LINEAR PROGRAMMING APPROACH TO LOSS MINIMIZATION AND CAPACITOR SIZING AND PLACEMENT

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Abstract

Loss minimization in distribution systems has assumed greater significance recently since the trend towards distribution automation will require the most efficient operating scenario for economic viability. System loss reduction is generally achieved via capacitor placement in the distribution network. Optimality is achieved for optimal sizing and placement conditions. Recently, researchers are concerned with development of algorithms that lead to the reduction of computational requirements.

In this paper, the problem of loss minimization, capacitor allocation and sizing is linearized. The linearization is achieved by linearizing the problem in terms of the reactive compensation and voltage changes. This eliminates the need for recomputing the load flow problem. The linearized formulae are derived as functions of voltage changes and capacitor settings. Transformers (voltage regulators) settings can be easily included in the derivations.

Introduction

Loss minimization in distribution systems has assumed greater significance recently since the trend towards distribution automation will require the most efficient operating scenario for economic viability [1]. System loss reduction is generally achieved via capacitor placement in the distribution network. Optimality is achieved for optimal sizing and placement conditions.

Numerous approaches to loss minimization and voltage regulation have appeared in the literature since early 1960 [2-18]. Almost all the algorithms for loss minimization use network models developed for transmission systems with bus voltages and their phase angles taken as variables. These algorithms, in most cases, involve repeated computation of the load flow calculations. In most cases optimization techniques are implemented using nonlinear programming algorithms, or at the best using quadratic programming methods.

These methods share a drawback in requiring heavy computations at times.

Schmill [2] considered the optimization problem in which the author examined the problem considering a uniformly loaded feeder. In this paper, [2], the problem is considered with limiting constraints. The necessary conditions for solving few problems on a case by case basis are also presented. Duran [3] proposed a dynamic programming algorithm. In this paper, the author realistically treats the sizes of the capacitors as discrete variables. As mentioned earlier, the capacitor allocation and sizing problem have been dealt with earlier [2-18]. However, the work presented in [2-7] suffers from many standpoints. In most of these papers, the problem has been solved on a case-by-case basis as is with [2]. Further, load distribution, in many cases, was treated as either uniformly distributed or a combination of uniformly and concentrated load distributions. The wire sizing is also assumed, in many of the analyses, to be uniform.

Taking the above argument into consideration, it can be easily concluded that most of the earlier work reported in the literature suffers from the lack of generality. Obviously, lack of generality in the problem provides some serious drawbacks to the approach. This should be clear since, if a particular approach works for one network configuration, it may not necessarily provide good results in another network configuration.

The generalized problem has been extensively dealt with in recent years [7-18]. In the recent literature the problem has been addressed, in most cases, using the nonlinear programming formulation. In addition, the switched type capacitor sizes have been used as continuous variables [9]. A voltage dependent model was introduced in [10] for loss reduction. The authors in [12] presented a formulation for optimal design and control scheme for continuous capacitive compensation. El-Kib, et al. [13], presented a methodology that encompasses unbalanced three-phase feeders. The formulation is extended to handle multilateral feeders in [14]. In [15] the problem is extended...
to the distribution system level. In this paper, the capacitor placement problem is combined with the voltage regulation problem. Also, decoupled solution approaches are proposed in [19]. Baran and Wu [16] consider the problem of finding the optimal size of capacitors placed on the nodes of a radial distribution system. In this paper the problem is formulated as a nonlinear programming problem. In [17] the authors consider the capacitor allocation problem to reduce peak power and energy losses. The cost of compensation is also taken into account. The problem in [17] is formulated as a mixed integer programming problem. Finally, in [18] Baldick and Wu propose approximation formulae for the distribution system. The problem in [18] is formulated as a quadratic programming problem.

In this paper the system of capacitor allocation and sizing is linearized. The linearization is achieved by linearizing the problem in terms of the reactive compensation and voltage changes. This eliminates the need of recomputing the load flow problem. In other words, the linearized formulae are derived as functions of voltage changes and capacitor settings. Transformer (voltage regulator) settings can be easily included in the derivations.

The approximate formulae proposed in this paper were derived from an approach that defines voltage magnitudes and section losses as variables presented in [19].

