Power System Transient Analysis Using Discrete Time Modeling

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Abstract

Static power system modeling methods, such as power flow and fault analyses, seek to describe steady state operating conditions of the system. However, practical systems are not always in a steady state mode. Uploading and down loading units, system failures and naturally instigated surges are all situations where steady state analysis techniques fail to accurately describe the system. In these situations transients are introduced which can range from minor ripples to significant spikes with the potential to disrupt system stability or even damage to subsystems. To investigate these occurrences, transient analysis techniques must be employed. This paper deals with the discrete-time modeling method which can be used to perform the transient analysis. The derivation of this technique as well as possible implementations are discussed.

1 Introduction

The Importance of transient analysis of power systems can not be overstated. Indeed, the results of these analyses determine the design of protection circuitry for individual as well as system safety. Although all possible disrupting events can not be anticipated, a range of acceptable deviation from the steady state mode can be defined by studying the effects of varying the system parameters (i.e. voltage, current, etc.). This can only be accomplished through simulation which requires an accurate system model.

The motivation here is to convert the continuous-time power system into a discrete-time equivalent model for simulation. Once this is done digital analysis techniques may be employed with respect given to the integrity of the actual system.

2 Determination of Element Models

The following discrete-time models are derived directly from their continuous-time elements with the assumption of zero initial conditions. This is acceptable since the concern is with the deviation from the static case whether it be normal operating conditions or simply zero.

2.1 Resistors, Capacitors and Inductors

Since resistors are linear and memoryless devices they convert directly to discrete elements as:

\[ V_r(t) = V_r[t] \]  \hspace{1cm} (1)

Capacitors are converted as follows:

\[ i_c(t) = C \frac{dV_c(t)}{dt} \]

which after integrating both sides yields

\[ \Delta t [i_c t + i_c t - \Delta t] = V_{ct} - V_c t - \Delta t \]

After some algebraic manipulation and redefining terms the result is

\[ I_c[t] = \frac{V_c[t]}{\Delta t/4C} - I_c[t - \Delta t] \]  \hspace{1cm} (2)

where '[t]' denotes discrete time value.
For inductors, following the same method as for capacitors yields

\[ I_l[t] = \frac{V_l[t]}{L/\Delta t} + I_l[t - \Delta t] \quad (3) \]

### 2.2 Transformer Model

The transformer model can be realized by using the derivations above directly if the actual inductances, resistances and conductances are known, or can be determined. However, most nameplates give the reactances, \( X \), which are generally treated as resistances with a 90° phase shift effect on the through variables \( v \) and \( i \). In this case,

\[ L = \frac{X}{2\pi f} \quad (4) \]

with \( f \) being the system frequency. Whenever possible it is advantageous to simplify the circuit as much as desired accuracy allows, which for large power transformers (several kilovoltamperes or over) the resistances can be neglected entirely, which leaves only an equivalent reactance, \( X_{eq} \), which has absorbed the primary and secondary reactances as well as the effects of the air gap [2]. Then (4) becomes,

\[ L_{eq} = \frac{X_{eq}}{2\pi f} \]

It should be noted that this assumption is valid only if the internal response of the transformer is of no interest. If this is not the case, the former method should be used (i.e. individual element conversion).

### 2.3 Transmission Lines

There are basically two applications of this type of system modeling to transmission lines: 1) Individual conversion using a standard model such as a \( \pi \) or \( T \) and 2) Partitioning. The first model follows directly from the previous discussion. Hence, only the second model is described here.

The partitioning method is built upon the derivation of the forward and backward traveling waves propagating along a single phase lossless transmission line which is given completely in [3]. This results in the following parameter equations.

\[ v(x,t) = v + (x - \nu t) + v - (x + \nu t) \quad (5) \]

which can be rewritten as

\[ i(x,t) = \frac{v + (x - \nu t) - v - (x + \nu t)}{Z_e} \quad (6) \]

where \( Z_e \) is the characteristic impedance of the line. Rearranging yields

\[ 2v + (x - \nu t) = v(x,t) + Z_e i(x,t) \quad (7) \]
\[ 2v - (x + \nu t) = v(x,t) - Z_e i(x,t) \quad (8) \]

Now, if the transit time from terminal \( k \) to terminal \( m \) is \( \tau = \frac{x}{\nu} \), then the value of \( v + Z_e i \), at \( t - \tau \), at terminal \( k \) is equal to the value of \( (v + Z_e i) \), at \( t \), at terminal \( m \). In equation form,

\[ V_k[t - \tau] + Z_e i_k[t - \tau] = V_m[t] \quad (9) \]

for the forward relation and for the backward relation

\[ V_m[t - \tau] - Z_e i_m[t - \tau] = V_k[t] \quad (10) \]

Solving for \( I_k[t] \) and \( I_m[t] \)

\[ I_k[t] = I_m[t - \tau] - \frac{2V_m[t]}{Z_e} \quad (11) \]

and

\[ I_m[t] = I_k[t - \tau] + \frac{2V_k[t]}{Z_e} \quad (12) \]

where \( V_k[t] \) and \( V_m[t] \) are the sending and receiving end voltages, respectively.

The above procedure is effectively breaking the line into two lines with the same (the original) characteristic impedance. Thus, the name partitioning. This method can be repeated for infinitely small variations in \( \Delta t \), which allows for the calculation of the system parameters at any point along the line or equivalently at any time, \( t \), between 0 and the maximum time delay of the line.

### 3 System Analysis

The system or subsystem can now be modeled in terms of (1), (2), (3), (11) and (12). The importance of this derivation lies in the complete representation of the original continuous-time system and the wide range of implementation methods which can be utilized to analyze the system.

### 4 Conclusion

This paper gives the motivation and derivation of the discrete-time modeling method of analysis for a power system. The intent was not to simulate any particular system, which is of little use generically, but to show the foundation and usefulness of such an
approach. From the previous discussion, it is apparent that the major components of a power system can be described by a series of linear equations which can be solved by iteration. Moreover, the range of analysis and degree of accuracy can be controlled by manipulation of a single variable, t. However, the power of this technique lies in the freedom of implementation.

References

