A Numerical Algorithm for Identifying Spread Functions of Shift-Invariant Imaging Systems

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Abstract—Numerical optimization techniques are applied to the identification of linear, shift-invariant imaging systems in the presence of noise. The approach used is to model the available or measured image of a real known object as the planar convolution of object and system-spread function and additive noise. The spread function is derived by minimization of a spatial error criterion (least squares) and characterized using a matric formalism. The numerical realization of the algorithm is discussed in detail; the most substantial problem encountered being the calculation of a vector-generalized inverse. This problem is avoided in the special case where the object scene is taken to be decomposable.

Index Terms—Image restoration, numerical deconvolution, spread-response function, system identification, Toeplitz matrices, vector-generalized inverse.

I. INTRODUCTION

A PROBLEM of considerable interest in the area of image processing by digital computer is the a posteriori correction or sharpening of blurred images [1]–[3]. This blurring or distortion arises in physical applications because of the limiting response of imaging instruments. It is customary to model this effect with a convolution-type integral equation involving the object scene f(x, y), formed image g(x, y), and point-spread function of the optics h(x, y)

\[
g(x, y) = \int_{-\infty}^{+\infty} f(x', y - y') h(x', y') \, dx' \, dy'.
\]

(1)

In such a characterization the imaging system is linear and shift invariant (that is, position invariant) and completely described by h(x, y). Because the shape of the point-spread response determines the form and degree of degradation in the imaging, it is necessary to have a quantitative description of h(x, y) in order to restore the quality of g(x, y). In the hypothesis of most useful formal restoration schemes, this description is assumed available a priori [4].

In principle, h(x, y) can be simply determined by a direct measurement of the image pattern associated with a point object of finite energy. Such an experimental determination is limited in practice by the lack of suitable point sources and photosensitive measurement devices, and often by the physical inaccessibility of the system to such test conditions. Alternate methods that have been applied frequently include model identification [5], [6] and edge measurement [7]. In the first case, h(x, y) is postulated to be a specific parametric model (deduced in theory from the physical mechanisms of the imaging) and in the second, h(x, y) is assumed isotropic and hence available from the response to an edge object. Evidently, in practice the underlying assumptions involved in applying both these methods noticeably limit the class of admissible systems whose point-spread responses can be obtained with these techniques.

In this paper such restrictions are avoided in a numerical algorithm for identifying point-spread responses of a general class of imaging systems. The routine is suitable for direct implementation on a digital computer, and it is an extension of the classical unidimensional system identification method of Levin [8]. The approach taken is to discretize the system model. The measured image array is modeled as the system response to a known object scene corrupted by additive independent noise. Applying the method of least squares, the point-spread response is then derived with a vector optimization procedure. Probabilistic interpretations of the method are discussed along with its extension to some special-case applications. The computational problems involved with implementing the algorithm are considered and some specific computing methods suggested.

II. PROBLEM HYPOTHESIS

The imaging systems under consideration are those whose point-spread responses h(x, y) are square integrable on the whole plane, differentiable to all finite orders, and for practical purposes, of finite bandwidth. The object scene f(x, y) in (1) is also assumed to satisfy these conditions. The discrete representations of the image forming process are obtained from (1) by quadrature approximation. One of the most useful representations involves sampling the continuous functions on a periodic, rectangular grid. It is well known that if the sampling mesh is suitably small, (1) can be suitably represented by the convolution summation [9]

\[
g(n, m) = \sum_{n', m'=0}^{+\infty} f(n - n', m - m') h(n', m')
\]

(2)

where, for convenience, the grid spacing has been normalized and n, n', m, m' are integers. We adopt this representation and in the sequel consider the original identification problem to be equivalent to determining \{h(n, m)\}.
EKSTROM: IDENTIFYING SPREAD FUNCTIONS

The hypothesis of the problem is as follows (see Fig. 1). The object scene \( \{f(n, m)\} \) is assumed known (that is, deterministic) and space limited to the rectangular region in the \( nm \) plane for which \( n \in [-P, +P], m \in [-Q, +Q]; \) otherwise, this array is completely arbitrary. It may be a test pattern, such as a resolution chart, or an actual object scene. For practical purposes the point-spread response \( \{h(n, m)\} \) is similarly taken to be of bounded support (this follows from physical arguments), nonzero in the region \( n \in [-N, +N], m \in [-M, +M]. \) Consequently, the output array \( \{g(n, m)\}, \) as given by (2), is space limited to the region \( n \in [-R, +R], m \in [-S, +S] \) where \( R = N + P \) and \( S = M + Q. \) The regions over which these arrays are defined to be nonzero are summarized in Fig. 2.

