical lens surface which brings rays passing through different annular zones of the surface to focus at different points along the lens axis. The aberrations of a lens are usually grouped into seven categories; five pertain to monochromatic imagery and two must be added when polychromatic radiation is passed through the lens. They are: spherical aberration; coma; astigmatism; curvature of field; distortion; longitudinal color; and lateral color.

There are many approaches to the lens design problem. One of the most powerful is that in which the design is obtained through a series of approximations to the trigonometric-ray-tracing equations. At an interface between any two refracting media the following equation, Snell's Law, holds:

\[ n \sin i = n' \sin r \]

where \( n \) and \( n' \) are constants of the two media, called indexes of refraction, and \( i \) and \( r \) are, respectively, the angle of incidence and the angle of refraction at the interface as shown in Fig. 6. If we use only the first terms in the series expansion for the sine function, we have for Snell's Law:

\[ ni = n'r \]

an approximation which is valid only for very small values of \( i \) and \( r \). Using this so-called "first-order" theory, one can determine the power (inverse focal length) and location for each of the several elements required to satisfy the conditions of object and image position and magnification, assuming no aberrations. When this first-order solution is obtained, the next step is to calculate the "third-order" aberrations of the system, aberrations calculated under the approximation:

\[ \sin X = X - \frac{X^3}{3!} \]

The usefulness of this approximation lies in the fact that the third-order aberrations (or primary aberrations) can be expressed as explicit quadratic functions of the lens constants; hence, one can solve for these constants as a function of the aberration limitations imposed on the image. It would be ideal, of course, if one could write an analogous set of equations for the exact aberrations, but this is not practical because the lens constants would not appear explicitly, and because the equations would be of very high order and would not be closed. Since the third-order solution uses the first two terms of the sine expansion it yields almost perfect results for a lens of moderate field and aperture and serves as a very useful guide in other cases. Part of the utility of this approach is in its determination of the contributions of each surface of a multielement lens to each of the lens aberrations, thereby pin-pointing the areas of weakness in the design. The final stage of the design process is accomplished with exact ray tracing through the third-order solution to find the exact aberrations and to make the necessary changes to reduce the higher order aberration effects. This step will vary from a very simple one in a narrow-angle lens, such as a telescope, to a lengthy and complex one in a wide-angle lens, such as the type used in aerial photography.

In understanding the foregoing approach to lens design it is important to note that each step is a necessary, but not a sufficient, condition for the following step. That is, the existence of a first-order solution by no means implies the existence of a third-order solution; and the existence of a third-order solution does not guarantee that there is an exact solution. What is true is the converse; if no third-order solution can be found, then no exact solution exists. This consideration is most important in the first-order design, since only a limited class of the infinite first-order solutions possible will yield a third-order solution. Thus, the designer must keep in mind the primary aberration requirements when laying out his first-order calculation if he is to avoid a mass of extraneous solutions; and he must, in a similar manner, be mindful of the effects of higher order aberrations in his third-order work. It is in this respect that previous experience and knowledge of the field play an important part in determining the efficiency of a designer.

Here we come to the major problem of lens design, that of relating the actual image quality obtained with a lens to the calculated aberrations. While the practice of defining five monochromatic and two chromatic aberrations is mathematically appealing as well as convenient to the designer, the final criterion of excellence in a lens is not the amount of spherical aberration, or coma, or astigmatism, but the quality of image resultant from the combination and interaction of these aberrations. The problem is compounded by the experimental fact that a lens which has satisfactory image quality for one class of objects may produce totally unsatisfactory quality for another class. One can show the superiority of lens A to lens B for a test object of black lines on a white background and then completely reverse the judgment for a test object of white lines on a black background. Of recent years there have been many criteria proposed for image evaluation, all having their merits and demerits, none completely accepted by even a majority of

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Fig. 3. Optical schematics of simple biconvex lens and Tessar lens

Fig. 4. Ray traces illustrating perfect imagery

Fig. 5. Ray traces illustrating spherical aberration

Fig. 6. Snell's law of refraction at a surface
optical workers. It is not within the scope of this paper to discuss the many criteria proposed, but some manner of explanation, if not justification, is called for in introducing the criterion of image quality to be used in our proposed system for automatic design, the contrast rendition criterion.

