The assertion proved below on the run time required by certain programs is a generalized version of the halting problem, and shares with it simplicity of proof and intuitiveness of the result.

Suppose $T(P)$ is a timing program which takes an arbitrary program P with no free variables as its argument, and that for all such P, $T(P) = $ TRUE if P terminates execution in time less than t when run, and $T(P) = $ FALSE if P runs for at least time t. Suppose further that all programs P as well as the timing program T ran on the same computer C, and that identical programs will always have equal run times on C.

Then for some argument program Q, $T(Q)$ must run at least for time $t - \epsilon$ where ϵ is some ("small") positive constant dependent on C but independent of t.

Liberally interpreted, this result says that the best one can do in general to estimate the run time of programs is to execute them. The ϵ in the assertion above is vexing, since one intuitively expects the stronger statement that $T(Q)$ must run at least for time t, or even $t + \epsilon$. However, it does not seem that the proof below can be sharpened to yield a stronger result. The proof is as follows.

Consider a program Q which, in a free notation, is expressed by Q: recursive procedure; if $T(Q)$ then run for time t; return; end. Notice that t is some fixed number, hence Q has no free variables and qualifies as an argument for T. Notice also that $T(Q)$ cannot have the value TRUE because if it had, Q would run for at least time t, contradicting our assumption on the behavior of T. Hence $T(Q)$ must be FALSE, and hence the run time $r(Q)$ of Q must be at least t. According to the definition of Q, its run time $r(Q)$ consists of two parts: 1) the time $r(T(Q))$ required to compute the value of $T(Q)$, plus 2) the time to enter the procedure Q, to branch on FALSE, and to return. The time required for the latter control operation clearly does not depend on T (and hence not on the time limit t), and we denote it by ϵ.

Hence we have

$$r(Q) = r(T(Q)) + \epsilon \geq t,$$

and

$$r(T(Q)) \geq t - \epsilon,$$

which was to be demonstrated.

J. Nievergelt
Dept. of Computer Science
University of Illinois
Urbana, Ill.

REFERENCE

[1] C. Strachey, "An impossible program," (Letter to the Editor), *Computer J.*, vol. 7, no. 4, p. 313, January 1965.

Comment on "The Determination of the Maximum Compatibility Classes"

A recent correspondence by Choudhury, Basu, and DeSarkar [1] describes an algebraic method for determining the maximal compatibles of an incompletely specified sequential machine. This method appears in an earlier paper by Marcus [3], who attributes the basic idea to Weissman [4]. One of this writer's students, Major T. G. Purnhagen, of the U. S. Air Force, points out that the method was also discovered by Lipovski [2].

Frank M. Brown
Dept. of Elec. Engrg.
Air Force Institute of Technology
Dayton, Ohio 45433

REFERENCES

[1] A. K. Choudhury, A. K. Basu, and S. C. DeSarkar, "On the determination of the maximum compatibility classes," *IEEE Trans. Computers* (Correspondence), vol. C-18, p. 665, July 1969.
[2] G. J. Lipovski, "An improved method of finding all largest combinable classes," Coordinated Science Laboratory, University of Illinois, Urbana, Rept. R-362, August 1967.
[3] M. P. Marcus, "Derivation of maximal compatibles using Boolean algebra," *IBM J. Res. and Develop.*, vol. 8, pp. 537–538, November 1964.
[4] J. Weissman, "Boolean algebra, map coloring, and interconnections," *Am. Math. Monthly*, vol. 69, pp. 608–613, September 1962.

Authors' Reply

Having gone through the paper by Marcus [1], we agree that our method [2] of finding the maximum compatibles is essentially similar to his approach. However, our method was developed independently. The different issues of the IBM journals are not available to us.

A. K. Choudhury
A. K. Basu
S. C. DeSarkar
Inst. of Radiophysics and Electronics
University of Calcutta
Calcutta, India

REFERENCES

1] M. P. Marcus, "Derivation of maximum compatibles using Boolean algebra," *IBM J. Res. and Develop.*, vol. 8, pp. 537–538, November 1964.
[2] A. K. Choudhury, A. K. Basu, and S. C. DeSarkar, "On the determination of the maximum compatibility classes," *IEEE Trans. Computers* (Correspondence), vol. C-18, p. 665, July 1969.