A NEW APPROACH TO VISION AND CONTROL FOR ROAD FOLLOWING

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ABSTRACT
This paper deals with a new, quantitative, vision-based approach to road following. It is based on the theoretical framework of the recently developed optical flow-based visual field theory. By building on this theory, we suggest that motion commands can be generated from a visual feature, or cue, consisting of the projection into the image plane of the tangent point on the edge of the road, along with the optical flow of this point. Using this cue, we suggest several different vision-based control approaches. There are several advantages to using this visual cue: (1) it is extracted directly from the image, i.e., there is no need to reconstruct the scene, (2) it can be used in a tight perception-action loop to directly generate action commands, (3) for many road following situations this visual cue is sufficient, (4) it has a scientific basis, and (5) the related computations are relatively simple and thus suitable for real-time applications. For each control approach, we derive the value of the related steering commands.

1. INTRODUCTION

Autonomous visually-guided road following by a ground vehicle requires two basic steps. The first step is to extract relevant road features from images taken by on-board cameras. This step involves finding road regions, road edges and boundaries, center lines, highway lane lines, etc. in complex real-world images. The second step is to determine how to steer the vehicle (as well as how to speed up or slow down) once visual road information has been extracted. Algorithms for both steps have recently been explored by many investigators [2, 3, 4, 9, 10, 11].

This paper approaches the road following problem by building on the theoretical framework of the recently developed visual field theory [7, 8]. This theory provides quantitative relationships between a stationary 3-D environment and a moving camera. The theory involves pre-computing the expected instantaneous optical flow values in the camera imagery arising from every point in 3-D space. The theory provides a theoretical and scientific basis for optical flow-based road following algorithms.

This paper is concerned with an analysis of the road following problem. Using the visual field theory, we develop geometric and motion-related relationships and constraints for road following. These then suggest partial control algorithms as well as visual cues that can serve as input signals to control algorithms. The approach taken here is known in the literature as a "purposive vision" [1] or "animate vision" [13] approach, since we are only looking for visual information that can be used specifically for the task of road following. Rather than suggesting that one control approach is superior to another, the paper details many different control options. Any one of these, or some combination, can be used in the development of the actual control algorithms. The control schemes presented are partial since only the kinematics of the vehicle and the camera are considered. Issues such as the dynamics of the vehicle, sensitivity, stability, robustness, and time delays are not considered in this paper. The visual cues suggested in this paper have been used to develop road following control algorithms for a real mobile robot [12].

This paper has suggestions for algorithms for both steps described above. For the first step, involving the extraction of road features, this paper suggests that for many road following situations a sufficient road feature (for curved, convex roads) is the tangent point on the road edge (i.e., the point on the road edge lying on an imaginary line tangent to the road edge and passing through the camera) and its optical flow. Therefore, all image processing effort may be directed towards reliably finding and tracking the tangent point and extracting its optical flow. This is in contrast to most current systems which attempt to find as much about the road as possible. Existing systems often ignore the tangent point when making steering decisions, and usually are not concerned with the optical flow values of points on the road.

For the second step that involves determining steering commands once visual information has been extracted, this paper suggests that fast, computationally inexpensive, and simple control algorithms can be used. Given the desired distance from the edge of the road, the only visual information these algorithms require is the location of the tangent point (in the image) and its optical flow. The steering commands are directly related to the change of the tangent point location. Other inputs to these algorithms would be the current steering angle, the current vehicle speed, etc. The output of these control algorithms is the change in steering angle for the next instant of time. (In this paper, we do not consider decisions about speed and acceleration of the vehicle.)

Most existing road following algorithms convert the information extracted from images into a 3-D, vehicle-centered cartesian coordinate system aligned with the ground plane. Steering decisions are then determined in this coordinate system. A 3-D reconstruction is therefore performed before steering decisions are made. We suggest that control algorithms can be developed which directly use observable image information represented in the 2-D image coordinate system (e.g., image position and image flow). Clearly, this approach is simpler and requires much less computation; it is therefore much faster.

This paper begins by outlining our assumptions and defining road following, the coordinate system, and the vehicle. We then review the visual field theory as it relates to road following. Next, we provide analyses and suggest partial control schemes for two road following scenarios, one for circular roads and the other for curved roads. Finally, we discuss directions for future work.

2. DEFINITIONS AND ASSUMPTIONS

2.1. ROAD FOLLOWING

We define a road as any continuous, extended, curvilinear feature. The goal of road following is to follow along this feature over an extended period of time. In what we normally think of as road following, a road is defined either by its boundaries or by an extended solid or dashed white line. Here, the goal is not only to follow along these features but also to stay within a constant lateral distance from these features. In general, the feature to be followed need not define a real road. For example, it could be a boundary line of vegetation, a stripe painted on the ground, or even a wall. For a low-flying air vehicle, the feature to be followed could be a river.

