Visualization of Second Order Tensor Fields and Matrix Data

Thierry Delmarcelle
Department of Applied Physics
W. W. Hansen Laboratories
Stanford University
Stanford, CA. 94305 - 4090

Lambertus Hesselink
Department of Electrical Engineering
Department of Aeronautics and Astronautics
Stanford University
Stanford, CA. 94305 - 4035

Abstract
We present a study of the visualization of 3-D second order tensor fields and matrix data. The general problem of visualizing unsymmetric real or complex Hermitian second order tensor fields can be reduced to the simultaneous visualization of a real and symmetric second order tensor field and a real vector field. As opposed to the discrete iconic techniques commonly used in multivariate data visualization, the emphasis is on exploiting the mathematical properties of tensor fields in order to facilitate their visualization and to produce a continuous representation of the data. We focus on interactively sensing and exploring real and symmetric second order tensor data by generalizing the vector notion of streamline to the tensor concept of hyperstreamline. We stress the importance of a structural analysis of the data field analogous to the techniques of vector field topology extraction in order to obtain a unique and objective representation of second order tensor fields.

1. Introduction

3-D tensor fields of various orders belong to a special class of multivariate datasets. They are particularly interesting not only because they represent important physical phenomena, but they have mathematical properties that simplify the task of visualizing them.

First order 3-D tensor fields (vector fields) are not made up of just three independent scalar functions. The individual components of a vector field obey a specific transformation law under a change of coordinates which allows association of a vector with a simple graphic primitive such as an arrow (having a magnitude and a direction). Vector fields are often visualized in terms of their streamlines, i.e. the lines obtained by spatially integrating the vector field at a fixed time [1]. This leads to the more abstract depiction of vector fields in terms of their topology [2-5] which is obtained by extracting the critical points [6] of the vector data and linking them with their connecting streamlines. The topological representation of vector fields produces a graph depiction of the data which is unique and objective, i.e. independent of the initial conditions of integration. Further, vector field topology extraction can be fully automated [2-5]. In what follows, the name tensor fields stands implicitly for second order tensor fields and first order tensor fields are simply called vector fields.

Visualizing 3-D tensor fields is difficult. The tensor data consist of six or nine scalar functions whose independent visualization is possible but meaningless. Except for the Stream Polygon [7], a 3-D vector field visualization technique which can be readily applied to the visualization of vector field gradients, the problem of visualizing general tensor fields has not yet been addressed. In section 2 we demonstrate how a broad class of tensor data, i.e. unsymmetric real and complex Hermitian tensor fields, can be decomposed into a real and symmetric tensor field and a vector field. This decomposition is the first step toward a structural depiction of tensor fields.

In section 3 we focus on the specific problem of visualizing real and symmetric tensor fields by introducing the concept of hyperstreamline which owes its name to the analogy with the conventional streamline used in vector field visualization. Ellipsoidal icons have been used to visualize real and symmetric tensor fields [8-9] but a discrete description does not emphasize the continuous character of the data and our visual system is not adapted at perceiving information in a 3-D array of ellipsoids. Tensor glyphs [10] are other discrete icons that were used to visualize the time-varying stresses at a given location in problems of fracture dynamics. As pointed out by Dickinson [11] most of the other visualization techniques consist in extracting a scalar function from the tensor data. This is sufficient for some applications like detecting the boundary of a plastic region in an elastic stress field. However, understanding the physics of fluids, of polymer solutions, or of materials in the plastic or thermoplastic regime, requires a far more complete depiction of the data. Dickinson [11-12] recognized the importance of recovering the directional information contained in the tensor data by introducing the...
concept of tensor field lines, i.e. lines that are everywhere tangent to one of the eigenvectors. The hyperstreamlines generalize this idea in order to represent all the information present in the tensor data along these trajectories. In section 4 we provide results concerning tensor fields in the air flow past a hemisphere cylinder.

Before concluding we stress in section 5 how the mathematical properties of tensor fields and the decomposition of section 2 can lead to a unique and objective representation of tensor data.

2. Reduction of the Dimensionality

Real unsymmetric tensor fields and complex Hermitian tensor fields can be reduced to a real and symmetric tensor field and a real vector field. Real unsymmetric tensor fields can have complex eigenvalues and complex non-orthogonal eigenvectors. For the purpose of visualizing these tensor fields it is necessary to reduce them to a real and symmetric tensor field that possesses real eigenvalues and real orthogonal eigenvectors.