In the usual measurement configuration we have the image array available as the system output in the presence of observation noise

\[
a(n, m) = g(n, m) + e(n, m)
\]

where \( e(n, m) \) is the noise term and is taken to be a random variable. Common physical sources of noise include background effects, sensor noise, uncertainties in the object scene, and limited precision instruments. Some knowledge of the probabilistic nature of \( e(n, m) \) is generally accessible in most experiments. Here we simply assume it is white.

III. DERIVATION OF THE IDENTIFICATION ALGORITHM

Given a known object scene \( \{f(n, m)\} \) and the available image it induces \( \{a(n, m)\}, \) our identification problem is to determine the spread function \( \{h(n, m)\} \) that minimizes the mean error between the formed image \( \{g(n, m)\} \) and \( \{a(n, m)\}. \) This is accomplished by applying the classical method of least squares. In its general form, the method involves minimizing a quadratic error

\[
J = \sum_{n=-R}^{-P} W(n, m) [a(n, m) - g(n, m)]^2
\]

where \( W(n, m) \) is a predetermined weight picked to emphasize the relative importance of each component in the summand.

If we employ matric notation, the error functional can be written in a more convenient form. First we consider the representation of the double convolution summation (2). This system of equations can be written using lexicographic ordering

\[
\begin{bmatrix}
F^{-P} \\
F^{-P+1} \\
\vdots \\
F^{-P+N-1} \\
F^{-P} \\
\vdots \\
F^{+P} \\
\vdots \\
h^{-N} \\
h^0 \\
h^{+N}
\end{bmatrix} =
\begin{bmatrix}
\begin{bmatrix}
F^P \\
F^P \\
\vdots \\
F^P \\
\vdots \\
F^P \\
\vdots \\
F^P \\
\vdots \\
h^{-N} \\
h^0 \\
h^{+N}
\end{bmatrix} \\
\begin{bmatrix}
F^P \\
F^P \\
\vdots \\
F^P \\
\vdots \\
F^P \\
\vdots \\
F^P \\
\vdots \\
h^{-N} \\
h^0 \\
h^{+N}
\end{bmatrix} \\
\begin{bmatrix}
F^P \\
F^P \\
\vdots \\
F^P \\
\vdots \\
F^P \\
\vdots \\
F^P \\
\vdots \\
h^{-N} \\
h^0 \\
h^{+N}
\end{bmatrix}
\end{bmatrix}
\]

where the subvectors and matrices are of the general form

\[
g^\lambda = \text{col} [g(\lambda, -S) \cdots g(\lambda, 0) \cdots g(\lambda, S)]
\]

\[
h^\lambda = \text{col} [h(\lambda, -M) \cdots h(\lambda, 0) \cdots h(\lambda, M)]
\]

and

\[
F^\lambda =
\begin{bmatrix}
f(\lambda, -Q) \\
f(\lambda, -Q + 1) \\
\vdots \\
f(\lambda, -Q + M - 1) \\
f(\lambda, +Q) \\
\vdots \\
f(\lambda, +Q)
\end{bmatrix}
\]

A vector \( a \) can be constructed similarly from \( \{a(n, m)\}. \) In terms of these quantities the weighted norm in (4) can be expressed as

\[
J = (a - Fh)^T W(a - Fh)
\]

1Some motivation for this choice of error criteria is suggested at the end of this section.
where the superscript $T$ indicates transposition; and the weight matrix $W$ is defined by

$$W = \text{diag} \left( W^{-R} \cdots W^0 \cdots W^{+R} \right)$$

(11)

where

$$W^k = \text{diag} \left[ W(\lambda, -5) \cdots W(\lambda, 0) \cdots W(\lambda, +5) \right].$$

(12)

Finding the spread-response vector $h$ which minimizes the error functional $J$ in (10) is an extreme norm problem that can be solved with gradient techniques [10]. The solution vector is given by

$$h = (F^TWF)^{-1} F^TWa.$$  

(13)

In the less general circumstance, where the errors in $J$ are equally weighted (that is, $W = I$), we have

$$h = (F^TF)^{-1} F^Ta$$  

$$= F^t a$$  

(14)

where $F^t$ is the vector-generalized inverse of $F$ [11].

