Visualizing Causal Effects in 4D Space-Time Vector Fields

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Abstract

In this paper, we present a method to juxtapose 4D space-time vector fields in which one contains a "source" variable and the other the "response" field. The technique helps highlight the topological relationship between the two in an effort to understand the causal connection. These concepts are applied to ongoing research in evolving fluid dynamical problems.

1 Introduction

The goal of large-scale simulations and experiments in computational science is a quantitative and mathematical understanding of the model being investigated. A key component in this process is visualization: the use of computer graphics techniques to depict and project the resulting data. Visualization provides access to new areas of exploration and allows us to study observed phenomena in greater detail. However, visualization alone is not sufficient. Quantitative techniques, based upon physical-mathematical properties, are needed to reduce the amount of data and highlight the areas of interest; for example, searching for and comparing four-dimensional space-time features as a function of model, parameter values and numerical resolution.

When a feature or subregion is undergoing interesting metamorphosis, it should be targeted for further study. Initial quantification involves tracking the kinematics and the morphological change of the feature. Once these are understood, one can delve deeper into the substructure. While the topological change occurs on a large scale level, there are many subprocesses occurring on a smaller scale, inside the region. Furthermore, fundamental issues as to the origins or causes of certain observed phenomena must be investigated. The interaction of the cause, or source of the change, with the original ongoing process can provide unique insights into properties of the underlying model. This causal effect is a co-process, and in many simulations can either be derived from the original data, or measured directly. The goal then, is to analyze the relationship between a temporal process and a related co-process (the source or cause). In 3D, the effect is topological, so visualization plays a key role. However, since both of these variables may be 4D vector fields (3D for each time step), the amount of information to visualize is overwhelming. Therefore, sophisticated model-related techniques are needed to reduce the data and highlight the causal connection.

In this paper, we present a method to juxtapose 4D space-time vector fields. Although the work is based upon numerical simulations of fluid flow, the concepts are applicable to magnetohydrodynamics and, in general, to the study of 4D vector fields.

The next section contains a brief mathematical background to the concepts being discussed. In Section 3, the visualization procedure is outlined and is followed by an example from ongoing studies in vortex reconnection.

2 Causal Connection

In physical processes, there are localized source and sink terms, i.e. variables that effect the overall behavior of the underlying evolution. In the case of vorticity, the important vortex stretching term may be considered as a source - creating the necessary conditions for the onset of a topological change (creating vorticity which may be considered the source of rotational flow).
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Figure 1: Source and Response

source response  

Time

Figure 2: Flow Process

If seen in time, the two complement each other as shown (Figure 1):

The source causes a response later in time. For fluid flow, a causal model can be represented as follows:

(In numerical simulations all of the variables listed in Figure 2 are available, however, this is not the case in experiments. The particle trajectories are obtained from flow visualization and the velocity from either hot wire devices or laser velocimetry. Vorticity is still difficult to measure.)

In this work, we are investigating the effects of the vortex stretching term on weak regions of vorticity. Mathematically, the source vorticity can be derived from the original vorticity equation. Applying the curl operation to the incompressible Navier-Stokes equation, yields an equation for the evolution of vorticity:

$$\frac{D\omega}{Dt} = \omega \cdot \nabla u + \nu \nabla^2 \omega.$$  (1)

The second term on the right hand side of the equa-

2.1 Motivation

The physical-space interactions which lead to the collapse and reconnection of vortex tubes and the accompanying cascade to small-scale vortex debris is a fundamental problem in 3D vortex dynamics and turbulence. They are observed in wing-tip vortices (contrails) of high-flying aircraft, in boundary layer “hairpin” vortices, and in simulations of liquid helium. To understand this process, numerical simulations of the evolution of two vortex tubes initially perpendicular to each other have been carried out [2]. The ratio of the strengths of the vortex tubes ($\Gamma_1/\Gamma_2$) is approximately 1. By examining the magnitudes of vorticity (scalar quantity) at a low threshold (27% of magnitude) the topology of the reconnection process is evident (see Plate 1 and [5]).

In the initial time sequences, two distinct orthogonal tubes are evident (t1). As the tubes evolve, the central portion of the tubes begin a process of reconnection (t5). Bridges form along this region, eventually connecting the top portion of the tubes to each other, and the bottom portion of the tubes to each other, forming two new horizontal tubes (t8). This is known as reconnection.

To understand the precise mechanism for reconnection, we are currently investigating the relationship between the time evolving vorticity and the stretching term (the source). Both of these variables are 4D fields (3D and time). It is the topological relationship which is fundamental to understanding the dynamics of the model. This type of relationship will help explain the highly intense reconnection process which occurs in intermittency, turbulence, “magnetic” storms, etc.

3 Visualization

To summarize the goal: given the topological nature of the causal relationship, what visualization tech-
niques can be used to accurately portray it? A major concern is the amount of data to be analyzed and displayed. There are two 3D vector data sets for each time step. Unfortunately, superimposing the datasets and using conventional methods to view the vector fields (e.g. with arrows or vortex lines) is overly confusing and will most likely obfuscate the focal regions. Furthermore, there is ongoing topological transformations occurring caused by the co-process. What is needed is a method to highlight the area undergoing evolution (from the original dataset) and juxtapose it with the source or cause directly effecting it. This is basically a two step process: first, to extract the topology from each data set, then to highlight the area of interaction and display the result. To do so, we have used a combination approach utilizing thresholding, ellipsoidal quantification, and vortex line generation. The entire process is outlined in Figure 3.