2.1. Reduction of real unsymmetric tensor fields

Depending on the physics involved, two reductions are possible. The first consists in decomposing the tensor field into the sum of a symmetric component and an antisymmetric tensor according to

\[ T(\tilde{x}) = \frac{T(\tilde{x}) + T^t(\tilde{x})}{2} + \frac{T(\tilde{x}) - T^t(\tilde{x})}{2} \]

where \( T^t(\tilde{x}) \) represents the transpose of \( T(\tilde{x}) \). For instance, the velocity gradient in fluids is the sum of the rate-of-strain tensor and the rate-of-rotation tensor or half the vorticity. The antisymmetric component is equivalent to a vector field.

The second reduction consists of carrying out a Polar Decomposition [13] of the tensor field:

\[ T(\tilde{x}) = Q(\tilde{x}) U(\tilde{x}) = V(\tilde{x}) Q(\tilde{x}) \]

where \( U(\tilde{x}) \) and \( V(\tilde{x}) \) are symmetric positive definite tensor fields and \( Q(\tilde{x}) \) is an orthogonal tensor field. This decomposition is unique and valid where \( T(\tilde{x}) \) is invertible. The Polar Decomposition owes its name to the analogy with the decomposition of a complex number in phase \( Q(\tilde{x}) \) and amplitude \( U(\tilde{x}) \) or \( V(\tilde{x}) \). The Polar Decomposition (2) conveys more structural information than the simple symmetric / antisymmetric decomposition (1). Indeed, if the tensor field \( T(\tilde{x}) \) is conceived as a deformation in a hypothetical 3-D space, \( Q(\tilde{x}) \) is the isometric component of the deformation and both \( u(\tilde{x}) \) and \( v(\tilde{x}) \) represent the stretching component. \( u(\tilde{x}) \) and \( v(\tilde{x}) \) are the square root of the right and left Cauchy-Green tensors [13], respectively. Representing the tensor field \( T(\tilde{x}) \) by one of its stretching components conserves the amplitude information. Indeed, \( Q(\tilde{x}) \) being orthogonal, the equality

\[ \| u(\tilde{x}) \| = \| v(\tilde{x}) \| = \| T(\tilde{x}) \| \]

holds for every vector \( \tilde{u} \). In particular, if \( \tilde{u} \) is the eigenvector of \( T(\tilde{x}) \) corresponding to the eigenvalue \( \lambda \) (real or complex)

\[ \| u(\tilde{x}) \| = \| v(\tilde{x}) \| = |\lambda| \| \tilde{u} \| \]

The tensor field \( Q(\tilde{x}) \) is equivalent to a rotation field in the regions of space where \( \det T(\tilde{x}) > 0 \) or to a rotation field multiplied by an inversion field where \( \det T(\tilde{x}) < 0 \). In both cases, \( Q(\tilde{x}) \) is representable like the vector field

\[ \tilde{r}(\tilde{x}) = \theta(\tilde{x}) \tilde{u}(\tilde{x}) \]

where \( \theta(\tilde{x}) \) is the local rotation amplitude and \( \tilde{u}(\tilde{x}) \) is the local unit axis of rotation or of rotation / inversion.

2.2. Reduction of complex Hermitian tensor fields

A practical case is the susceptibility tensor in photorefractive materials or in optically active media. Generally, these tensors transform a complex vector \( \tilde{e}(\tilde{x}, t) \) into another complex vector \( \tilde{d}(\tilde{x}, t) \) according to

\[ \tilde{d}(\tilde{x}, t) = H(\tilde{x}) \tilde{e}(\tilde{x}, t) \]

Since \( H(\tilde{x}) \) is Hermitean, it is the sum of a real and symmetric part \( H_r(\tilde{x}) \) and an imaginary antisymmetric part \( H_i(\tilde{x}) \):

\[ H(\tilde{x}) = H_r(\tilde{x}) + iH_i(\tilde{x}) \]

where \( i = \sqrt{-1} \), \( H_r(\tilde{x}) = H_r^t(\tilde{x}) \), and \( H_i(\tilde{x}) = -H_i^t(\tilde{x}) \). \( H_r(\tilde{x}) \) is equivalent to a vector field. The decomposition of \( H(\tilde{x}) \) into a real and symmetric tensor field and a vector field corresponds to the physical decomposition into in-phase and quadrature components.
3. Real and symmetric tensor fields: hyperstreamlines

Visualizing real and symmetric tensor fields is an important problem per se. Also, the reduction presented above shows that it is a necessary step in order to visualize broader classes of tensor fields. From now on, we focus on a specific methodology to visualize real and symmetric tensor fields.

