The CUBE Language *

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Abstract

CUBE is a three-dimensional, visual, statically typed, higher-order logic programming language. CUBE will eventually be embedded into a virtual-reality-based programming environment that allows a user to manipulate a CUBE program simply by grabbing, placing, and moving its components.

1 Introduction

CUBE is a new programming language which combines several innovative features, namely
- a visual, three-dimensional syntax, which shall eventually make it possible to edit CUBE programs in a virtual-reality based programming environment,
- a static, polymorphic type system, as it is used by many functional languages
- a Horn-Logic based semantics which is higher-order in that it treats predicates as first class values

As the semantics of Horn-Logic is inherently parallel, so is the semantics of CUBE. We hope to come up with a parallel implementation of CUBE.

The visual aspect of CUBE has been strongly influenced by Kimura's Show-and-Tell [9] and by work on a type system for Show-and-Tell [15]. Much work has been done on polymorphic type systems for functional languages [3], and, more recently, for logic languages [13][17], which applies almost directly to our work. A lot of work has been done as well on languages based on (first-order) Horn-Logic [3][10], and some on higher-order variants thereof [11]. Finally, much attention has been devoted recently to parallel implementations of logic-based languages [4][15].

2 An example program

The program shown in Fig. 1 gives the flavor of a CUBE program. It shows the recursive definition of the factorial predicate. This predicate is represented by a predicate definition cube with two ports, an input and an output port. Values enter the cube through the port on the left side and get distributed over the two inner planes. Planes are evaluated independently and concurrently.

If the input value \( i \) is not 0, and thus does not unify with the value 0 inside the lower left holder, the lower plane fails, otherwise, the unification succeeds, and, as there are no other conditions to satisfy, the value 1 inside the lower right holder cube will “flow” to the output port (be unified with it), and thus will be returned as a result.

The value \( i \) of the input port will also be distributed to the upper plane. Here it will flow into the predicate \( greater \), which also receives the value 0 from the holder cube to its right as a second argument. The predicate succeeds if \( i > 0 \), otherwise, it fails and thereby causes the entire upper plane to fail. \( i \) also flows into the \( minus \) predicate, together with the value 1 from the holder cube right of \( minus \). The result of the subtraction flows into a recursive occurrence of the factorial cube. Its result flows in turn into a multiplication cube, which also takes \( i \), and returns the result to the right port.

In the next chapter, we will present the syntax and the semantics of CUBE.

3 The Language

In the following section, we will define the CUBE language in an informal fashion. In order to give readers who are familiar with traditional textual functional and logic programming languages a better understanding of the semantics of our language, we will also define a textual version of CUBE, which is similar to many functional and logic languages, and point out the correspondence between visual CUBE and textual CUBE.

The basic syntactic elements of CUBE are cubes, planes, pipes, and icons. We can categorize cubes into transparent holder cubes (or holders for short), transparent abstraction cubes, and opaque reference cubes. We can also categorize cubes into grey type cubes, and blue value cubes (we omit a discussion on the role of colors in execution animation). Type cubes hold, define, or refer to types, value cubes hold, define, or refer to values. Values are constructors, predicates, and variable instances.

A holder cube either holds or will eventually hold a value (or type). Two (or more) holder cubes may be connected by a pipe, if the types of the values (or types) inside them are compatible (where the type of a type is TYPE). If two holder cubes are connected, then the values (or types) they contain are unified. If a holder cube does not hold a value yet, CUBE instead fills it with the cube representing the inferred type of the value-to-be.

A naming cube is a special kind of holder cube, which carries an icon on its top. It defines a name for the value (or type) inside it. The scope of this name is the inside of the enclosing definition cube. Within this scope, each occurrence of a reference cube carrying the same icon will refer to the value (or type) inside the naming cube.

So, the two principal mechanisms in CUBE to propagate values (and types) are pipes, which are direct and explicit point-to-point links, and for which the issue of scope does not arise, and iconic names, which "broadcast" values from a naming cube to all corresponding reference cubes within a given scope.