Our identification algorithm as given in (13) is characteristic in form to the solution of many least squares problems. This, of course, is a consequence of the linearity of the imaging and the error criterion used. The version in (14) is notionally similar to the classic result of Levin, with the appropriate vector matrices substituted for the scalar ones (see [8, eq. (13)]). The utility of the representation derives both from its notational simplicity and the insight it affords in organizing the subsequent computation of $h$.

The deterministic optimization procedure used in deriving (14) can also be interpreted probabilistically. With $\{f(n, m)\}$ known, a rational statistical approach to determining $\{h(n, m)\}$ would be to pick the spread response that gives the available image $\{a(n, m)\}$ a high probability of occurrence. This is the method of maximum likelihood [12]. It is well known that under the hypothesis of Gaussian noise $N(0, a^2)$ the maximum likelihood and least squares estimates of $\{h(n, m)\}$ coincide. In this case, the weights in (13) are given by $W(i, j) = 1/a^2 \delta_{ij}$.

Alternatively, $\{f(n, m)\}$ can be considered as a sample array from a homogeneous random field with covariance $\{R_f(n, m)\}$, and cross covariance with the available image $\{C_{fa}(n, m)\}$. If we consider the matric products in (13) to be estimates of these covariance arrays, formed by spatial averaging, and let $W$ be the vector-covariance matrix of a white noise process; then $h$ as given in (14) is the linear minimum-variance unbiased estimate of the spread-response array [10].

Both of the above interpretations are useful to the extent that they provide some formal justification for the method and an alternate perspective of the quantities occurring in the identification algorithm.

IV. EXTENSIONS OF THE BASIC SCHEME

A useful feature of the basic identification scheme is that with little analytical inconvenience, it can be extended to accommodate a number of applications involving variations of the stated hypothesis. The following two cases are of specific interest because of their frequent occurrence in experimental applications.

A. A Priori Constraints

In identifying physical imaging systems, we require that the estimated point-spread response satisfy a number of engineering constraints. These constraints most often involve limitations on the gain and bandwidth of $h(x, y)$ and arise from a priori knowledge of the specific system to be identified. This information can be incorporated into the identification technique in the following way. First, we assume that the system gain is known a priori. This gain can be expressed mathematically as

$$\sum_{m=-M}^{+M} h(n, m) = C_G$$

(15)

or as an inner product with the response vector and a vector of ones,

$$(h, 1) = C_G.$$  

(16)

Now, our new identification problem is to find the $h$ that minimizes $J$ in (4) while satisfying (16). This is most easily accomplished using the method of Lagrange multipliers and involves minimizing the new error functional

$$J_G = [a - Fh]^T W[a - Fh] + \lambda(h, 1).$$

(17)

The minimizing vector is given by

$$h = (F^TWF)^{-1} F^TWa - (F^TWF)^{-1} \frac{\lambda}{2} 1.$$  

(18)

where $\lambda$ is the Lagrange multiplier and is chosen so that (16) will be satisfied.

Similarly, the gain-bandwidth product of the system may be known. It can be written in the form

$$\sum_{n=-N}^{+N} \sum_{m=-M}^{+M} h^2(n, m) = (h, h) = C_{BW}.$$  

(19)

Following the above procedure, the minimization of $J$ subject to this constraint results in

$$h = (F^TWF + \lambda I)^{-1} F^TWa,$$  

(20)

$\lambda$ is chosen in this case so that (19) will be satisfied.

The two preceding constraints are members of the general class of constraints that admits to an inner product representation. (Another example of this class is the constraint on the second difference of $\{h(n, m)\}$ that results from a priori knowledge of the smoothness of the spread function.) Identification involving this class of constraints can be handled formally as above, with combinations of such constraints requiring the introduction of multiple Lagrange multipliers. Interestingly, the numerical calculations involved are not substantially more complicated than those in the basic algorithm (13) and may be somewhat better conditioned, as in the case of (20).

The constraints can also be interpreted in the context of the probabilistic estimation procedures mentioned in Section III. For example, the inclusion of the gain-bandwidth constraint gave rise to the additional term $\lambda I$ in (20). This term increases the origin point in the pseudocovariance array. In the deterministic approach this is called damping the least squares.
estimate, while in minimum-variance estimation, this step is commonly called “whitening” the object scene.