Vision-based road following requires the ability to continuously detect and track features in imagery obtained from an onboard camera, and to make steering decisions based on visual properties of these features.
Figure 1 shows a vehicle and the left-hand side road edge. The unit vector $\mathbf{f}$ is the instantaneous heading of the vehicle, $\mathbf{o}$ is the instantaneous center of curvature of the vehicle path, and $\mathbf{i}$ is the instantaneous radius of curvature of this path. We define road following as an activity that involves servoing $\mathbf{f}$ such that it follows the road edge. It is desired that $\mathbf{f}$ be servoed such that the vehicle is always parallel to the tangent to the local curvature of the road edge (Figure 1b), and such that the distance $\mathbf{o}$ of a point on the vehicle from the road edge is maintained at a constant value. In other words, the instantaneous center of curvature of the road edge and the instantaneous center of curvature of the vehicle path should coincide, and the tangent to the edge of the road at the intersection point B should be parallel to $\mathbf{f}$. Normally, these constraints will not be met if the vehicle is attempting to avoid an obstacle or if the vehicle is changing lanes on a road. If a road has two boundaries, then road following will normally involve staying within these boundary edges. In this paper, we assume that the road is always curved and we do not discuss the two-boundary case.

![Figure 1. Road following: (a) 3-D (b) Top view.](image)

2.2. COORDINATE SYSTEM

The equations in this paper will be defined in a coordinate system which is fixed with respect to the camera on board the vehicle. This coordinate system is shown in Figure 2. We assume that the camera is mounted on a vehicle (later we explain how) moving in a stationary environment. Assume a pinhole camera model and that the pinhole point of the camera is at the origin of the coordinate system. This coordinate system is used to measure angles to points in space and to measure optical flow at these points. We use spherical coordinates $(\theta, \phi, \psi)$ for this purpose. In this system, angular velocities $(\theta, \phi, \psi)$ of any point in space, say $P$, are identical to the optical flow values at $P'$ in the image domain. Figure 3 illustrates this concept: $\theta$ and $\phi$ of a point in space are the same as $\theta$ and $\phi$ of the projected point $P'$ in the image domain, and therefore there is no need to convert angular velocities of points in 3D space to optical flow. In Figure 3 the image domain is a sphere. However, for practical purposes the surface of the image sphere can be mapped onto an image plane (or other surface; see [8]).

![Figure 2. Coordinate system fixed to camera.](image)

![Figure 3. Image domain.](image)

2.3. TWO-WHEELED VEHICLE

For our analysis, we use a theoretical two-wheeled vehicle as illustrated in Figure 4. A rigid frame of length $h$ holds both wheels. A steering wheel angle is applied to both wheels simultaneously, i.e., if one wheel is rotated by an angle $\phi$ relative to the frame, the other wheel will rotate by the same angle. This apparatus assures that both wheels will always stay at the same distance from the instantaneous center of curvature of the vehicle's path. The camera is mounted such that its pinhole point is located above the front wheel center, and it rotates with the front wheel. The optical axis of the camera coincides with the instantaneous translation vector (heading) of the front wheel. Note that the heading vector of other points on the vehicle will, in general, not be the same as that of the front wheel.

![Figure 4. Two-wheeled vehicle with camera.](image)
The frame length \( m \) is usually known. Thus the instantaneous radius of curvature \( r \) of the vehicle path can be determined by measuring the steering angle \( \beta \).

Figure 5 is an overall description of the system including the spherical coordinate system, and Figure 6 is the related top view. For convenience we chose to have the \( z \) axis pointing down. However the same coordinate system as described in Figure 2 is used here. The camera is mounted at some height above the ground and rotates with the front wheel. The position of any point on the road can be expressed with the coordinates \( R, E, \) and \( \theta \), as shown in Figure 5.

