Optimal Task Distribution in a Multi-Robot System Handling a Common Object

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Abstract

One of the most important issues in a multi-robot system working on a common object is task distribution among the robots. By extending the recent work described in [8], this paper presents a strategy that achieves optimal task distribution. The algorithm is obtained by solving a constrained optimization problem and ensures the force constraints for fine manipulation.

Keywords: Robotics, adaptive path tracking, constrained optimization, fine manipulation.

1. INTRODUCTION

The number of robots currently being used in industrial and commercial applications is increasing. However, they currently perform simple jobs, such as pick-and-place tasks, machine loading and unloading, spray painting, and spot welding. The complexity of most assembly tasks motivates the use of two or more robots operating in a coordinated fashion. One of the most important issues in a multi-robot system is the task distribution among the robots. Considerable work has been reported along this line [1-7].

This paper is an extension of the recent work described in [8], where a system of d robots handling a common unknown object was investigated, and an adaptive control algorithm was developed. It is suggested in [8] that the task distribution among the robots in \( \Omega_9 \) (the set of robots that have spare capacity) can be arbitrarily chosen as long as the null space property of \( W \) is satisfied. Here the matrix \( W \) maps the contact forces applied by the various robots to the object's center of mass [8].

In this paper, the optimal distribution of the task among the robots in \( \Omega_9 \) is studied. By using an instantaneous energy cost function, the task is shown to be optimally shared by the robots in the sense that the cost function is minimized. The algorithm is obtained by solving a constrained optimization problem.

A design example is presented which demonstrates the application of the method.

2. BACKGROUND

In [8], by introducing task coordinates, the combined payload/robot dynamic model was derived as,

\[
H^* \ddot{X}_0 + C^* \dot{X}_0 + G^* = F^*,
\]

where

\[
F^* = A^T WJ^{-T} T,
H^* = A^T WJ^{-T} H J^{-1} S A + A^T \left( \begin{array}{cc}
mE_3 & 0 \\
0 & Rl_0 R^T T_0
\end{array} \right),
C^* = A^T WJ^{-T} H (J^{-1} S A + J^{-1} \dot{S} A + J^{-1} S \dot{A})
+ A^T WJ^{-T} C J^{-1} S A
\]

\[
G^* = A^T WJ^{-T} G + \left[ \begin{array}{c}
mg \\
0
\end{array} \right] \triangleq G_0^* + \left[ \begin{array}{c}
mg \\
0
\end{array} \right].
\]

(See [8] for complete descriptions of the terms.)

The motion control problem, "design the control torque \( T \), such that the actual path of the payload's mass center \( X_0 \) asymptotically tracks the desired path \( X_d \) in the face of unknown payload parameters," is solved by the algorithm presented next.

Adaptive Tracking Algorithm[8]

Define the tracking error as

\[
\epsilon = X_0 - X^d
\]

\[
= \left[ \begin{array}{c}
\dot{x}_0 - \dot{x}_d^d \\
\phi_0 - \phi_d^d
\end{array} \right] \triangleq \left[ \begin{array}{c}
\epsilon_1 \\
\epsilon_2
\end{array} \right]
\]
The control input \( F^* \) is designed as

\[
F^* = H_0^*[\dot{X}_0^d + (\lambda + \beta)\dot{c} - \lambda\beta c] + C_0^*[\dot{X}_0^d - \beta c]
\]

\[
+ G_0^* + \left[ \sum_{m=1}^{m} \sum_{i=1}^{n} I_{0ij}^i \Psi_{ij} \right] - K\Phi,
\]

where \( \beta > 0, \lambda > 0, K = K^T > 0 \) are pre-selected parameters and the parameter estimates are

\[
\dot{m} = -k_1\Phi^T T_{ij} - k_2 \frac{d}{dt}(\Phi^T T_{ij})
\]

\[
\dot{I}_{0ij} = -k_{1ij}\Phi^T T_{ij} - k_{2ij} \frac{d}{dt}(\Phi^T T_{ij})
\]

where \( i = x, y, z \) and \( j = i, ..., z \), \( k_1 > 0, k_2 > 0, k_{1ij} > 0 \) and \( k_{2ij} > 0 \) are estimation algorithm design parameters,

\[
\Psi_m = \dot{x}^d_0 - (\lambda + \beta)\dot{c}_1 - \lambda\beta c_1 + g,
\]

\[
\Psi_{ij} = \dot{X}_0^d \left[ E_{ij} (\dot{X}_0^d - (\lambda + \beta)\dot{c}_2 - \lambda\beta c_2) + \left[ \omega_0 \times E_{ij} T_0 + E_{ij} \dot{T}_0 \right] (\dot{X}_0^d - \lambda\beta c_2) \right]
\]

\[
\Phi_1 = \dot{c}_1 + \beta c_1, \quad \Phi_2 = \dot{c}_2 + \beta c_2
\]

and

\[
\Phi = (\Phi_1^T, \Phi_2^T)^T.
\]