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There are two kinds of abstraction cubes: type abstraction cubes, and predicate abstraction cubes (predicates are, as we will see, a special kind of value). Abstraction cubes must be surrounded (and thereby named) by a naming cube. For convenience, we usually show the two cubes as one, and refer to them as a definition cube. Definition cubes define "functions" which take a number of arguments, and return a result. In traditional textual languages, arguments are bound to formal parameters by position: The first argument is bound to the first parameter, etc. In CUBE, however, binding is done by name: Formal parameters are represented by naming cubes (called ports) inside the definition cube, such that their icons touch its walls. A reference cube corresponding to the named definition cube will have the transparent ports set into its opaque body. These ports can be moved over the surface of the reference cube, and are identified by their icon. They can be filled or supplied through a pipe with values, i.e. arguments. Inside the definition cube, one can either use pipes to propagate the values of ports, or use reference cubes to refer to them.

Note that we somewhat mix two concepts here: given a definition cube A with port B, we use B's icon to refer to B within the body of A. But, given a reference to A, we use B's icon to identify a port (we could not refer to this port, though, as B is local to the scope of A). So, we use internal names as external labels. This mixing of concepts, however, reduces the visual complexity of programs.


3.1 Types

In the following, we assume the reader to be familiar with the Hindley-Milner type system [5], which is used in many functional languages, like ML [12]. The fundamental ideas of this type system have also been applied to logic languages, namely Prolog [13][16]. Table 1 shows the textual syntax of types and type definitions (which is identical to the standard format, except that arguments are associated with parameters by name rather than by position). A type definition is of the form

\[ K = \lambda (X_1: \text{TYPE}, \ldots, X_n: \text{TYPE}). V_1 + \ldots + V_m \]  

\( (m \geq 0, n > 0) \)

is represented by a type definition cube (i.e. a type abstraction cube), representing the \( \lambda \), together with a type naming cube with icon \( \lambda \), naming a type constructor, and ports with names \( X_1, \ldots, X_n \). These ports take types, so their arguments are of type TYPE. TYPE is represented as an opaque grey cube
xi.

naming the constructor. Inside the plane below). Fig. 2.b shows a variant.

represented by a plane with a transparent icon

parameters on the top of the definition cube. Inside the type
cubes o ...)

holder connected (directly or indirectly) to the port icon). of

pdf

holder connected (directly or indirectly) to the port iconj.of

Types

tdf :: K = \Lambda (X_1:TYPE,...,X_m:TYPE), V_1 + ... + V_n

(m \geq 0, n > 0)

V :: k \{x_i:u_1,...,x_n:u_n\} (n \geq 0)

σ :: X_1 \rightarrow \sigma \{x_i:u_1,...,x_n:u_n\} (n \geq 0)

τ :: \alpha_1 \rightarrow \tau \{x_1:u_1,...,x_n:u_n\} \rightarrow \tau_0 (n \geq 0)

The relationship between pipes, reference cubes, naming cubes, holders, and textual variables is as follows: Given a (visual) CUBE program, assign a variable iconi to each naming and each reference cube. To each pipe or unnamed holder connected (directly or indirectly) to the port iconi of a definition cube, assign the same variable. Assign a variable x_i to each pipe and holder cube not labelled yet; such that (directly and indirectly) connected pipes and holders have the same name, and unconnected pipes and holders have different names.

Table 1: A textual syntax for CUBE
without any icons. By convention, we place ports for type parameters on the top of the definition cube. Inside the type definition cube are m different planes V_1,...,V_m, which represent the m different variants of the sum type. Fig. 2.a shows a type definition.

A variant V has the form k \{x_i:u_1,...,x_n:u_n\} (n \geq 0). V is represented by a plane with a transparent icon k on top, naming the constructor. Inside the plane are m opaque type cubes \sigma_1,...,\sigma_n. Above each type cube \sigma_i is a transparent icon x_i, which serves as a parameter name for the constructor (see below). Fig. 2.b shows a variant.

A type σ can have two forms:
(a) K \{X_1:σ_1,...,X_n:σ_n\} (n \geq 0), where K is a type constructor defined by a type definition cube within the current scope. The type is represented by a type reference cube with icon K and ports named X_1,...,X_n, and filled with types σ_1,...,σ_n. Fig. 2.c shows this form.
(b) X, which can be represented either by a type reference cube referring to a port (a type parameter bound in the \Lambda abstraction), or by a type holder cube connected (directly or indirectly) to a port by a pipe. Fig. 2.d shows this form.