![Figure 5. Overall description of system.](image)

Similarly, to convert from \( \text{XYZ} \) to \( \text{RCO} \) coordinates we use:

\[
X = R \cos \theta \sin \phi
\]

\[
Y = R \cos \theta \cos \phi
\]

\[
Z = R \sin \theta
\]

In order to find the optical flow of a 3-D point in \( \text{RCO} \) coordinates, we use the following relations and transformations (see [6] and Figure 2):

\[
V_{\text{RCO}} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}
\]

Then:

\[
V_{\text{XYZ}} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}
\]

We start with the derivation of the velocity of a 3-D point in the \( \text{XYZ} \) coordinates (Figure 2). Let the instantaneous coordinates of the point \( P \) be \( \mathbf{r} = (x, y, z)^T \) (where the superscript \( T \) denotes transpose). If the instantaneous translational velocity of the camera is \( \mathbf{v} = (u, v, w)^T \) and the instantaneous angular velocity is \( \mathbf{m} = \omega = (\omega_x, \omega_y, \omega_z)^T \) then the velocity vector \( \mathbf{v} \) of the point \( P \) with respect to the \( \text{XYZ} \) coordinate system is:

\[
\mathbf{v} = -\mathbf{m} \times \mathbf{r}
\]

or:

\[
v_x = -u \cdot z + v \cdot x
\]

\[
v_y = -u \cdot y
\]

\[
v_z = -w \cdot z
\]

The equations of motion and optical flow are described by:

\[
\mathbf{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}
\]

\[
\dot{\mathbf{r}} = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} u \\ v \\ w \end{bmatrix}
\]

\[
\mathbf{v} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} \omega_x & \omega_y & \omega_z \\ y & -x & 0 \\ z & 0 & -y \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}
\]

And:

\[
\mathbf{a} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} \omega_x & \omega_y & \omega_z \\ y & -x & 0 \\ z & 0 & -y \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}
\]

Also (see [6]):

\[
\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} u \\ v \\ w \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}
\]

In the following analysis, we assume a moving vehicle in a stationary environment. The road is assumed to be planar, and road edges are assumed to be extractable. Figure 14 shows examples of road images obtained from a camera mounted on a vehicle.

3. VISUAL FIELD THEORY

We have recently developed a new visual field theory that relates six-degree-of-freedom camera motion to optical flow for a stationary environment [7, 8]. The theory describes the structure of a field in 3-D space consisting of surfaces surrounding the moving camera. If static objects are placed anywhere in the surrounding space, the optical flow produced by these objects in the camera is predicted by the field theory. The field is always centered at the camera pinhole point and moves with the camera. The structure of the field changes as a function of the instantaneous camera motion.

This theory provides us with a theoretical and scientific basis for developing constraints, control schemes, and optical flow-based visual cues for road following. This section reviews this theory as it relates to the road following problem.

3.1. EQUATIONS OF MOTION AND OPTICAL FLOW

First we describe the equations that relate a point in 3-D space to the projection of that point in the image for general six-degree-of-freedom motion of the camera. Some of the equations can be found in many books, e.g., see [5].
\[ \dot{v}_y = \dot{e} \]  
\[ \dot{v}_x = \dot{\omega}_x \cos \phi \]  
\[ \dot{v}_x = \dot{\omega}_x \sin \phi \]  
where \( \dot{\} \) denotes first derivative with respect to time.

Using equations (3)-(19) yields the following expressions:

\[ \begin{bmatrix} \dot{e} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} -\sin \phi & \cos \phi & 0 \\ -k_{xy} \sin \phi & k_{xy} \cos \phi & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} -U \dot{z} + CT \\ -U \dot{z} + CT \end{bmatrix} \]  
\[ \begin{bmatrix} \dot{e} \\ \dot{\phi} \end{bmatrix} = \frac{X}{V^2 + X^2 + Z^2} \begin{bmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi \cos \phi & -\sin \phi \sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} + \begin{bmatrix} -U \dot{z} + CT \\ -U \dot{z} + CT \end{bmatrix} \]  
(20)

As mentioned earlier, \( \dot{\} \) and \( \dot{\phi} \) of a point in space (i.e., the angular velocities in the camera coordinate system) are the same as the optical flow components \( \dot{e} \) and \( \dot{\phi} \) (Figure 3).

Suppose that we want to determine the locus of points in 3-D space that produce constant optical flow values \( \dot{e} \) and constant optical flow values \( \dot{\phi} \) in the image for a given arbitrary six-degree-of-freedom camera motion. To do so we simply set \( \dot{e} \) and \( \dot{\phi} \) in equation set (21) to the desired constants and solve for \( x, y, \) and \( z \). All points in 3-D space that satisfy this solution are called equal flow points. However, the solution to these two equations is not unique since there are three unknowns and two equations. In general, there is an infinite number of solutions.

3.2. A SPECIAL CASE

In this section we analyze a specific motion in the instantaneous \( xy \) plane of the camera coordinate system.