A real and symmetric tensor field \( U(\vec{x}) \) has three real eigenvalues and three real and orthogonal eigenvectors at every point \( \vec{x} \) in space. Instead of visualizing the individual components of \( U(\vec{x}) \) we consider the orthogonal vector fields

\[
\vec{v}^{(i)}(\vec{x}) = \lambda^{(i)}(\vec{x}) \vec{e}^{(i)}(\vec{x})
\]

for \( i = 1, 2, \) or \( 3 \). \( \lambda^{(i)}(\vec{x}) \) are the eigenvalues and \( \vec{e}^{(i)}(\vec{x}) \) the associated unit eigenvectors. The spectral decomposition of \( U(\vec{x}) \) in a sum of projectors on its eigenspaces [14] leads to a simple relation between the individual components of \( U(\vec{x}) \) and its eigenvectors:

\[
U_{pq}(\vec{x}) = \sum_{i=1}^{3} \lambda^{(i)}(\vec{x}) e_p^{(i)}(\vec{x}) e_q^{(i)}(\vec{x})
\]

where \( p \) and \( q \) vary from 1 to 3. Thus, at every point \( \vec{x} \) the component \( U_{pq}(\vec{x}) \) is a sum of all the eigenvalues multiplied by the \( p \) and \( q \) components of the corresponding unit eigenvectors. Visualizing a real and symmetric tensor field \( U(\vec{x}) \) is fully equivalent to visualizing simultaneously the three orthogonal vector fields (8). We are displaying the information contained in the three vector fields in a unique display in order to enlighten the correlations between the different vector fields. For this purpose we introduce the concept of a hyperstreamline: as opposed to the conventional streamlines used in vector field visualization, a geometric primitive having a finite size sweeps along one of the eigenvector fields \( \vec{v}^{(i)}(\vec{x}) \) while stretching in the transverse plane under the combined action of the two other orthogonal eigenvector fields. The surface obtained by linking the stretched primitives at the different points along the trajectory is called a hyperstreamline and is color-coded according to the algebraic amplitude of the longitudinal eigenvalue and the hue (magenta, blue, cyan, green, yellow, red) maps linearly the longitudinal eigenvalue in the range \([-3,0]\). All the lines converge towards the regions near the points on the surface where the forces are applied. For the B2 problem, note the surface of separation in the middle where two trajectories very close to each other suddenly diverge towards one or another region of high stress. Similarly, trajectories along the two other eigenvectors (8) delineate a basket-shaped surface that is everywhere perpendicular to the most compressive direction.

The tensor data are diagonalized in physical space using an iterative Jacobi algorithm. The eigenvectors are transformed to computational space using the grid Jacobian and integrated by means of a fifth order adaptive Runge-Kutta algorithm. At the end of the computation, the trajectories are transformed back to physical space using the inverse grid Jacobian. Integrating tensor field lines is a far more complicated problem than computing the streamlines of common vector fields. Difficulties arise from the fact that 1) the sign of the eigenvectors is not determined and 2) degeneracies can occur along the trajectory and in between the sampling points requested by the integration routine. A simple continuity check of the longitudinal
eigenvector along the trajectory takes care of the sign indeterminacy. In order to deal correctly with degeneracies, it is assumed that the tensor field is smooth, i.e. that the direction (in physical space) of the longitudinal eigenvector is not likely to vary by more than a user-predefined angle between two successive sampling points unless the trajectory just crossed a degeneracy involving the longitudinal eigenvalue. In this case, the program detects the degeneracy between the last two sampling points. The points where the transverse eigenvalues vanish are also detected and included to the curve in order not to miss a singularity of the cross-section of the hyperstreamline.

3.2. Cross-section of a hyperstreamline

The geometry of the cross-section of the hyperstreamlines codes the two eigenvector fields orthogonal to the trajectory. The choice of the geometric primitive sweeping along the trajectory depends on what kind of information the user is interested in. We consider two types of primitive: 1) a circle; stretching into an ellipse while sweeping and generating a hyperstreamline called a tube; and 2) a cross, generating a hyperstreamline called a helix. In a tube, the principal semi-axes of each elliptical cross-section are along the direction of the transverse eigenvectors and have a length proportional to the transverse eigenvalues. The same property holds for a helix, whose arms are proportional to the transverse eigenvectors. This encodes both directional and amplitude information along the trajectory. The local sign of the transverse eigenvalues can be understood by detecting the singularities of the cross-section of the hyperstreamline. Indeed, they are indicative of a sign change of one of the transverse eigenvalues (assuming that there is no even zero along the trajectory). In such points the cross-section degenerates into a line or a point.