In general, the user never has to declare the type of an expression, types can be inferred automatically. Type inference algorithms are described in [5] [16].

If a holder cube does not hold a value yet, CUBE will instead show the inferred type of the value-to-be inside the holder. A type expression τ can have three forms:
(a) A type constructor application K \{X_1:τ_1,...,X_n:τ_n\} (n \geq 0), where K is a type constructor defined by a type definition cube within the current scope. The type expression is represented by a type reference cube with icon K and ports named X_1,...,X_n, and filled with type expressions τ_1,...,τ_n.
(b) an uninstantiated type variable α, represented by an opaque grey cube (the representation of TYPE) with the grey number i in its top left corner.
(c) a function type \{x_1:τ_1,...,x_n:τ_n\} \rightarrow τ_0, represented by the type cube representing τ_0, with n ports named x_1,...,x_n and filled with types τ_1,...,τ_n set into its side.

Fig. 3 shows the different forms of type expressions.

<table>
<thead>
<tr>
<th>Fig. 3: Visual syntax of type expressions</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) K {X=T}</td>
</tr>
<tr>
<td>(b) \sigma</td>
</tr>
<tr>
<td>(c) {x:S} \rightarrow T</td>
</tr>
</tbody>
</table>

Fig. 4: Some predefined types

<table>
<thead>
<tr>
<th>Fig. 4: Some predefined types</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Int</td>
</tr>
<tr>
<td>(b) o</td>
</tr>
<tr>
<td>(c) TYPE</td>
</tr>
</tbody>
</table>
Fig. 4 shows the representation of some predefined types. Fig. 5 shows the definition of a polymorphic tree type, and instances of it.

\[
\text{Tree} \triangleq \lambda (X:\text{TYPE}). \\
\text{tree} \{\text{root}:X, \text{left}: \text{Tree} \{X-X\}, \text{right}: \text{Tree} \{X-X\}\} + \text{emptytree}
\]

Fig. 5: Tree Type

3.2 Uninstantiated Variables

In Horn Logic variables can be either bound to values, or they can be left uninstantiated. We treat uninstantiated variables as first-class-values. An uninstantiated variable \(a_i\) of type \(\tau\) is represented by the grey opaque type cube representing \(\tau\), with the blue number \(i\) in its bottom right corner. Fig. 6 shows two uninstantiated variables.

\[
a_3: \text{Int} \quad a_3: \alpha_1
\]

Fig. 6: Uninstantiated Variables

3.3 Constructors

Type definitions give rise to constructors. Constructors are used to construct values, and are first-class-values themselves. A type definition

\[
K = \Lambda (X_1:\text{TYPE}, \ldots, X_m:\text{TYPE}) \ldots + k \ (x_1:\tau_1, \ldots, x_n:\tau_n) + \ldots
\]

gives rise to a constructor \(k\) of type

\[
(x_1: \tau_1, \ldots, x_n: \tau_n) \rightarrow K (x_1: \alpha_1, \ldots, X_m: \alpha_m)
\]

where \(\alpha_1, \ldots, \alpha_m\) are new type variables and \(\tau_i = \sigma_i[[\alpha]/[X]]\) for all \(i\).

A constructor can be applied to values by filling (some of) its ports with values or by supplying them through pipes. An application \(k \ (x_1: \tau_1, \ldots, x_n: \tau_n)\) of terms \(x_1', \ldots, x_m'\) of types \(\tau_1', \ldots, \tau_m'\) to a constructor \(k\) is well-typed if \(k\) is of type \(\{x_1: \tau_1, \ldots, x_n: \tau_n\} \rightarrow \tau_0 \ (n \geq 0)\), each \(x_i'\) corresponds to an \(x_i\), and \(\tau_i\) and \(\tau_i'\) are compatible types, yielding a most general type unifier \(\theta\). The type \(\tau\) of the result is

\[
(\tau_i: \tau_0 \theta \mid x_j \text{ does not match any } x_i') \rightarrow \tau_0 \theta
\]

or, if \(\tau = \{\} \rightarrow \tau_0 \theta, \tau = \tau_0 \theta\).