In the recognition contrast rendition apparatus (RCR) 1 a test object is used which is a series of slits separated by opaque regions, the width of successive slits and opaque bands decreasing according to a power series (Fig. 7). This test object moves transverse to the optical axis of a telescope system that images the axial portion of the object onto a slit behind which a photoelectric cell is located (Fig. 8). The telescope system is an aberration synthesizer and can be made to introduce calibrated amounts of each of the primary lens aberrations (Fig. 9). If the output of the photoelectric cell is fed to a strip recorder, then a curve similar to that shown in Fig. 10 is obtained as the test object is moved across the optical axis. As the spacing between slits becomes smaller and smaller, the aberrations introduced by the aberration synthesizer tend to blur together the bright and dark bands in the image until a point is reached at which the bands are no longer resolved and the recorded oscillations cease. The quantity RCR is defined as

$$RCR = \frac{B_{\text{max}} - B_{\text{min}}}{B_{\text{max}}} \times 100 \text{ per cent}$$

where $B_{\text{max}}$ is the average brightness of two consecutive slit images and $B_{\text{min}}$ is the brightness of the image of the dark band between them. Thus, there is an RCR value corresponding to each pair of consecutive slits in the test object. When the peak to trough amplitude of the recorded oscillations becomes zero, $B_{\text{max}}$, equals $B_{\text{min}}$ and the RCR is zero. The image quality factor $F$ is measured as the integrated area within the envelope of the recorded oscillatory curve.

It is possible, then, with this apparatus to synthesize any combination of the primary aberrations one wishes to study in a lens and to obtain a single quantity $F$ which is representative of the image quality to be expected from a lens system having these aberrations. If this apparatus is combined with high-speed computing equipment programmed to compute the lens aberrations for a lens whose curves, thicknesses, and indexes are fed into it, then we have the basis for an automatic calculation which proceeds as follows: the designer selects the form of the optical system and specifies nominal values for the curves, thicknesses, and indexes. With these data, the equipment performs an iterative process of successively making a small change in one of the lens parameters, computing the aberrations, setting these aberrations into the synthesizer, and measuring the quality factor $F$. If for the $n$th change $F_n > F_{n-1}$, then one makes the next step a larger change in the same direction. If $F_n < F_{n-1}$, then the $(n + 1)^{\text{th}}$ must be in the opposite direction. This iteration is continued for variations in each parameter until the image quality is maximized. Fig. 11 shows schematically the equipment used in this scheme. The electronic calculator is an International Business Machines Corporation (IBM) type 607 and the card punch and tabulator are also standard IBM units.

Our present plan is to work with third-order aberrations, because they are easier to calculate than the exact aberrations, and because we know how to synthesize them in the relatively simple arrangement shown in Fig. 9. Assuming that it is possible to design a synthesizer to include the effects of the higher order aberrations, this same approach could be used to reduce the exact total aberrations of a lens. It should be noted that in order to start the iterative scheme described one must have already arrived at the general form of the lens to be used. That is, the scheme is useful only for relatively small changes in a form. While this is far from the goal of completely automatic design it will provide us with a means for eliminating a large portion of the lens designer’s work, the tedious and time-consuming task of reducing the aberrations of a lens to the point of acceptability.

It would appear that the logical extension of this work would be to perform the entire design process automatically. While a scientific worker is seldom on safe ground when he dubs a task impossible, one can certainly say that the prospects for achieving this goal are quite discouraging. One way we might attack the problem is to define a criterion of image quality at each point of the field to be covered by a lens, $F(x,y)$, or $F(\theta, \phi)$ if we chose to work with angular field. The nature of this $F(x,y)$ or $F(\theta, \phi)$ will vary from application to application. For example, the limitation on distortion is much more severe in an aerial mapping lens than in a motion picture camera lens; and spherical aberration must be extremely well corrected in a microscope objective, while a certain amount of this aberration is sometimes desirable in a por-

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*Fig. 7. Test target used in RCR apparatus*

*Fig. 8. Schematic of RCR apparatus with aberration synthesizer*

*Fig. 9. Schematic of aberration synthesizer*

*Fig. 10. Typical curve recorded by RCR apparatus*
trait or landscape lens. Having defined the \( F \) function, the problem would be reduced to finding a lens having image quality at least as good as \( F \) at each point in the field. Even to begin to handle this problem, we must relate the quantity \( F \) to the lens constants of thicknesses, surface curves, and indexes; and this must be done by establishing some system such as the RCR apparatus with its aberration synthesizer working in reverse. That is, we must find a mechanism for going from each \( F \) to the aberrations which produce it, and thence to the lens constants. We know no way to do this, yet; and even if we did, we know of no easy way to get from the exact aberrations to the lens constants, since this would involve solving an open system of higher order equations. What would have to be done instead is to calculate the exact aberrations for all points of the field to be covered and for all possible lenses. Having done this we would have a multitude of lenses from which we would have to "sort out" those which satisfied the restrictions on \( F(x,y) \) or \( F(\theta, \phi) \). This is not an encouraging approach even for high-speed digital computers, small or large.