The tube and the helix encode the same information about the tensor field, but some aspects are easier to perceive with one hyperstreamline than with the other. For instance, the tube shows better where the tensor is degenerate in the transverse plane, since recognizing that an ellipse is circular is easier than comparing the length of two perpendicular line segments. If the tensor field is transversely degenerate in a whole region of space, helices are not adequate since the direction of the transverse eigenvectors is not determined. On the other hand, the helix gives better clues to perceive the direction of the transverse eigenvectors. By using equation (9) the user can mentally infer the relative values of the individual components of the tensor with little or no training. This is particularly clear when looking at the bottom-right picture in figure 1. The vertical hyperstreamline is a tube of circular cross-section propagating along the most compressive eigenvector. The tensor is degenerate along the trajectory and using a helix would be meaningless. The other hyperstreamline is along the intermediate eigenvector field and follows a circular trajectory. The long arm of the helix corresponds to the most compressive direction and the short arm is along the major eigenvector. Equation (9) implies that, along the circle, $U_{22}$ is large and constant (it is the sum of a large DC component produced by the most compressive eigenvector and of a smaller one produced by the least compressive eigenvector), whereas $U_{12}$ and $U_{23}$ are smaller and vary sinusoidally.

Figure 2 shows hyperstreamlines for the B3 problem. The color-coding is as in figure 1. The top-left picture exhibits a tube (front) and a helix (back). The other pictures show the tube at three different stages during its development. Top-right: the cross-section is circular and the tensor field is transversely degenerate; bottom-left: the cross-section degenerates into a straight line, the tensor field is locally singular, and the stresses are locally 2-D; bottom-right: the stresses are 3-D again, and the eigenvectors underwent a rapid rotation and a substantial stretching which reveals a fairly important gradient of shear and pressure in the region.

4. Example of the flow past a hemisphere cylinder

Figures 3 and 4 show hyperstreamlines of the reversible momentum flux density tensor and the stress tensor in an incompressible air flow past a hemisphere cylinder. The flow is symmetric with respect to the horizontal plane containing the axis of the cylinder. The direction of the incoming air flow is $5^\circ$ to the left of the cylinder axis in the plane of symmetry and the Reynolds number is 14,000.

4.1. Reversible momentum flux density tensor

The reversible transfer of momentum density in a fluid is characterized [16] by the tensor field

$$\Pi_{ik}(\mathbf{x}) = p(\mathbf{x}) \delta_{ik} + \rho(\mathbf{x}) v_i(\mathbf{x}) v_k(\mathbf{x})$$  \hspace{0.5cm} (10)$$

where $p(\mathbf{x})$ is the pressure, $v_i(\mathbf{x})$ and $v_k(\mathbf{x})$ are velocity components, and $\rho(\mathbf{x})$ is the mass density (constant in this case). $\Pi_{ik}(\mathbf{x})$ represents the component $k$ of the reversible (i.e. non viscous) momentum density that is transferred at $\mathbf{x}$ per unit time through a unit surface element oriented in the direction $i$. $\Pi_{ik}(\mathbf{x})$ is symmetric and the eigenvector field corresponding to the major eigenvalue is in the direction of the velocity field. Indeed, $\Pi_{ik}(\mathbf{x})$ can be diagonalized as
where $v(\vec{x})$ is the velocity magnitude. Equation (11) shows that $\Pi_{\alpha\beta}(\vec{x})$ is degenerate in the whole space. Only tubes along the major eigenvector field can be used. The cross-section of the tubes is circular everywhere and its size codes the local pressure. Results are shown in figure 3. The color mapping function is chosen as

$$
\lambda_1(\vec{x}) - \frac{\lambda_2(\vec{x}) + \lambda_3(\vec{x})}{2}
$$

where $\lambda_1(\vec{x}) \geq \lambda_2(\vec{x}) \geq \lambda_3(\vec{x})$, so the hue maps linearly the density of kinetic energy $0.5 \rho(\vec{x}) v^2(\vec{x})$. Magenta, blue, cyan, green, yellow and red represent increasing values of the density of kinetic energy in the interval $[0.05, 0.87]$ (dimensionless units). This example shows that hyperstreamlines are useful in correlating different physical quantities of the flow, in this case the pressure and the density of kinetic energy. In this incompressible flow this is equivalent to correlating the pressure and the velocity magnitude.