Fig. 7 shows the tree constructor and its type, Fig. 8 the value tree \(\{\text{root}:1, \text{left}:\text{emptytree}, \text{right}:\text{emptytree}\}\) and its type. 1 is a nullary constructor belonging to the type \(\text{Int}\). Fig. 9 shows a curried application of a constructor.

\[
\text{tree} \{\text{root}:1, \text{left}:\text{emptytree}, \text{right}:\text{emptytree}\} \rightarrow \text{Tree} (\lambda X_1: \alpha_1) \\
\{\text{root}:1, \text{left}:\text{emptytree}, \text{right}:\text{emptytree}\} \rightarrow \text{Tree} (\lambda X_1: \alpha_1)
\]

Fig. 7: tree and its type

3.4 Predicates

In the following, we assume the reader to be familiar with Horn Logic [10] and Prolog [3]. Formally, an \(n\)-ary predicate is an \(n\)-ary relation, i.e. a set of \(n\)-tuples. For a given \(n\)-tuple \(x\), the predicate \(P\) is said to hold if \(x \in P\). For our purposes, it is more convenient to think of \(P\) as an \(n\)-ary function, which, when applied to \(x\), returns either success or failure, depending on whether \(P\) holds for \(x\) or not. success and failure belong to the predefined type \(o\) (see Fig. 4b), i.e. \(o = \text{success} \lor \text{failure}\).

So the type of an \(n\)-ary predicate \(P\) which takes arguments of type \(\tau_1, \ldots, \tau_n\) at parameters \(x_1, \ldots, x_n\) is \(\{x_1: \tau_1, \ldots, x_n: \tau_n\} \rightarrow o\).

The type rule for applying constructors to values holds as well when applying predicates to values (with \(\tau_0\) being \(o\)).
Let us establish a relation between CUBE and Prolog. The Prolog n-ary predicate \( p \) defined by
\[
p(t_{1},...,t_{n}) \leftarrow g_{1},...,g_{m_{1}},
\]
\[
p(t_{1},...,t_{n}) \leftarrow g_{k_{1}},...,g_{k_{m_{k}}},
\]
can be normalized to the (Prolog) predicate
\[
p(x_{1},...,x_{n}) \leftarrow (x_{1}=t_{1},...,x_{n}=t_{n},g_{1},...,g_{m_{1}}),...,
\]
\[
(x_{1}=t_{1},...,x_{n}=t_{n},g_{k_{1}},...,g_{k_{m_{k}}}).
\]
In CUBE, we \( \lambda \)-abstract the predicate to
\[
p = \lambda x_{1},...,x_{n},
\]
\[
(x_{1}=t_{1} \land ... \land x_{n}=t_{n} \land g_{1} \land ... \land g_{m_{1}}) \lor ... \lor
\]
\[
(x_{1}=t_{1} \land ... \land x_{n}=t_{n} \land g_{k_{1}} \land ... \land g_{k_{m_{k}}}).
\]
We also use binding-by-name instead of binding-by-position, and we infer types for parameters. In addition, we allow for local type and predicate definitions within a predicate (i.e., we introduce nesting). Finally, we treat predicates as first-class values, meaning that they can be passed around through pipes, and supplied as arguments to constructors and predicates. In this sense, CUBE is a higher-order Horn Logic language. Note, however, that CUBE uses only first-order unification (higher-order unification has been shown to be undecidable [8]). Unifying two predicates does not mean to test if they describe the same relation (which is undecidable), but just to test whether they syntactically unify (which is simple structural unification). There are other, more powerful — and more expensive — higher-order variants of Horn Logic (cf. [11]).

Given two holder cubes connected by a pipe, such that one is inside a plane and the other one is outside, we refer to the cube inside the plane as to the inner cube. If two holder cubes are not separated by a plane, we say they are on the same level.

\[
\text{plus}(arg_{1},arg_{2}) \leftarrow \text{res}=y_{2},
\]
\[
y_{1} \{ \text{arg}_{1}=y_{1}, \text{arg}_{2}=y_{2}, \text{res}=y_{4} \}
\]

Fig. 10 : Predicate Application

\[
y_{2} = \text{tree}\{ \text{left}=y_{1}, \text{right}=y_{2} \}
\]

Fig. 11 : Explicit unification

Atomic formulas or goals in CUBE can have two forms:
(a) A predicate application \( z \{ x_{1}=t_{1},...,x_{n}=t_{n} \} \) (\( n \geq 0 \)). \( z \) can be either a reference cube referring to a predicate definition cube or a predicate-taking port in the current scope, or a holder cube taking a value of a predicate-type.