The approach that makes more sense to many of us is that of breaking the problem down into two phases: choice of a general lens form and optimization of that form. The first phase involves the decision to use a doublet or a triplet; a Petzval lens or a Tessar. It is now based on the basis of past experience and general knowledge of the designer; but we can well imagine an experience box of some kind into which we fed such information as focal length, total angular field, \( f/ \) number, and average resolving power, and out of which came recommendations for lens forms worth investigating in detail. The second phase is the one which we would like to attack with our 607 calculator, aberration synthesizer, and RCR apparatus; even the modest attempt we are making to solve the problem for third-order aberrations is fraught with the difficulties of having to make many arbitrary assumptions which can only be partially justified.

Probably the best way to sum up all that has been discussed is to admit that the physics and mathematics of lens design have not been developed to the point where we even could say what we would like a calculating machine to do in order to make completely automatic optical design possible. Until we can solve such basic problems as that of image evaluation we can only continue as we are now doing, making assumptions and attacking pieces of the job with the hope that we may discover how to reformulate the problem so that it may be solved in its entirety.

**Reference**


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**Discussion**

Gordon L. Walker (American Optical): What do you mean by "exact aberrations"?

N. A. Finkelstein: What we mean there is the aberrations calculated by exact ray tracing through the lens system; in other words, by tracing rays through each surface using the full series in the sine expression, the exact ray trace. From that we calculate the exact aberration.

Mr. Casey (General Electric Company): Have you run into any problems of divergence in the iterative machine procedure you described?

N. A. Finkelstein: I would like to point out that we have not yet completed this iterative scheme. What we have now is the RCR apparatus and a method of calculating the third-order aberrations on the IBM 607. I think I can answer your question in the light of the work that has been done by others on the CPC. I believe Dr. Hopkins, who is the University of Rochester, has used the CPC at Cornell University's calculating laboratory actually to iterate third-order aberrations. I think he has found that in most cases the iteration does converge and in the cases that it diverges he usually knows what is wrong.

One of the interesting characteristics of lens design I will point out here is that a good lens designer usually knows what is going to happen before it happens. One of the rules to which we like to adhere—and Dr. Hopkins I know feels the same way—is that you should never trace a ray through a lens without knowing where it is going to land before you traced it. You should always have a pretty good idea of what is happening in the lens system because, since they are nonlinear systems, they tend to get out of hand very quickly and unless you keep very close to the right solution, you lose it entirely. Ordinarily, Dr. Hopkins has found that when he does use an iteration process to solve third-order aberrations, when it does diverge he knows why and it is very easy to remedy. We have not had any personal experience with that phase of the problem.

Franz Edelman (Radio Corporation of America): Could you amplify again briefly your remarks concerning the flow of information in the system? Has anything been done to apply your ideas to problems of continuously varying refractive index (electron optical systems)?

N. A. Finkelstein: I would like to answer the second question first. That is an easy one. Because we have not been able to solve our own problems we are not going to worry about the electron optical man's problem.

Interestingly enough, we have thought seriously of using some of the techniques the electron optical designers can use because his medium is continuous. We have thought of trying to adapt some of those to our problem, which is something that very often happens. Just as a side remark, I know we had this same experience in filter design. We tried to adapt electric filter design—transmission-line design theory to the design of interference filters for optical work and we discovered that some people in the electric filter design group of one large company were trying to do the same thing in reverse. We have not touched continuous media.

As for the flow of information, what we do is to have the lens designer choose a rough form. In other words, he looks at the requirements of the lens and decides whether it can be solved by a doubler or a tripler, or some other general form. As I said, we could conceive of this being done by some type of experience box, a box into which we put quantities like \( f/ \) number and field and which come out with a general class of forms we could use.

Once having chosen this form, the designer makes a first-order solution (in other words, a solution which has the right powers in it), puts it into the 607, and has the 607 calculate the third-order aberrations. These third-order aberrations answers are fed over to the aberration synthesizer, which was that optical system on which we could put in calibrated amounts of each of the aberrations. He then uses this optical system to image the target I have shown and to calculate a quality factor which is equal to the integrated area.
under the envelope of the oscillatory curve I have shown. He then goes back to the 607 with this information, goes back automatically, and makes a small change in one of the lens constants and calculates a new quality factor and he uses this iterative scheme to make changes in each constant until he has achieved the optimum quality factor for the over-all scheme.