The bottom picture shows the transfer of momentum along the body when starting the integration 20 grid planes away from the cylinder. The detachment around the end of the cylinder is well visible. Note the change in pressure (diameter) and in kinetic energy (color) associated with a sudden change in the direction of the three foremost tubes. The hyperstreamlines indicate that the momentum is transferred from the tip to the end parallel to the body in a fairly uniform fashion. The top picture shows the momentum transfer when starting the integration 10 grid planes away from the body. The pattern is different since there is a region close to the tip where the momentum recirculates and is not transferred towards the end of the cylinder. In both plots the hyperstreamline trajectories are similar but not identical to the streamlines of the velocity field. The local derivative along the trajectory is along the direction of the velocity but has a different magnitude.

4.2. Stress tensor

In an isotropic Newtonian viscous fluid the stress tensor [16] is given by

$$
\sigma_{ik}(\vec{x}) = -p(\vec{x}) \delta_{ik} + \lambda \frac{\partial}{\partial x_i} v_1(\vec{x}) \delta_{ik} + \mu \left( \frac{\partial}{\partial x_i} v_k(\vec{x}) + \frac{\partial}{\partial x_k} v_i(\vec{x}) \right)
$$

where $\lambda$ and $\mu$ are the viscosity coefficients. In an incompressible flow the divergence term vanishes and the stress tensor is reduced to an isotropic pressure component and a viscous term proportional to the strain tensor. The latter determines the direction of the eigenvectors but the eigenvalues involve a contribution of both the pressure and the viscous stresses. In this dataset, the three eigenvalues are negative or zero.

Hyperstreamlines of the stress tensor field are shown in figure 4. The tubes in front are along the eigenvector field corresponding to the major eigenvalue, i.e. the least compressive one. The direction of these tubes show how the forces propagate from the region in front of the cylinder to the surface of the body. The color codes the (algebraic) intensity of the tracked eigenvalue along the trajectory in the range [-1.2, -0.7] (dimensionless units). It is interesting to note that all the tubes end up in approximately the same section of the body. Away from the body, the cross-section of the tubes is almost circular, indicating, as expected, that the stresses induced by pressure are dominant. Closer to the body, the viscous stresses progressively increase the anisotropy of the cross-section of the tubes. The yellow surface shown in figure 4 is the isosurface

$$
e(\vec{x}) = 1 - \left( \frac{\lambda_2(\vec{x})}{\lambda_3(\vec{x})} \right)^2 = 0.1
$$

which represents the locus of the points where the eccentricity of the cross-section of the tubes along the least compressive direction is equal to $10\%$.

The helices are along the eigenvector field corresponding to the intermediate eigenvalue. The integration starts 20 grid planes away from the body and the helices propagate mostly parallel to the cylinder. However, the helices further along the cylinder suddenly detach from the body. The third helix ends at a triple degeneracy of the intermediate eigenvector field and exhibits a strong variation in color when detaching from the body, indicating that both the direction and the magnitude of the intermediate eigenvector field vary significantly in the zone of detachment. The other helices have a much more uniform color, suggesting that the amplitude of the intermediate eigenvector field varies little along the trajectory. By comparing the color of different helices, it appears that the intermediate principal stress is less compressive in the region of contact between the tubes and the body. The arms of the helices are along the two other eigenvector fields, i.e. the least and the most compressive directions. The arms have a fairly constant direction along the trajectories except in the detachment region of the third helix, suggesting that the stress tensor is less uniform in this region of the flow than in other parts. Figure 5 shows a
5. Toward a structural depiction of tensor fields

Two factors limit the practicality of hyperstreamlines: 1) the resulting display depends on the initial conditions of integration and 2) a large number of hyperstreamlines produces a visual clutter. The same problem arises in 3-D scalar and vector field visualization. When visualizing a scalar field with isosurfaces, the final image depends on the particular isosurfaces chosen and only a few isosurfaces can be displayed simultaneously. The conventional streamlines used in vector field visualization lead to a display depending on the initial conditions of integration and too many streamlines clutter the display. In the latter case, these problems are overcome by algorithms extracting automatically the vector field topology [2-5]. In tensor field visualization, objective features can also be extracted that provide a structural depiction of the data.