Problem Formulation

Figure 1 shows the general ith section of a distribution feeder:

\[
\begin{align*}
V_{i+1} & \quad R_i + jX_i \\
V_i & \quad  \Delta P_i, \Delta Q_i, \Delta C_i \\
\end{align*}
\]

Figure 1 The General ith Section

where

- \(V_{i+1}\) source end voltage
- \(V_i\) load end voltage
- \(P_i\) real power at node i
- \(Q_i\) reactive power at node i
- \(\Delta P_i\) real losses at node i
- \(\Delta Q_i\) reactive losses at node i
- \(\Delta C_i\) reactive compensation at node i
- \(R_i\) resistance of the ith section
- \(X_i\) reactance of the ith section

The general loss equations for the ith section are given by

\[
\Delta P_i = \frac{\left[ P_i + \sum_{j=1}^{i-1} \Delta P_j \right]^2}{V_i^2} + \frac{\left[ Q_i + \sum_{j=1}^{i-1} \Delta Q_j - \sum_{j=1}^{i} \Delta C_j \right]^2}{R_i}, \quad (1)
\]

and

\[
\Delta Q_i = \gamma_i \Delta P_i, \quad (2)
\]

where \(\gamma_i = \frac{X_i}{R_i}\).

The general voltage drop equation for the ith section is given by

\[
V_i^2 + V_{i+1}^2 - 2 \left( \left[ P_i + \sum_{j=1}^{i-1} \Delta P_j \right] R_i + \left[ Q_i + \sum_{j=1}^{i-1} \gamma_j \Delta P_j - \sum_{j=1}^{i} \Delta C_j \right] X_i \right)
\]

\[
- V_{i+1}^2 \right) + Z_i^2 \left[ \left[ P_i + \sum_{j=1}^{i-1} \Delta P_j \right]^2 + \left[ Q_i + \sum_{j=1}^{i-1} \gamma_j \Delta P_j - \sum_{j=1}^{i} \Delta C_j \right]^2 \right] = 0. \quad (4)
\]

where \(Z_i^2 = R_i^2 + X_i^2\).

Let

\[
V_i^2 = Y_i - \Delta V_i. \quad (6)
\]

and

\[
S_i^2 = P_i^2 + Q_i^2. \quad (7)
\]

Substituting (6) into (1) and (4) with the definitions (3), (5) and (7) in mind yield a quadratic formulation for the problem. If the binomial series expansion is used in (5) and keeping only 1st order variables, a linearized formulation of the problem is obtained. A point worth noting is that only section losses, \(\Delta P_i\), reactive compensation, \(\Delta C_i\), and voltage changes, \(\Delta Y_i\), are considered as variables in the problem.

With the above discussion in mind, the linearized version of equations (1) and (4) are:
For the loss equation we have:

\[
\Delta P_i = \frac{R_i}{Y_1} \left[ S_i^2 + 2 \sum_{j=1}^{i-1} (P_j + Q_j) \Delta P_j - 2Q_i \sum_{j=1}^{i} AC_j + \frac{S_i^2}{Y_1} \Delta Y_i \right].
\]

(8)

For the voltage equation we have the following approximation:

\[
Y_i \left[ Y_i + 2(P_i R_i + Q_i X_i) - Y_{i-1} \right] + Z_i^2 S_i^2 + 2 \sum_{j=1}^{i-1} (R_i + X_i Y_j) + \sum_{j=1}^{i} \left( P_i + Q_i Y_j \right) \Delta P_j - 2 \sum_{j=1}^{i} (X_i Y_j + Z_i^2 Q_j) \Delta C_j
\]

\[
= 0
\]

(9)

The details of the linearizations are given in the Appendix.

To illustrate the formulation, consider a two-section feeder distribution system.

Two-Section Case

Consider the following 2-section network

\[
V_0 \rightarrow R_2 + jX_2 \rightarrow V_2 \rightarrow R_1 + jX_1 \rightarrow \rightarrow V_1
\]

\[
P_2 + jQ_2 \rightarrow \rightarrow P_1 + jQ_1
\]

Figure 2 Two-Section Network

To obtain the loss and voltage drop equations for each section, we simply replace i by the number of the section. Note if i = 2, then Y_{i-1} = V_2^2 and \Delta Y_{i-1} = 0. The loss and voltage equations for nodes 1 and 2 are given respectively.

(i) Loss Equation at Node 1

\[
\Delta P_i = \frac{R_i}{Y_1} \left[ S_i^2 - 2Q_i \Delta C_i + \frac{S_i^2}{Y_1} \Delta Y_i \right]
\]

(10)

(ii) Loss Equation at Node 2

\[
\Delta P_i = \frac{R_i}{Y_1} \left[ S_i^2 + 2P_i R_i Q_i \Delta P_i - 2Q_i \Delta C_i + \frac{S_i^2}{Y_1} \Delta Y_i \right]
\]

(11)

(iii) Voltage Equation at Node 1

\[
Y_i \left[ Y_i + 2(P_i R_i + Q_i X_i) - Y_{i-1} \right] + S_i^2 Z_i^2 - 2(X_i Y_i + Z_i^2 Q_i) \Delta C_i
\]

\[-2(P_i R_i + Q_i X_i) - Y_{i-1} + 2Y_i \Delta Y_i + Y_i \Delta Y_{i-1} = 0
\]