Let the camera motion vectors \( \mathbf{t} \) and \( \mathbf{a} \) be given as follows:

\[ \mathbf{t} = U \mathbf{v} \mathbf{n} \mathbf{k}^T \]  
\[ \mathbf{a} = \omega_0 \mathbf{c} \mathbf{k} \mathbf{n} \]  
(22)

(23)

This means that the translation vector may lie anywhere in the instantaneous \( xy \) plane while the rotation is about the \( z \)-axis. Substituting these motion vectors into equation set (21) yields:

\[ \begin{bmatrix} \dot{e} \\ \dot{\phi} \end{bmatrix} = \frac{X}{V^2 + X^2 + Z^2} \begin{bmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi \cos \phi & -\sin \phi \sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} + \begin{bmatrix} -U \dot{z} + CT \\ -U \dot{z} + CT \end{bmatrix} \]  
(24)

Setting \( \dot{e} \) and \( \dot{\phi} \) in equation set (24) to constants will result in a set of equal flow points for this specific motion.

Consider the case where the optical flow value of \( \dot{\phi} \) is constant. From equation set (24), the points in space that result from constant \( \dot{\phi} \) (regardless of the value of \( \dot{e} \)) form a cylinder of infinite height whose equation is:

\[ \left( \frac{x}{2C + b} \right)^2 + \left( \frac{y}{2C + b} \right)^2 = \left( \frac{U}{2C + b} \right)^2 \left( \frac{1}{\dot{\phi}} \right) \]  
(25)

Figure 7 shows a horizontal section of the cylinder of Equation (25). The section is a circle that lies in the \( xy \) plane. This plane is perpendicular to the axis of symmetry of the cylinder. The radius of the circle is \( \left( \frac{U}{2C + b} \right)^2 \) and its center is at \( \left( \frac{U}{2C + b} \right)^2 \) and its center is at \( \left( \frac{U}{2C + b} \right)^2 \).

The circle is tangent to the camera translation vector at the origin.

The meaning of equation (25) is the following: all points in 3-D space that lie on the cylinder described by Equation (25) and which are visible (i.e., unoccluded and in the field of view of the camera) produce the same instantaneous horizontal optical flow \( \dot{\phi} \). We call the cylinder on which equal flow points lie the equal flow cylinder. A section of a set of equal flow cylinders is illustrated in Figure 9 for the case where the camera undergoes instantaneous translation and rotation. The label of each circle represents the horizontal optical flow \( \dot{\phi} \) in the image that corresponds to points on this circle. Here, there is a circle with finite radius that produces zero horizontal flow (\( \dot{\phi} = 0 \) in the image domain).

3.3. ZERO FLOW CYLINDERS

One of the equal flow cylinders corresponds to points in 3-D space that produce zero horizontal flow. We call this cylinder a zero flow cylinder. The equation that describes the zero flow cylinder can be obtained by setting \( \dot{\phi} = 0 \) in Equation (25), i.e.,

\[ \left( \frac{x}{2C + b} \right)^2 + \left( \frac{y}{2C + b} \right)^2 = \left( \frac{U}{2C + b} \right)^2 \]  
(26)

We have shown [7] that if the \( z \) component of the camera rotation vector \( \mathbf{a} \) is positive (i.e., \( C > 0 \)), then visible points in the \( xy \) plane that are inside the zero flow cylinder produce positive horizontal optical flow (\( \dot{\phi} > 0 \)), while visible points outside the zero flow cylinder produce negative horizontal optical flow (\( \dot{\phi} < 0 \)) in the image (see Figure 10). If \( \mathbf{a} \) is negative (i.e., \( C < 0 \)) then the opposite is true.

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3.4. EQUAL FLOW CYLINDERS AS A FUNCTION OF TIME

As the camera moves through 3-D space, the equal flow cylinders move with it. Figure 11 shows sections of equal flow cylinders as a function of time. At each instant of time, the radii of the equal flow cylinders are a function of the instantaneous motion parameters \( t \) and \( m \). The locations of the equal flow cylinders are such that they always contain the origin of the camera coordinate system (the same as the camera pinhole point), are tangent to the instantaneous translation vector \( t \), and their symmetry axes are parallel to the instantaneous rotation vector \( m \). (In Figure 11, the direction of \( m \) varies over time.) Each zero flow cylinder lies to the left or right of the translation vector depending on whether the instantaneous rotation is positive or negative, respectively.

4. ANALYSIS OF ROAD FOLLOWING

We describe two road following scenarios. The first one is for a circular road, where we outline basic geometric and motion-related relationships. Using this relatively simple case, we explain the problem of following a road using a vision sensor, problems associated with it, and relate it to the visual field theory described above. We also suggest several road following control approaches. The second road following scenario is for an arbitrary convex curved road, where we also suggest some control approaches. We show several approaches, some of which are related to the theory that was previously described. Others are presented to show alternative partial control schemes. Some combination of these would be used in the development of actual control algorithms.