(b) An explicit unification \( z = t \). This is represented by a holder cube representing \( z \) filled with a value cube representing \( t \) (see Fig. 11). If \( t \) is of type \( \tau \), then \( z \) is of the same type. \( z = t \) is of type \( \sigma \). If the unification fails, the cube fails. In Fig. 9 we have seen how this kind of atomic formula is used to describe curried application of constructors. The same technique can be used for predicates, as shown in Fig. 12.

\[
y_{2} = \text{plus}(arg_{1},y_{1}) \land y_{3} \{ \text{arg}_{1}=1, \text{res}=y_{3} \}
\]

Fig. 12 : Curried predicate application

Pipes are used to connect holder cubes (or ports as special cases of holder cubes). Two connected holder cubes must be of compatible type. Upon execution, their values are unified (one can think of unification as of bidirectional dataflow). If they do not unify, and they are both on the same level, they both fail, if they are on different levels, only the inner one fails.

A plane is of the form
\[
\text{con} :: df_{1} \land ... \land df_{m} \land af_{1} \land ... \land af_{n}
\]
It is represented by a plane surrounding the local definition cubes \( df_{1},...,df_{m} \) and the \( n \) cubes representing the atomic formulas \( af_{1},...,af_{n} \). These cubes may be connected by pipes, and pipes may leave the plane as well. Each plane corresponds to a clause in a Prolog program. A plane fails if any of the cubes inside it fail. It succeeds if all of them succeed. Fig. 13.a shows a plane.

\[
\text{af}_{1} \land \text{af}_{2}
\]
\[
p = \lambda (x:T) df_{1} \land df_{2} \land (\text{con}_{1} \lor \text{con}_{2})
\]

Fig. 13 : Plane and Predicate Definition

Predicates are defined by predicate definition cubes. A predicate definition \( pdf \) is of the form
\[
p = \lambda (x_{1}:T_{1},...,x_{2}:T_{2}) df_{1} \land ... \land df_{m} \land (\text{con}_{1} \lor ... \lor \text{con}_{n})
\]
It is represented by a predicate definition cube with icon \( p \) on its top, and ports named \( x_{1},...,x_{2} \) on its walls. The type inference system will determine the types of the ports, and fill
filter = \( \lambda (\text{in} : \text{List} (X = a_1), \text{out} : \text{List} (X = a_1), P : (\text{in} : a_1) \rightarrow o : (\text{in} = \text{nil} \land \text{out} = \text{nil}) \)

\lor \ (\text{in} - \text{cons} (h - y_1, t - y_2) \land P \ (\text{in} - y_1) \land \text{filter} (\text{in} - y_2, \text{out} - y_3, P) \land \text{out} - \text{cons} (h - y_4, t - y_5))

\lor \ (\text{in} - \text{cons} (h - y_4, t - y_5) \land \text{not} (\text{in} - P \ (\text{in} - y_4) \land \text{filter} (\text{in} - y_5, \text{out} - o, P - P)))

Fig 14 : filter — a higher-order predicate

them with type cubes \( \tau_1, \ldots, \tau_k \). A predicate definition cube can contain other local (predicate or type) definition cubes \( df_1, \ldots, df_n \), which are arranged horizontally. It will also contain a number of disjoint “clauses” \( con_1, \ldots, con_m \), represented by planes. Fig. 13.b shows a predicate definition cube. Given above predicate definition, an application \( p \ \{x_1 - t_1, \ldots, x_n - t_n\} \) succeeds if, upon unifying each \( x_i \) with \( t_i \), any plane \( con_i \) succeeds. It fails if all \( con_i \) fail.

A CUBE program has the form \( df_1 \land \ldots \land df_m \land af_1 \land \ldots \land af_r \). It is represented like a plane, except that there is no plane surrounding the local definitions and the atomic formulas. It succeeds if all \( af_i \) succeed, and it fails if any of them fails.

Note that, unlike in Prolog, in CUBE there is no ordering between atomic formulas or between planes. They are not evaluated in sequence, but rather concurrently.