(12)

(iv) Voltage Equation at Node 2

\[
Y_i \left[ Y_i + 2(P_i R_i + Q_i X_i) - V_0^2 \right] + S_i^2 Z_i^2
\]

\[+2(R_i + X_i Y_i) Y_i + Z_i^2 (P_i + Q_i Y_i) \Delta P_i
\]

\[-2(X_i Y_i + Z_i^2 Q_i) \Delta C_i - 2(X_i Y_i + Z_i^2 Q_i) \Delta C_i
\]

\[-2(P_i R_i + Q_i X_i) - V_0^2 + 2Y_i \Delta Y_i = 0
\]

(13)

With equations (10)-(13) the loss minimization problem can be formulated as a linear programming problem as follows:

Minimize \( \Delta P_1 + \Delta P_2 \)

subject to

\[ \Delta Y_i \leq \epsilon \]

\[ \Delta C_i \leq \mu_i \]

and the linear constraints given by equations (10)-(13). The variables to be determined are the losses \( \Delta P_i \) and \( \Delta P_2 \) and the voltage changes \( \Delta Y_1 \) and \( \Delta Y_2 \).

The solution to the problem stated above will provide, wherever a feasible solution exists, the optimal values for \( \Delta P_1 \) and corresponding values of voltage drop, \( \Delta Y_1 \), and the compensation, \( \Delta C_1 \).

Numerical Example

To illustrate the features of the approach presented in this paper, a simple two-section balanced radial system is considered. Figure 2 shows the example system. The system parameters are given in Table 1. In Figure 2, the source voltage is assumed to be applied at the beginning of line section 1. For simplicity, all line sections are assumed to be of the
same conductor. \( \epsilon \) was set to 0.05 p.u. and \( \mu \) set to 10% of \( Q_i \).

<table>
<thead>
<tr>
<th>Wire Size</th>
<th>Section #1</th>
<th>Section #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>R/1000 ft. ohm</td>
<td>0.035</td>
<td>0.035</td>
</tr>
<tr>
<td>X/1000 ft. ohm</td>
<td>0.1148</td>
<td>0.1148</td>
</tr>
<tr>
<td>length ft.</td>
<td>1000</td>
<td>1000</td>
</tr>
</tbody>
</table>

Table 1

To fully illustrate the potentials of the approach presented in this paper, two load cases are simulated. The details of the various load cases are given in Table 2.

<table>
<thead>
<tr>
<th>Wire Size</th>
<th>Section #1</th>
<th>Section #2</th>
<th>Power Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load #1 pu</td>
<td>1.28</td>
<td>3.84</td>
<td>0.64</td>
</tr>
<tr>
<td>Load #2 pu</td>
<td>0.9381</td>
<td>2.8143</td>
<td>0.64</td>
</tr>
</tbody>
</table>

Table 2

The simulation results in each case were represented by the changes in the voltage profiles, compensation requirements, and the losses associated with each section. Further, the optimal section losses obtained in this paper are compared with the optimal section losses \( \Delta P_1^* \) and \( \Delta P_2^* \), for the same network, obtained using the nonlinear-programming approach [20]. These results are listed in Table 3.

Although, in the second case, the optimal value for losses on the second section obtained using the nonlinear programming approach, however, the optimal value for total losses, \( \Delta P_1 + \Delta P_2 \), obtained using the linear programming approach is less.

Conclusion

Distribution loss minimization is effected through a linear programming approach to the system model. The method presented in this paper solves the compensative reactive power sizing and location in a unified approach. A numerical example was considered to illustrate the advantages of the proposed compensation technique. A comparison between the nonlinear and linear programming approaches was illustrated through a numerical example. This example illustrated the abilities of the proposed approach to reduce the losses in electrical distribution circuits. The numerical results presented in this paper were obtained using actual utility data.

<table>
<thead>
<tr>
<th>Load Cases</th>
<th>( \Delta Y_1 )</th>
<th>( \Delta Y_2 )</th>
<th>( \Delta C_1 )</th>
<th>( \Delta C_2 )</th>
<th>( \Delta P_1 )</th>
<th>( \Delta P_2 )</th>
<th>( \Delta P_1^* )</th>
<th>( \Delta P_2^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>0.0225</td>
<td>0.05</td>
<td>0.12</td>
<td>0.4</td>
<td>0.00676</td>
<td>0.05599</td>
<td>0.0764</td>
<td>0.12574</td>
</tr>
<tr>
<td>#2</td>
<td>0.00</td>
<td>0.00</td>
<td>0.0573</td>
<td>0.0405</td>
<td>0.013568</td>
<td>0.122453</td>
<td>0.09705</td>
<td>0.12398</td>
</tr>
</tbody>
</table>

Table 3

(All values are in p.u.)

References