4.1. CIRCULAR ROAD

In this section, we consider following along a circular road. Given visual cues, a goal of a control system is to find the steering angle. If the vehicle is already on a path that follows the road, then only changes in steering angle are necessary. Figure 12 shows a vehicle moving around a circular road of radius \( r \). The path traversed by the vehicle is a circle of radius \( r \). Let the unit vector \( o \) indicate the direction of the tangent line, a line that contains the camera pinhole point and is tangent to the road edge.

We will now show that the tangent point \( T \) lies on the instantaneous zero flow cylinder if the camera orientation is fixed relative to the vehicle. The proof for this is as follows. Let us consider the planar case first, as shown in Figure 12. Given that the line \( AT \) is tangent to the road edge, then \( AT \) is perpendicular to \( OT \). We will now show that \( OA \) is the diameter of the section of the zero flow cylinder displayed in Figure 12. From Equation 25, we can see that the center of the zero flow cylinder section is at \( \frac{V}{3C-e} \). Further, the location of the camera is at \( 0 \). Now the center of rotation of the camera’s circular path must lie on the zero flow cylinder section. To see why, we first note that the vector from the camera to the center of rotation \( o \) is always perpendicular to the camera heading vector. (Remember that the heading vector and camera optical axis coincide.) Therefore the center \( c \) can be considered as a fixation point for the camera during its motion, i.e., the position of point \( c \) does not change relative to the camera’s coordinate system [7, 8]. This means that this point must theoretically produce zero horizontal optical flow in the camera (assuming the camera has a wrap-around lens). Therefore, point \( c \) must lie on the zero flow cylinder section. Then the center of rotation of the camera’s circular path must be at \( \frac{V}{3C-e} \) (Figure 8). Therefore line \( OA \) is the diameter of the zero flow cylinder. Since the angle \( O TA \) is a right angle, point \( T \) must lie on the circle whose diameter is \( OA \). Since this circle is a section of the zero flow cylinder, point \( T \) must lie on this cylinder. For the case where the road edge does not lie in the \( X-Y \) plane of the camera, all points above or below the point \( T \) (including the one on the edge of the road) lie on the zero flow cylinder.

Notice that this proof holds no matter what the diameter of the circular road edge. This means that no matter how far the vehicle is from the road edge (Figures 13 and 14a), the tangent point lies on the zero flow cylinder. Thus the horizontal component of optical flow of
Differentiating Equation (27) with respect to time:

\[ i = \dot{r} \sin \theta + r \dot{\theta} \cos \theta \]  

(29)

where \( \dot{r} \) denotes derivative with respect to time. For a circular road, \( i \) is constant, and thus \( \dot{r} \) can be set to zero in Equation (29):

\[ \dot{r} = -r \dot{\theta} \cos \theta. \]  

(30)

When the vehicle is moving on a perfect circular path both \( \dot{r} \) and \( \dot{\theta} \) are equal to zero. However, suppose the vehicle’s path is not a perfect circle. Since \( r \) is the instantaneous radius of curvature of the vehicle motion, \( \dot{r} \) is the rate at which the curvature changes. Equation (30) suggests a way of controlling the vehicle motion so as to achieve a constant circular motion. Consider the two-wheeled vehicle described in Section 2.3. From Equation (1), we can derive the following:

\[ \theta = \frac{\sin^{-1}(e)}{r}, \]  

(31)

Equation (31) gives a value of the steering angle \( \theta \) as a function of the instantaneous radius of curvature \( r \) and the distance \( 2z \) between the two wheels. Normally the value \( e \) is known. For a more realistic vehicle (such as a four-wheeled vehicle with front-wheel steering), some other relationship may hold.

In Equation (30), \( r \) is the rate at which the radius of curvature of the vehicle motion is changing. We can express \( i \) as a function of the steering angle \( \theta \) by differentiating Equation (1) with respect to time:

\[ \dot{i} = -r \cos \theta \dot{\theta}. \]  

(32)

Substituting Equations (32) and (1) into (30) and solving for \( \dot{\theta} \):

\[ \dot{\theta} = \frac{\dot{i}}{r \sin \theta}. \]  

(33)

Equation (33) suggests a partial control scheme whose inputs are the current steering angle \( \theta \), the current angle \( \alpha \) of the tangent line relative to the \( x \)-axis, and the optical flow \( \dot{e} \) of the tangent point. All of these inputs can be measured. The variable being computed is the rate of change of the steering angle, \( \dot{\theta} \). Equation (33) provides the gain \( \frac{\dot{e}}{r \sin \theta} \) by which \( \dot{\theta} \) should be multiplied in order to get the correct change in steering wheel angle. This gain depends on the current steering wheel angle \( \theta \) and the angular location \( \alpha \) of the tangent point in the image. Note that the change in steering control command should be the negative of the value of \( \dot{\theta} \) derived in Equation (33).