3. OPTIMAL TASK DISTRIBUTION

What is actually needed to guarantee the path tracking of the multi-robot/payload system are the control torques for each robot (the elements of \( T \)). Fortunately, since \( W \) is full rank [8], there exists a matrix

\[
W^+ = W^T(WW^T)^{-1} \in R^{6d \times 6}
\]

such that the total control torque \( T \) becomes

\[
T = J^T F_{\text{end}},
\]

where

\[
F_{\text{end}} = W^+ A^{-T} F^* + F_I
\]

\[
= F_P + F_I.
\]

In these equations, \( F_P \) is the force causing the motion of the payload (\( F^* \) is computed by (2.2) ) and \( F_I \in \text{Null Space}(W) \) represents an internal force vector. A well known formulation for \( F_I \),

\[
F_I = (E_{0d} - W^+ W)\mu \quad \forall \mu \in R^{6d},
\]

has been the basis for much work dealing with load distribution. However, it is noted in the work [5] that, although \( F_I \) in (3.2) satisfies \( WF_I = 0 \), it is an inadequate definition for internal force from a physical point of view. A recent work [6] also pointed out that (3.2) does not completely define the internal loading.

In the following this problem is treated using an alternate approach. First let

\[
\Omega_1 \triangleq \{ \text{a set of robots working on the task}\}
\]

\[
\Omega_2 \triangleq \{ \text{a set of robots needing help}\}, \text{ and}
\]

\[
\Omega_3 \triangleq \{ \text{a set of robots with spare capacity}\}.
\]

Assume \( \Omega_1 = \Omega_2 \oplus \Omega_3 \) and \( \Omega_2 \neq \{ 0 \} \) and \( \Omega_3 \neq \{ 0 \} \), which implies that each robot either needs help or provides help, and at least one robot needs (or provides) help. It is further assumed that the number of the robots with spare capacity is \( r \) and these robots are able to provide the required forces.

It is noted that in practical applications, constraints are generally imposed on the manipulating forces/moments at the grasp points due to the limited control energy, i.e.,

\[
|F_{\text{end}}(i)| = |F_P(i) + F_I(i)| \leq \mu_j(i),
\]

\[
\text{if } j \in \Omega_1, i = 1, 2, ..., 6, (3.3)
\]

where \( F_{\text{end}}(i), F_P(i) \) and \( F_I(i) \) are the \( i \)-th elements of the \( j \)-th partitions of \( F_{\text{end}}, F_P \) and \( F_I \), respectively, and \( \mu_j(i) \) are given positive numbers. Such constraints are also necessary to achieve fine manipulation.

So an interesting problem is how to distribute the task among the robots such that the constraints (3.3) and \( WF_I = 0 \) are satisfied. The following strategy provides a solution to this problem.

The strategy basically consists of two steps. Step 1 checks which robots need help and step 2 arranges the help. In the proposed strategy, the first step is done by using the quantity \( F_P(i) \) as a criterion. That is,

\[
\text{STEP 1: If } |F_P(i)| \geq \mu_j(i), \text{ then } j \in \Omega_2.
\]

In such a case \( F_I(i) \) have to be adjusted such that (3.3) is guaranteed. This is achieved by choosing \( F_I(i) \) as

\[
F_I(i) \overset{\text{def}}{=} F_I(i)
\]

\[
= \left\{ \begin{array}{ll}
\mu_j(i) - F_P(i) & \text{if } F_P(i) \geq \mu_j(i); \\
-\mu_j(i) - F_P(i) & \text{if } F_P(i) \leq -\mu_j(i).
\end{array} \right. (3.4)
\]

The second step is motivated by the following observations.

First it is noted that in order to make the payload asymptotically track the desired path, the total control force \( F^* \) must be equivalently generated by the total joint torque. Thus \( T \) should satisfy,

\[
A^T W J^T T = A^T W F_{\text{end}} = F^*.
\]
With $F_{I_k}$ specified as in (3.4), the condition (3.5) may not be satisfied. Furthermore, (3.4) may also cause the null space property of $F_{I_k}$ to be violated. Hence we need to seek for help from the other robots. Clearly such help should completely compensate the load that the robot $j$ ($j \in \Omega_2$) cannot supply. This is ensured if the payload lies within the loading capacity of the robots (otherwise, more robots should be assigned to the task). Once $F_{I_k}$ ($k \in \Omega_3$) is specified according to (3.4), $F_{I_k}$ ($k \in \Omega_3$) must be chosen such that the null space condition holds. This is ensured if $F_{I_k}$ is determined by

$$
\sum_{k=1}^{k_r} W_k F_{I_k} = - \sum_{j \in \Omega_2} W_j F_{I_j}^* \tag{3.6}
$$

where $F_{I_j}^*$ is given by (3.4).