5.1. Real and symmetric tensor fields

A vector field topology extraction can be seen as a process of coding the collective behavior of a large number of streamlines. Similarly, a structural depiction of real and symmetric tensor fields is a process of coding the collective behavior of a collection of hyperstreamlines. The structure of a real and symmetric tensor field could be objectively represented by the three orthogonal topologies of the eigenvector fields (8) but this would require an unambiguous sign convention in order to suppress the intrinsic sign indeterminacy of the eigenvector fields. Assuming that this can be done, two categories of points have to be considered: 1) the critical points (6) of each of the vector fields (8) and 2) the location of the degeneracies acting as connecting points of the orthogonal topologies. The surface \( \lambda^{(i)}(\vec{x}) = 0 \) is the locus of the critical points of the corresponding eigenvector field \( \vec{v}^{(i)}(\vec{x}) \) and the isosurface \( \det(U(\vec{x})) = 0 \) is the locus of the critical points of the three eigenvector fields (8). The isosurface \( \lambda^{(i)}(\vec{x}) \lambda^{(k)}(\vec{x}) = 0 \) is the locus of points where the cross-section of the hyperstreamlines along the eigenvector field \( \vec{v}^{(i)}(\vec{x}) \) is singular, i.e. reduced to a straight line or a point. An example for the B2 problem is given in figure 6. The green surface is the locus of the critical points of the intermediate eigenvector field and the yellow surface represents the critical points of the major eigenvector field. The two surfaces together represent the locus of points where the cross-section of the hyperstreamlines along the most compressive direction is singular. In the case of fluid flow tensor fields, isolated critical points are also expected.

Other surfaces characterize the collective behavior of hyperstreamlines. For example, the surface \( \lambda^{(j)}(\vec{x}) - \lambda^{(k)}(\vec{x}) = 0 \) is the locus of points where the cross-section of the hyperstreamlines along the eigenvector field \( \vec{v}^{(j)}(\vec{x}) \) is degenerated into a circle. Unfortunately, this kind of surface is very difficult to extract since it is the solution of a problem of extremum tracking and standard techniques of isosurface extraction like the Marching Cube algorithm [17] can not be applied to this case. The isosurface of constant eccentricity in figure 4 is another example of coding the collective behavior of hyperstreamlines.

5.2. Real unsymmetric and complex Hermitian tensor fields

In section 2, broader classes of tensor fields are reduced to a real and symmetric tensor field and to a vector field \( \vec{v}(\vec{x}) \). In addition to the surfaces described above, the locus of the points where the tensor data are symmetrical (i.e. real for complex Hermitian tensor fields) is the locus of the critical points of the vector field \( \vec{v}(\vec{x}) \). If the tensor field is reduced by means of a Polar Decomposition, it is also the isosurface \( \theta(\vec{x}) = 0 \).

6. Conclusions

Visualizing tensor fields is a specific problem of multivariate data visualization. As opposed to a component per component approach or a discrete iconic treatment of the raw data, we exploit the mathematical properties of tensors in order to facilitate their visualization. The particular problem of visualizing real and symmetric tensor fields is important per se and appears as an essential step in visualizing broader classes of tensor fields. Hyperstreamlines are an adequate tool for sensing and exploring real and symmetric tensor data. They generalize to tensor fields the notion of conventional streamlines used in vector field visualization. The limitations of the hyperstreamline technique are pointed out and we emphasize the need for a structural depiction of tensor fields aimed at providing a unique and objective characterization of the data. We consider the equivalent problem of characterizing the collective behavior of hyperstreamlines. Much more research remains to be done in order to be able to fully visualize a tensor field in terms of a structural depiction. The needs and the techniques used in tensor field visualization are similar to those encountered in vector field visualization. This lays out a coherent picture for the visualization of tensor fields of general order.
Acknowledgments

We wish to thank Yuval Levy for providing us with interesting datasets and Paul Ning for his initial implementation of the Marching Cube algorithm upon which our isosurface extraction algorithm is based. This work is supported by NASA under contracts NAG 2-701 and NCA 2-579, including support from the NASA Ames Numerical Aerodynamics Simulation Program and the NASA Ames Fluid Dynamics Division, and by NSF under grant ECS 8815815.

References

Figure 1: Hyperstreamlines in the B1 (bottom) and the B2 (top) problems. Color coding: see text.

Figure 2: (Top-left) hyperstreamlines in the B3 problem. (Others) the tube at three different stages.

Figure 3: Pattern of reversible momentum density transfer in the airflow past a hemisphere cylinder.

Figure 4: Stress tensor in the airflow past a hemisphere cylinder. Color coding: see text.

Figure 5: Closer view of the third helix of figure 4.

Figure 6: Structural depiction of the stress tensor field in the B2 problem. Color coding as in figures 1 and 2.

(See color plates, p. CP-34.)