Figure 14b shows a sequence of images taken from a camera mounted on a vehicle. The images in the figure are numbered in the same order in which they were taken. The road is almost circular. Note that the tangent point stays (almost) at the same location in each image in the sequence. If the road were perfectly circular and the vehicle were moving on a perfect circular path, then the position of the tangent point would not change from image to image. However, if the vehicle’s path were not a perfect circle, then its steering can be controlled by measuring horizontal changes in the position of the tangent point. These changes are the horizontal component of optical flow at that point, and can be used to generate changes (\( \delta \)) in the steering wheel command \( \dot{\theta} \).

Note that Equations (1) and (31) hold only for certain types of vehicles. Vehicles with other wheel and steering configurations will result in different expressions relating steering angle to the radius of curvature of motion. In all such expressions, however, there should be a one-to-one relationship between \( \dot{\theta} \) and \( r \). These expressions can then be substituted into Equation (30) to derive the relevant control signals. It is important to emphasize that the derivation of \( \dot{\theta} \) takes into account the kinematics of the system but not the dynamics. This is also the reason why we emphasize that the control scheme is not complete.

If the rate of change of the steering angle, \( \dot{\theta} \), is the only variable being controlled (as indicated in Equation (33)), then in practice the vehicle may not maintain a constant distance from the edge of the road. Therefore, in addition to Equation (33), Equation (28) can also be used to control the vehicle to achieve a constant circular motion. Substituting Equation (1) into (28):
The control signals (δ and δ̇) and partial control schemes suggested above assume that the road is circular, that the center of curvature of the desired vehicle path coincides with the center of curvature of the circular road, and that the road is planar. It is also assumed that the tangent point (in the image) can always be determined, and that the vehicle heading coincides with the camera optical axis. The main significance of this approach is that (1) it shows that the tangent point and its optical flow are important visual control signals for road following, and that potentially they are sufficient to achieve road following, (2) a tight perception-action loop is possible for road following which is simple and therefore computationally inexpensive, (3) image information can potentially be used directly as input into a control algorithm, (4) only a few measurements may be needed to control the vehicle, (5) the approach is independent of the speed of the vehicle, (6) it is independent of the camera height above the road, and (7) only a very small portion of the image -- the portion around the tangent point -- may need to be analyzed, in principle. (Of course, item (7) may not be true in practice since larger portions of the road may have to be extracted in order to reliably find the tangent point.)

A different approach for circular road following is based on the simple fact that the height of the camera above the road is constant during driving. Refer to Figures 5 and 12. Let \( h_v \) be the height of the camera above the ground. For the tangent point \( T \) we can write:

\[
h_v = r \cos \theta v \tag{35}
\]

or

\[
r = \frac{h_v}{\cos \theta v} \tag{36}
\]

The value \( h_v \) is usually a known constant, while \( s \) and \( \theta \) for the tangent point can be measured in the image. Thus the distance \( r \) of the camera from the center of curvature of the circular road can be determined. Equations (36) and (31) allow computing the steering wheel angle \( \delta \) using visual information without measuring it directly.

Taking the derivative of equation (35) with respect to time yields:

\[
h_v' = i \cos \theta v + r (-\sin \theta v) \delta' + r \cos \theta v \frac{d}{dt} \theta \tag{37}
\]

Since \( h_v \) is constant, then \( h_v' = 0 \) in Equation (37), and solving for \( i \):

\[
i = (\delta' v - \frac{d}{dt} \theta) r \tag{38}
\]

The corresponding change in steering angle is (using equations (32) and (38)):

\[
\delta = \frac{1}{s} \frac{h_v}{\cos \theta v} (\delta' v - \frac{d}{dt} \theta) \tag{39}
\]

Substituting \( \delta = \omega \delta' \) (Equation (31)) and then \( r = \frac{h_v}{\cos \theta v} \) (Equation (36)) in Equation (39) results in an expression for \( \delta \) as a function of the visually measured parameters \( s, \theta, \delta' \) and the known constants \( h_v \) and \( \theta_v \).