In helping robot $j$ ($j \in \Omega_2$), there is no constraint on how much effort each robot in $\Omega_3$ should provide. Hence one generally has infinite choices for $F_{I_k}$, as long as the resultant $F_j$ lies in the null space of $W$. But what we are interested in is the optimal choice for such $F_{I_k}$. This brings us to step 2.

STEP 2: Determine the $F_{I_k}(i)$, $i = 1, 2, ..., 6$, $k \in \Omega_3$, such that, under the constraints (3.3) and (3.6), the cost function

$$
J_c(F_{I_k}) = \frac{1}{2} \sum_{k=1}^{k_r} \sum_{i=1}^{6} \rho_k(i) F_{I_k}^*(i) = \frac{1}{2} \chi^T P \chi, \tag{3.7}
$$

is minimized. In this equation, $\rho_k(i) > 0$ is a weighting parameter, $\chi = [F_{I_k}^*, F_{I_2}^*, ..., F_{I_6}^*]^T \in R^{6r}$ and $P = \text{diag} [\rho_k(i)] \in R^{6r \times 6r}$ is a symmetric positive definite matrix.

By denoting

$$
\Gamma = - \sum_{j \in \Omega_2} W_j F_{I_j}^* \in R^{6},
$$

the constraints (3.6) can be rewritten as

$$
Q \chi = \Gamma, \tag{3.8}
$$

where $Q = [W_{k_1}, W_{k_2}, ..., W_{k_r}] \in R^{6 \times 6r}$. Therefore, the optimal task distribution problem under force constraints becomes

minimize \quad $J_c(\chi) = \frac{1}{2} \chi^T P \chi$

subject to \quad $Q \chi = \Gamma$

The Lagrangian multiplier method is used to solve this problem. Using $\nu_i$, $i = 1, 2, ..., 6$ as the Lagrange multipliers, the Lagrangian function is

$$
L(\chi, \nu) = J_c(\chi) + \frac{1}{2} \sum_{i=1}^{6} \nu_i \left( [Q \chi]_i - \Gamma_i \right) = \frac{1}{2} \chi^T P \chi + \nu^T [Q \chi - \Gamma].
$$

The necessary conditions for the optimal solution can be found from,

$$
\frac{\partial L(\chi, \nu)}{\partial \chi} = \frac{\partial}{\partial \chi} \left[ \frac{1}{2} \chi^T P \chi \right] + \frac{\partial}{\partial \chi} \left[ \nu^T [Q \chi - \Gamma] \right] = P \chi + Q^T \nu = 0
$$

and

$$
\frac{\partial L(\chi, \nu)}{\partial \nu} = Q \chi - \Gamma = 0,
$$

which gives

$$
\nu^* = - [QP^{-1}Q^T]^{-1} \Gamma \tag{3.9}
$$

$$
\chi^* = P^{-1}Q^T [QP^{-1}Q^T]^{-1} \Gamma. \tag{3.10}
$$

Correspondingly the minimum cost function is

$$
J_{\text{optimal}} = L(\chi^*, \nu^*) = \frac{1}{2} \chi^T P \chi + \nu^T [Q \chi - \Gamma]. \tag{3.11}
$$

Notice that the inverse of the matrix $[QP^{-1}Q^T]$ should exist in order to obtain (3.9), (3.10) and (3.11). Since this is an important issue concerning the existence of the optimal solution, a rigorous proof of the invertibility of the matrix $QP^{-1}Q^T$ is worth investigating. For simplicity, let $P = E_{6r}$, a unit matrix. In view of the definition of $Q$, it is seen that

$$
QQ^T = W_{k_1} W_{k_1}^T + W_{k_2} W_{k_2}^T + ... + W_{k_r} W_{k_r}^T. \tag{3.12}
$$

Since [8]

$$
W_i = \begin{bmatrix} E_3 & 0 & E_3 \\ B_i & 0 & 0 \\ 0 & 0 & E_3 \end{bmatrix}
$$

where

$$
B_i = (Re_i) \times,
$$

it is derived that

$$
W_{k_i} W_{k_i}^T = \begin{bmatrix} E_3 & B_{k_i}^T \\ B_{k_i} & E_3 + B_{k_i} B_{k_i}^T \end{bmatrix}.
$$