3.1 Ellipsoidal Quantification

The topology of the dataset can be analyzed and extracted using a thresholding procedure (with isosurfaces [4]). However, because the two processes will be studied together, it is important that the complexity of each does not overshadow the underlying interaction. The key is to depict these extracted regions simply, i.e. an abstraction is needed to capture the essential features (and reduce the clutter).

First, the magnitudes of the vector field are computed, to obtain a scalar quantity. Thresholding and isosurface generation are simple methods which determine regions in a scalar field, since they define a subset of the field constituted by areas in which the associated scalar quantity is equal to or larger than the threshold value. Varying the thresholds separates the field into substructures [3]. Combining this with other diagnostic functions allows one to measure and track the dynamic ranges of the data (the range of values at which structure appears).

During the process of thresholding, generic shapes appear. By further thresholding, we find that physical space moments about extrema can be used for abstraction. Thus, by isolating regions of the field and then computing the second moment, essential information at a first level of quantification is obtained. The second moments defines a tensor, which can be associated with an oriented ellipsoid. Furthermore, ellipses show up in analytical solutions to some problems in incompressible fluid dynamics (2D). Thus the use of the ellipsoids (ellipses) can be a step towards the mathematization process.

The ellipsoids are described by the centroid of the object, and the eigenvalues and eigenvectors of the tensor of moments. (See [6] [7] for more detail.) They are simple abstraction, yet provide a sense of position, orientation, and relative weight to the underlying region. In addition, the ellipsoids offer a “point of contact” to the 3D vector datasets, i.e. instead of random field lines, the field lines are generated emanating from the ellipsoids.

3.2 Vortex Lines

After extracting the ellipsoids, vortex lines are now computed from the vector dataset. The vortex lines are generated along the minor elliptical cross-section of the ellipsoid (minor ellipse). These lines connect the various ellipsoids clarifying their topological link. This entire process is done to both datasets in an effort to understand the individual dynamics. By mapping the resulting information onto the same physical space, the causal connection can now be investigated. Because the data has been significantly reduced, the topological interaction of the process and its co-process is highlighted.

A crucial part of this procedure is the ability to interact real-time with the data, i.e. stepping between threshold values (for the ellipsoids and the isosurfaces), vortex lines, and time for each dataset, all while viewing the two variables juxtaposed.
4 Example

The entire reconnection process can be seen from Plate 1, which represents the isosurface of the magnitude of the vorticity field (see Section 2.1). In Plates 2 and 3, the magnitudes and the vector fields of both the vorticity and normalized stretching term, \((\omega \cdot \nabla u)/|\omega|\), are displayed. In Plate 2, the time is between \(t_1\) and \(t_2\). We see a wireframe of the isosurface of \(|\omega|\) at a threshold value of about 65\%, the yellow ellipsoids (inside the tubes) with a higher threshold value (85\%), and the green ellipsoid from the magnitude of the stretching field. The black vortex lines are emanating from the minor ellipse of the yellow ellipsoid (vorticity), and the red from the green ellipsoid (stretching term). The negative strain causes the hole to appear in both structures as is evident by the increase in area of the vortex line 5-bundle through the yellow ellipsoids (since the circulation is constant in a vortex tube). On the vertical tube, the upward going vortex lines move toward the green ellipsoid and away toward the horizontal vortex indicating the incipient reconnection. The red lines of the stretching term are opposite to the direction of the vorticity (black lines) in the main tube at the hole and in the same direction where the fingers are forming.

Plate 3 is at a later time (\(t_4\)). Here we see the yellow ellipsoid associated with \(|\omega| = 60\%\). Permeating each are the black 5-bundle showing the vortex direction and "flare". The green vortex stretching ellipsoids (\(|\omega \cdot \nabla u|/|\omega| = 46\%\)) are in regions where reconnection is going to occur (the response). Here, the coherent field pattern is a 'torus'-like object whose upper and lower transversals are in the place of the out-going bridges of vorticity which will soon emerge. Note also that the longer sides of the 'torus' are on the outside of the yellow ellipsoids, indicating that there will be a stretching 'reduction' of vorticity in these locations. The reconnection torus is a signature of the reconnection event and was discovered by this visiometric process [8] [1].

5 Conclusion

We have demonstrated how thresholding, ellipsoid fitting, and vortex line generation have all been used to reduce the amount of information and help analyze the relationship between two 3D vector variables evolving in time, one involving a source term and the other the response vector field. Topological relationships are fundamental elements of the ongoing process. This type of data juxtaposition will help explain the intense reconnection process which is important to understanding turbulent intermittency, "magnetic" storms, and other bursting or collapse phenomena. We advocate this approach for obtaining deep insights into intermittent phenomena in turbulent and chaotic fluid motions.

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References


Plate 1: Vorticity isosurface, vortex reconnection-bursting.

Plate 2: Juxtaposition of vorticity and vortex stretching, $t_1 < t < t_2$.

Plate 3: Juxtaposition of vorticity and vortex stretching $t = t_4$.

(See color plates, page 406.)