Yet another approach for circular road following is depicted in Figure 15. A circular path will be maintained by the vehicle if the heading vector \( \vec{s}_h \) at point \( A \) is smoothly servoed so as to result in a heading vector \( \vec{s}_h \) at point \( B \). It is desired that the heading vector always be tangent to the circular path concentric with the circular road and at a radial distance \( s \) from the road edge. Let \( \theta \) be the angle between the camera x-axis and the tangent line, and let \( \alpha = \frac{s}{r} = \theta \). Notice that since it is desired that \( \vec{s}_h \) be parallel to the tangent line from point \( A \), the change in direction between \( \vec{s}_h \) and \( \vec{s}_h \) is \( \alpha \).

We define the turning rate as the amount the heading vector direction changes (or turns) per unit time in the world coordinate system.

Since the circular arc length between points \( A \) and \( B \) is \( (\alpha + r) s \), the turning rate is \( \frac{d}{dt} \alpha \) amount of time to travel the length of the arc, where \( v \) is the speed of the vehicle (assuming constant speed). Therefore

\[
\text{Turning Rate} = \frac{\alpha}{\sqrt{r^2 + (\frac{s}{r})^2}} = \frac{v}{r} \tag{40}
\]

For the two-wheeled vehicle in Figure 4, the turning rate is a function of the steering angle \( \delta \).

4.2. CURVED ROAD FOLLOWING

In this section, we consider road following for the case where the curvature of a convex road is not constant. Figure 16 shows two cases. In Figure 16a the radius of curvature increases as the vehicle moves. In Figure 16b, the radius of curvature decreases. Figure 17 shows a detailed version of Figure 16a. Let the current instantaneous center of curvature of the vehicle path be at \( o \). If the road curvature were constant, then the point of tangency of the vector \( T \) would be at \( A \) and this point would lie on the zero flow cylinder. However, because the road's curvature is changing, the point of tangency is at \( A \). The center of curvature of the curve at \( T \) is at \( A \). Notice that the point \( T \) lies on some equal flow cylinder whose \( \phi \) optical flow is negative (\( r \) lies outside the zero flow cylinder). If the radius of curvature were decreasing (Figure 16b), the tangent point would lie inside the zero flow cylinder, and its \( \phi \) optical flow would be positive. Therefore, intuitively, if the horizontal component of the optical flow, \( \delta \), at the tangent point is measured, then its value can be used as a control signal for steering the vehicle. If \( \delta \) is negative (Figure 16a) then the steering command is to increase the radius of curvature of the vehicle's current motion. If \( \delta \) is positive (Figure 16b), then the steering command is to decrease the radius of curvature of the vehicle's current motion by sharpening the turn.

Another approach to curved road following is to extend the technique shown in Figure 15. Consider Figure 18, which shows the case of a road whose radius of curvature is increasing. As before, the goal is to smoothly serve the heading vector \( \vec{s}_h \) at point \( A \) so as to result in a heading vector \( \vec{s}_h \) at point \( B \). The vector \( \vec{s}_h \) is parallel to the tangent line from point \( A \), and is at a distance \( r \) from the line. If \( \alpha = \frac{s}{r} = \theta \) is the angle between \( \vec{s}_h \) and the tangent line, then the change in direction between \( \vec{s}_h \) and \( \vec{s}_h \) is \( \alpha \). Let the distance between the vehicle and the tangent point \( T \) be \( s \). Later we will discuss how \( s \) might be computed. The arc between point \( A \) and \( B \) can be very closely approximated by an arc of a circle. The straight line distance between \( \alpha \) and \( \beta \) is \( \sqrt{s^2 + r^2} \). The ratio between the arc \( AB \) and the line \( AB \) is \( \frac{\pi s}{2 \omega} \) (assuming arc \( AB \) is a circle). Then

\[
\text{Turning Rate} = \frac{2 \omega \sin \theta - \frac{s}{2}}{\sqrt{s^2 + r^2}} \tag{41}
\]

A way to compute \( s \) in Equation (41) is as follows. Suppose the cam-
era is located at a height $h_c$ above the ground (Figure 19). The angle between the horizontal and the tangent line is $\theta$. Then, from Figure 19,

$$ s = \frac{h_c}{\tan \theta}. \quad (42) $$

Since angle $\theta$ can be measured in the image and $h_c$ is a known constant, it is easy to compute $s$ from Equation (42).

Equation (41) gives the turning rate to achieve road following. The quantities that need to be measured to achieve this are the angle $\theta$ between the $x$-axis and tangent vector, the value $s$, and the velocity $v$ of the vehicle. The value $s$ is assumed to be given.