It is a distinguishing feature of logic programming that a predicate application can succeed several times, each time obtaining different argument instantiations. For example, given a predicate \( p = \lambda (\text{out} : \text{Int}) . \text{out} - 1 \lor \text{out} - 2 \), the application \( p \ \{\text{out} - x\} \) will succeed once with \( x \) bound to 1, and once with \( x \) bound to 2.

Fig. 14 shows an example of a higher-order predicate: the logic equivalent of the well-known filter functional, which takes a list \( l \) of \( ts \) and a predicate \( P \) of type \( (\text{in} : \tau) \rightarrow o \), and returns a list of all those elements of \( l \) for which \( P \) holds.

As said before, the name of the port of a predicate (or constructor) is part of its type. This raises one problem. Suppose we want to apply a higher-order predicate, say filter, to a predicate \( P \) whose type is not \( (\text{in} : \tau) \rightarrow o \), but \( (\text{in} : \tau) \rightarrow o \). We could of course define a new predicate \( P' = \lambda (\text{in} : \tau) . P \ (\text{in} : \text{in}) \). In order to avoid this extra definition, we introduce the concept of a port renaming.

Given a term \( t \) of type \( \{x_1 : \tau_1, \ldots, x_n : \tau_n\} \rightarrow \tau_0 \), the type of \( t \ \{x' - x_1, \ldots, x'_n - x_n\} \) is \( \{x'_1 : \tau_1, \ldots, x'_n : \tau_n\} \rightarrow \tau_0 \). A port renaming is visualized by a transparent cube surrounding the cube representing \( t \), and a transparent icon \( x'_i \) over each port name \( x_i \). Fig. 15 shows a port renaming.

CUBE has a number of predefined predicates, e.g. greater, plus, ... and not. Some of these predicates will, when applied, suspend until their arguments are sufficiently
known. For instance, not takes a nullary predicate \( p \) (a value of type \( o \)) as argument, but will resolve only when \( p \) is ground. An application of a non-constant predicate will resolve only when the predicate is ground. \( \text{plus} \) takes three arguments \( a, b, c \) of type \( 
\text{Num} \), and computes \( c = a + b \), but will resolve only when at least two of them are ground. Similar mechanisms have been used in Concurrent Prolog [17] and Parlog [7].

It is worth mentioning that the language we described so far is not only much more expressive than Prolog, but also considerably cleaner. It does not rely on clause- or goal-ordering, it is side effect-free, and does not contain extra-logical constructs like Prolog's "cut". We believe that it is expressive enough to serve as a high-level general-purpose language.

Currently we are investigating how to include modules into CUBE, and how to incorporate a file system without sacrificing the purity (namely the confluence) of the language.

4 The Programming Environment

CUBE programs shall eventually be developed in a Virtual-Reality programming environment, which allows the user to directly manipulate the objects constituting the program, simply by grabbing, moving, and positioning them. Virtual Realities were first explored by researchers at NASA Ames. An overview of some of their research can be found in [6]. The hardware of a Virtual-Reality (VR) system consists of a set of position- and orientation-sensitive twin computer monitors mounted in front of the user's eyes that provide a stereoscopic view of computer-generated images, and a pair of gloves that can detect both their spatial position and orientation and the bending of the individual fingers. The software of a VR system generates a three-dimensional visual representation (called the Virtual Reality) of some objects, and allows the user to interact with this VR either by directly grabbing, moving, or otherwise manipulating the virtual objects, or by giving commands using a defined gesture protocol. Virtual Reality hardware is commercially available by now, and a number of applications have been developed for them [1][2].

5 Implementation

The implementation of CUBE is so far still in its infancy. At this point, we have implemented (in Prolog) a translation scheme which translates a subset of CUBE into a textual representation, and we have a working interpreter for an untyped version of the textual version of CUBE (written in Lazy ML). We are currently implementing a renderer to visualize CUBE programs, and plan next to integrate the different components.

6 Conclusion

We have informally described CUBE, a new language with a three-dimensional visual syntax, designed for a virtual-reality based programming environment. CUBE has a static polymorphic type system, and supports type inference. It is based on higher-order Horn Logic, thereby treating predicates as first-class values and allowing for higher-order predicates. It also allows the definition of local types and predicates, i.e. brings the concept of scope and nesting to logic programming. CUBE's semantics is inherently concurrent, allowing a parallel implementation.

Bibliography