B. Multiframe Imaging

Frequently, a situation arises where multiframe imaging data involving the system to be identified are available. This data set may be comprised of multiple exposures of the same object scene and/or single exposures of multiple objects. It is generally true that an increase in data results in improved estimates. In this section we show how our basic routine is adapted to use this enlarged data set.

We assume the general case of $K$ district available images $\{a_i(n, m)\} i = 1, 2, \cdots , K$ obtained by imaging of the object scenes $\{f_i(n, m)\} i = 1, 2, \cdots , K$ according to (2) and (3). As in Section II, we can associate a vector $a_i$ with each image and a matrix $F_i$ with each object scene. Now our identification problem is to determine the spread-response vector $h$ which simultaneously minimizes the following $K$ quadratic forms:

$$J_i = (a_i - F_i h_i)^T W_i (a_i - F_i h_i)$$

where $i = 1, 2, \cdots , K$.

By using a stacking operator $S(\cdot)$, which we define with the relations

$$S(a) = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_K \end{bmatrix} \quad S(F) = \begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_K \end{bmatrix}$$

we can also write the error functional associated with this problem as a weighted inner product

$$J' = [S(a) - S(F) S(h)]^T W' [S(a) - S(F) S(h)]$$

where

$$W' = \text{diag}(W_1, W_2, \cdots , W_K).$$

This follows from the existence of a linear mapping between the images and parameters to be estimated [as in (5)]. As a consequence of (23), it should be clear that the optimization procedure used here in determining $h$ is structurally identical to that used in deriving the basic algorithm, (13). In this case the optimum $h$ that minimizes $J'$ in (23) is given by

$$h = [(S(F))^T W' S(F)]^{-1} (S(F))^T W' S(a).$$

By carrying out some of the matrix products in (23), we can write $h$ in a somewhat less compact but more familiar form as

$$h = \left[ \sum_{i=1}^{K} F_i^T W_i F_i \right]^{-1} \left[ \sum_{i=1}^{K} F_i^T W_i a_i \right].$$

In either form, it is easy to see that (13) is a special case of (23) and (25). It also follows that for the case of multiple imaging of a single object, (25) reduces to (13) with $a$ replaced by its arithmetic mean, $(1/K) \sum_{i=1}^{K} a_i$. For multiple objects, the averaging process in (25) is considerably more subtle.

V. Computational Considerations

In this section we consider the computational aspects of implementing the basic identification algorithm. According to (13), identification of a point-spread response of support $N \times M$ requires the calculation of a weighted, vector-generalized inverse. For even moderate-sized $NM$, this may be a substantial numerical problem. Although special algorithms for calculating the weighted inverse exist [13], they require sizable data stores and tend to be numerically unstable for large matrices. This is an apparent limitation of the general identification algorithm; however, it can be avoided in two special-case applications.

A. Decomposable Object Scene

In this first case the object scene is taken to be decomposable. Thus, at each point in the object scene array we have

$$f(n, m) = f_A(n)f_B(m).$$

As a consequence of this decomposability, the object matrix $F$ can be written as

$$F = \begin{bmatrix} f_A(-P)F_B \\ f_A(-P + 1)F_B \\ \cdots \\ f_A(-P + N - 1)F_B \\ f_A(-P)F_B \\ \cdots \end{bmatrix}$$

where

$$F_B = \begin{bmatrix} f_B(-Q) \\ \cdots \\ f_B(-Q + M - 1) \\ f_B(-Q) \\ \cdots \end{bmatrix}$$

$$f_B(Q)$$

and

$$f_A(P)$$

for $P = 0, 1, 2, \cdots , N - 1$ and $Q = 0, 1, 2, \cdots , M - 1$. These vectors may be generated using the relations

$$f_A(-P) = f_A(-P + 1) \quad f_A(-P + 1) = f_A(-P + 2) \quad \cdots \quad f_A(-P + N - 1) = f_A(-P),$$

$$f_B(-Q) = f_B(-Q + 1) \quad f_B(-Q + 1) = f_B(-Q + 2) \quad \cdots \quad f_B(-Q + M - 1) = f_B(-Q).$$