A somewhat different method of controlling the vehicle can also be derived from Figure 18. This method assumes that the control scheme is to continuously compute a desired future location for the vehicle (point $A$ in Figure 18) relative to the vehicle's current location (point $a$) and to the tangent point (point $T$). This can be done as follows. From Figure 18:

$$ s = \tan \theta \tan \phi. \quad (43) $$

Combining Equations (42) and (43):

$$ s = \frac{h_c}{\tan \theta \tan \phi}. \quad (44) $$

Equation (44) gives the direction $\theta$ of the desired point $A$ relative to the direction of the tangent point $T$. The control approach is then to continuously measure $\theta$ and servo the heading vector in the direction $\theta$. This approach has the advantage of observing a visible point (the tangent point) and servoing toward a nearby point.

To determine the rate $\frac{d\theta}{dt}$ at which to servo the heading vector, we derive the following. From Equation (44):

$$ \tan \theta \tan \phi = \frac{s}{h_c}. \quad (45) $$

Taking the derivative with respect to time:

$$ \frac{d}{dt} \left( \tan \theta \tan \phi \right) = \frac{s}{h_c} \frac{ds}{dt} \cos \phi \cos \theta. \quad (45) $$

Here $\phi$ is the optical flow of the tangent point in the $\phi$ direction. The value $s$ can be directly measured in the camera, while the value $\theta$ is computed from Equation (44). The values $s$ and $h_c$ are a priori known constants. Thus the only measurements needed for computing $\frac{d\theta}{dt}$ are visual, i.e., they can be extracted from the image.

A final scheme that can be useful for road following is depicted in Figure 20. The goal is to servo the heading vector $\theta$ so as to maintain the vehicle at a constant distance $s$ from the edge of the road. The method here is to consider an imaginary line on the ground which is perpendicular to the heading vector (and to the camera $y$-axis) and is at a constant distance $s$ from the vehicle. We measure the angle $\theta$ between the $x$-axis and the vector from the camera to the point of intersection of the imaginary line and the road edge (point $r$ in Figure 20). The distance $s$ from the road edge to the heading vector, measured along the imaginary line, is
If the heading vector is servoed so as to maintain the angle $\theta$ constant, then, because $a$ is a constant, $s$ will also be maintained at a constant value.

$$s = \frac{d}{\tan \theta}$$

Figure 20. Road following: analysis.

Figure 21. Road following: analysis.

Figure 21 shows how this imaginary line can be determined from a tilt angle $s$, measured in the camera $yz$-plane. From Figure 21:

$$d = \frac{s}{\tan \theta}$$

This equation shows how the value of $s$, which can be measured in the image, defines the distance $d$ of the imaginary line.

From Figure 20, we see that the angle $\theta$ should always be smaller than the tangent angle $\theta_T$ if the road edge is on the left of the vehicle, and greater than $\theta_T$ for a road edge on the right. Since $\theta_T$ is continuously changing for a non-circular road, we may want to have several imaginary lines at distances $\delta_1, \ldots, \delta_n$, and store several corresponding values $s_1, \ldots, s_n$. These distances correspond to several tilt angles $\theta_1, \ldots, \theta_n$.

This last control approach is the only one in this paper that requires information about the road at points other than the tangent point. We discuss this kind of approach to indicate that the approaches based on the tangent point can be combined with other approaches in developing robust control algorithms.

5. CONCLUSION

This paper has presented a new approach to vision and control for road following. The approach is based on an analysis of the geometric and motion-related relationships and constraints for road following. This analysis builds upon the visual field theory previously developed by the authors. From this initial analysis, we derived visual cues and control approaches for road following.

The most significant points that we have shown are (1) for following curved, convex roads, the position of the tangent point on the road edge and its optical flow are important visual control signals, and they may in fact be sufficient, (2) control approaches are possible that directly use image information represented in the 2-D image coordinate system; a 3-D reconstruction of the scene is not required, (3) such approaches are simple and fast and result in a tight coupling of perception and action. Although we have focused on the tangent point, larger portions of the road may have to be extracted in practice in order to reliably find the tangent point.

The partial control schemes presented in this paper have not been implemented yet. Current and future work will be directed towards implementing control algorithms that use the approaches suggested in this paper [12]. Issues such as the dynamics of the vehicle, sensitivity, stability, robustness, and time delays must be considered when developing control algorithms for real vehicles.

Another area for future work is to extend the ideas in this paper to other types of roads. Issues that will need to be addressed include concave roads, roads with two boundaries, where the left and right boundaries are alternatively convex, and roads that contain both straight and curved portions. Of course, to follow real-world roads, the ideas in this paper will have to be integrated with systems that find road edges in real images of highways.

REFERENCES