W. D. White (Airborne Instrument Laboratory): Why not compute the aberration synthesis?

N. A. Finkelstein: I assume you mean by this, why we bother doing this synthesizing optically.

The reason we do is because we are afraid of getting too far away from the problem, because we understand so little about how the quality of the image is related to the aberrations.

As I have said, this idea, this approach of breaking down the defects in an image into spherical and coma and all these other terms, is quite arbitrary, and because of this we are not confident that we know in any sense the word the relation between these aberrations and the image quality. That is the thing we would like to know, but because we don’t like to be able to synthesize this optically; in other words, not get too far from the problem. We even like to have a microscope there, as you can see, so we can look into and see what the image looks like. We know from past experience what an image that has a lot of coma in it, for example, looks like, and if the 607 calculates a lot of coma and we see a comatic flare we feel a little bit happy. The main reason we don’t compute this aberration synthesis is because we really don’t know how and we would have to use another rather arbitrary step there, and there are enough arbitrary steps, as you can probably judge, so that we don’t want to add anything more.

William Kegelman (Philadelphia Electric Company): Are actual sine values loaded into the computer for exact design?

N. A. Finkelstein: In exact ray tracing we try to use an algebraic expression of the trigonometric function. We try to use a formulation which is similar to that of Smith’s, if you are familiar with the papers he wrote in the Proceedings of the Royal Society in the last 10 years, in which he formulated ray-tracing equations without using sine or cosine function. We try to do that only because the 607 is not a convenient machine with which to use trigonometric functions.

W. A. Malthaner (Bell Telephone Laboratories): Have you considered the analogue model approach to these design problems?

Donn Combelic (Computer Control Company): In your proposed scheme does the designer enter the feedback loop involved in estimating what parameters are to be varied and how much?

N. A. Finkelstein: Have we considered the analogue model approach to these design problems? We have and the difficulty with analogue models is that in lens design calculations we are usually dealing with very small differences between very large numbers and because of that we usually have to work with at least 5-figure accuracy and, preferably, in many wide-angle lenses, with about 7-figure accuracy. We haven’t felt that an analogue approach would be of enough accuracy so that we could obtain good results from it.

In answer to the second question, the designer does not enter into the feedback loop in terms of estimating what parameters are to be varied and how much. All he does is to put the general form of the lens in and then go out to lunch and come back and hope it has converged.

Franz Edelman: Judging from the volume of your computations could the work be done economically on a CPC?

N. A. Finkelstein: The answer to that is ‘yes’ but we have found in our work that the 607 is a much more economical instrument than the CPC. In other words, we found that the cost per ray surface for ray tracing, for example, in the 607 under the schemes we have worked out is less than for a CPC. This is probably because we have programmed boards, wired-up boards, and as we do such a large amount of ray tracing this is one of the major jobs of our calculating equipment.

The ElectroData Computer in a Data-Reduction System

K. L. Austin

HIGHER speeds, temperatures, and pressures, greater power, less allowable weight, and other requirements make it increasingly necessary for engineers to employ more accurate design techniques. Data-processing instrumentation has an important role to play in the development and improvement of these techniques.

For example, a government vendor who fails to meet the deadline for submitting performance figures on a weapon he has contracted to build forfeits money and reputation. A manufacturer whose engineers are not provided with enough interpretable performance data during design stages so that required changes can be made soon enough, finds himself falling behind in the competitive race for sales. The validation of theory, the discovery of new techniques, and the proof of satisfactory design in a sufficiently short time are possible today only through the interpretation of data gathered and made meaningful by a data-processing system. Stated simply, the problem is to gather data describing both static and dynamic performance of a product or process and reduce them to comprehensible, meaningful form in whatever interval of time is required. Satisfactory solutions for this data-processing problem have become available only recently because:

1. The need has become critical only within the last few years.
2. The technology necessary for a solution has been, relatively speaking, in its infancy.

The need for data-processing systems will increase and the necessary technological advances will be made to surmount the technical difficulties as they arise.

Plotters, computers, readers, counters, transducers, etc., have been designed to solve parts of the problem but few attempts have been made to link all of the elements necessary for data reduction into one integrated system. For such an attempt the co-operation of several manufacturers is likely to be necessary, and the manufacturers must agree to modify their components wherever modification is necessary to avoid incompatibilities when the system is assembled. The data-reduction system described in this paper is made up of products manufactured or supplied by two corporations modified for integration into the system.

A typical system for data processing contains instrumentation for performing the following functions:

1. Measuring physical phenomena such as