The remaining steps in the algorithm for the decomposable case may be performed using the relations

$$U_{p+1} = \hat{A} U_p$$

where

$$\hat{A} = \begin{bmatrix} A_{0} & \cdots & A_{N-2} & A_{N-1} \\ \cdots & \cdots & \cdots & \cdots \\ A_{N-1} & \cdots & A_{0} & \cdots \end{bmatrix}$$

and

$$U_{p+1} = \begin{bmatrix} U_{0} \\ \cdots \\ U_{N-1} \end{bmatrix}.$$
Now $F$ in (27) is in the form of a direct or Kronecker product of matrices, denoted by [11]

$$F = F_A \otimes F_B$$

(29)

where

$$F_A = \begin{bmatrix}
    f_A(-P) \\
    f_A(-P + 1) \\
    \vdots \\
    \vdots \\
    f_A(-P + N - 1) \\
    f_A(-P) \\
\end{bmatrix}$$

Similarly, if we assign the error weights in (4) in a decomposable form

$$W(n, m) = W_A(n) \cdot W_B(m)$$

(31)

we have

$$W = W_A \otimes W_B$$

(32)

and

$$W_A = \text{diag}(W_A(-R) \cdots W_A(0) \cdots W_A(R))$$

(33)

Using (29), (32), and the following direct product identities [11]

$$(A \otimes B)^T = A^T \otimes B^T$$

(35a)

$$(A \otimes B)(C \otimes D) = AC \otimes BD$$

(35b)

$$(A \otimes B)^{-1} = A^{-1} \otimes B^{-1}$$

(35c)

we can express (13) in the form

$$h = [F^T W F]^{-1} F^T W a$$

$$= [(F_A \otimes F_B)^T (W_A \otimes W_B)(F_A \otimes F_B)]^{-1}$$

$$\cdot (F_A \otimes F_B)^T(W_A \otimes W_B)a$$

$$= \{(F_A^T W_A F_A)^{-1} F_A^T W_A\} \otimes \{(F_B^T W_B F_B)^{-1} F_B^T W_B\} a.$$  (35)

Thus, the weighted vector-generalized inverse reduces in this case to the direct product of two scalar component-generalized inverses. Consequently, we have the identity

$$(A \otimes B)^I = A^I \otimes B^I$$

which is an apparent generalization of (35c).

The computational differences in calculating (13) and (35) are quite substantial. All calculations and storage requirements in the direct product form are of a lower order of magnitude prior to the last matrix product. Difficulties are avoided in this last step by a convenient partitioning of the matrices and the subsequent block computation of each $h^A$, where $\lambda = -N, \cdots, +N$. As a result the conditioning of the calculation is improved dramatically. We sacrifice very little in real world applicability for this numerical convenience. Although the object scene is required to be decomposable, the system to be identified may, of course, be of full spatial coherence.

B. Uniformly Weighted Error Criterion

In (14), we gave the identification algorithm for the case of uniformly weighted errors. The computational usefulness of this simplification derives from the nature of the products in (14). The object matrix $F$ is a so-called vector Toeplitz matrix (that is, it has equal vector entries on each of its diagonals) with scalar Toeplitz submatrices [14]. Although we omit details, it can be shown by construction that $F^T F$ is also vector Toeplitz and that the product of such a matrix multiplied by a vector is equivalent to a discrete convolution. It follows that products $F^T a$ and $F^T F$ can be computed with a two-dimensional finite Fourier transform.

Thus, by partitioning, it is possible to efficiently reduce (14) to the inverse of a vector Toeplitz matrix multiplied by a vector. The justification of this step is the computability of the final form, since an algorithm is available that is suitable for inverting vector Toeplitz matrices of quite large dimension [15]. Thus, the dimensional restriction associated with use of the weighted error identification algorithm is evidently removed in the uniformly weighted version.

VI. EXAMPLE OF APPLICATION

We now demonstrate use of this procedure for identifying spread functions by considering the following numerical example. The system response and images considered in this example are illustrative (both in structure and duration) of those encountered in actual applications. The point-spread response $\{h(n, m)\}$ of the system we wish to identify is shown in Fig. 3. This system characteristic arises when imaging through the earth's atmosphere and models the effects of inhomogeneities in its refractive index. The square pillar in Fig. 4 is the test object $\{f(n, m)\}$ and could represent, for example, a bar on an optical resolution chart. The image $\{a(n, m)\}$ formed with the above object and system [according to (2)] and contaminated by wide-band noise is shown in Fig. 5. Each noise point is a uniformly distributed random variable with a zero mean and a 0.1 unit peak-to-peak variation. Because the object and point spread are of comparable spatial duration, the noisy image is a reasonably poor replica of the actual point spread. Our identification problem is to take this available image $\{a(n, m)\}$ and, knowing the test object $\{f(n, m)\}$, estimate $\{h(n, m)\}$.

We introduce the requirement that the point spread satisfy a gain-bandwidth constraint. Thus, we use the identification
Fig. 3. Point-spread response, \( \{h(n, m)\} \): Gaussian with full width at half maximum = 8. (20 x 20 grid.)

Fig. 4. Object scene, \( \{f(n, m)\} \): a 10-square pillar. (32 x 32 grid.)

Fig. 5. Noisy, available image \( \{g(n, m)\} \): peak-to-peak noise variation is 0.1. (32 x 32 grid.)

Fig. 6. Identified point-spread response: bell-shaped with full width at half maximum = 9. (20 x 20 grid.)

algorithm in (20). A uniform weighting is assumed for all errors and the matric calculations performed as discussed above. Notice that while we have chosen the object scene (hence \( F \) and \( F^T F \)) to be decomposable, we have not eliminated all computational problems in this example since \( F^T F + \lambda I \) is not decomposable. Thus, for the point-spread support of 20 x 20, the algorithm requires inversion of a vector Toeplitz matrix of dimension 400.

The point-spread response obtained by this method is shown in Fig. 6. The norm of the error between this array and the true Gaussian is only on the order of 25 percent that of the true spread response. In addition to the tolerable error norm, the important features of \( \{h(n, m)\} \) (its Gaussian shape, peak amplitude, and full width at half maximum) are satisfactorily reproduced in the identified response. The numerical computations were apparently well conditioned although the computation time on a CDC 7600 was on the order of 2 min.

VII. CONCLUDING SUMMARY

The method of least squares estimation was applied to the system identification problem of planar, shift-invariant imaging systems. A basic identification algorithm was presented using a matric formalism and was interpreted in terms of two classical estimation procedures. Generalizations of the basic scheme were considered along with the problems associated with its numerical realization.

It was shown that under fairly general conditions, the significant computational difficulties encountered in applying the algorithm could be eliminated by proper choice of the test object scene and/or weighting array. In the author’s opinion, it is this interplay between the formalism of the algorithm and its numerical feasibility that form the basis for its utility in practical applications.

REFERENCES


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New Hardware Realizations of Nonrecursive Digital Filters

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Abstract—Analysis of the bit-level operations involved in the convolution realizing a nonrecursive digital filter leads to hardware designs of digital filters based on the operation of counting.

Two distinct designs are outlined: the first one is capable of very high speed but is rather expensive; the second is quite slow but has the advantages of low cost and high flexibility.

The basic designs considered utilize fixed-point representation for the data and filter coefficients. Variants allowing floating-point representation of the coefficients are also described.

Index Terms—Convolution, correlation, digital filters, fast counting, negative radix application, real-time digital filters.

I. INTRODUCTION

Consider a filter that transforms its input time function \( \xi(t) \) to the filtered time function \( \eta(t) \). A nonrecursive digital filter simulates such a filter by computing samples of the filtered signal

\[
\eta_k = \eta(kT_s)
\]  

from samples of the input signal

\[
\xi_k = \xi(kT_s)
\]  

according to the formula

\[
\eta_m = \sum_{k=0}^{K-1} \xi_m - k \alpha_k
\]  

in which the coefficients \( \alpha_k \) are obtainable from the filter transfer function.

There is a growing need for special-purpose machines that would implement such a filter. Particularly attractive here are machines that produce filtered output samples at the rate input samples are generated in the data acquisition environment.

An obvious strategy in realizing such a machine is to compute \( \eta_m \) via the following rephrasing of (1.3):

\[
\eta_m = \sum_{k=0}^{K-1} p_{mk} \]

That is, the desired output \( \eta_m \) is evaluated through the intermediate entities \( p_{mk} \) that require the use of a multiplier.

Here we propose a different strategy in which the intermediate entities are obtained by counting. Our main purpose in this paper is to survey the various possible designs implementing this strategy. Interesting promising designs are indicated in the high-speed high-cost category as well as the low-speed low-cost class.

The paper is organized along the following lines. A master design is developed in Sections II and III. This design serves as the starting point for developing two different basic designs in Sections IV and V. Section IV presents a high-speed design involving high cost. Section V is devoted to an inexpensive