Therefore

$$
QQ^T = \begin{bmatrix} \sum_{i=1}^{k_r} B_{k_i} B_{k_i}^T, & \sum_{i=1}^{k_r} B_{k_i} B_{k_i}^T, & \cdots \\ \sum_{i=1}^{k_r} B_{k_i} B_{k_i}^T, & \cdots \end{bmatrix}.
$$

The Schur formula,

$$
\det \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \det(A) \det(D - CA^{-1}B)
$$

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\[ \det(QQ^T) = r \det[e_3 + A], \]
where
\[ A = \sum_{i=k_1}^{k_r} B_i B_i^T - \frac{1}{r} \left[ \sum_{i=k_1}^{k_r} B_i \right] \left[ \sum_{i=k_1}^{k_r} B_i^T \right] \]

With a little manipulation, it can be shown that
\[
\begin{bmatrix} \sum_{i=k_1}^{k_r} B_i \end{bmatrix} \begin{bmatrix} \sum_{i=k_1}^{k_r} B_i^T \end{bmatrix} = \sum_{i=k_1}^{k_r} B_i B_i^T + \sum_{i=k_1}^{k_r} \sum_{j=i+1}^{k_r} (B_i B_j^T + B_j B_i^T).
\]
This relation reduces \( \Lambda \) to,
\[ \Lambda = \frac{1}{r} \left[ (r-1) \sum_{i=k_1}^{k_r} B_i B_i^T - \sum_{i=k_1}^{k_r} \sum_{j=i+1}^{k_r} (B_i B_j^T + B_j B_i^T) \right] \]
Also noting that
\[ (r-1) \sum_{i=k_1}^{k_r} B_i B_i^T = \sum_{i=k_1}^{k_r} \sum_{j=i+1}^{k_r} (B_i B_j^T + B_j B_i^T) \quad \forall r \geq 1, \]
it is derived that
\[
\Lambda = \frac{1}{r} \sum_{i=k_1}^{k_r} \sum_{j=i+1}^{k_r} (B_i - B_j)(B_i - B_j)^T,
\]
which shows that \( \Lambda \) is at least semi-positive definite. Therefore \( rE_3 + \Lambda \) is positive definite and \( QQ^T \) is invertible. The same conclusion can be drawn for a general diagonal \( P \) with more effort. Based on this discussion, following results can be claimed.

**Theorem 3.1**

If \( T \) is generated by
\[
T = J^T W^+ A^{-T} F^* + J^T F_I, \tag{3.14}
\]
where \( F^* \) is from (2.2), \( F_I \) (\( j \in \Omega_2 \)) is specified by (3.4) and \( F_h \) (\( k \in \Omega_3 \)) is computed by (3.10), then;
1. asymptotically stable path tracking is ensured,
2. internal forces are non-zero at the contact points,
3. manipulating force constraints are guaranteed and
4. optimal sharing of the task is achieved.

Result (1) is true because such a \( T \) leads to the equivalent control force \( F^* \). Results (2), (3) and (4) hold because the choice \( F_I \) satisfy (3.4), (3.6) and (3.10). The property of non-zero internal force is of particular interest in many advanced applications where no slippage and effective manipulation are required. It is seen that with this strategy, whenever \( |F_{P_j}| > \mu_j(i) \), help from other robots is provided. Thus the given task is shared in a colleague-like manner in the sense that robots help each other when necessary. Furthermore, with \( F_{k_1} \) (\( k \in \Omega_3 \)) determined by (3.10), the task is shared among the robots in \( \Omega_3 \) optimally in that the cost function (3.7) is minimized.

**4. Design Example**

The case of three robots (each with three joints) transferring a point-mass payload is considered. Note that no rotations are involved in this case. Assume that the force constraints at the grasp point for robot \( l \) (\( l = 1, 2, 3 \) in the \( x, y \) and \( z \) directions are
\[
|F_{\text{end}}(x)| \leq 120 \text{ (N)}
\]
\[
|F_{\text{end}}(y)| \leq 150 \text{ (N)}
\]
\[
|F_{\text{end}}(z)| \leq 150 \text{ (N)}
\]
Since the payload is a point-mass with no rotation, \( A = E_3 \in R^{3 \times 3} \), a unit matrix, and \( W = [E_3 E_3 E_3] \in R^{3 \times 9} \). Hence
\[
W^+ = \frac{1}{3} \begin{bmatrix} E_3 \\ E_3 \\ E_3 \end{bmatrix}, \quad F_P = \frac{1}{3} \begin{bmatrix} F^* \\ F^* \\ F^* \end{bmatrix}, \quad Q = [E_3 E_3],
\]
where \( F^* \in R^3 \) is computed by (2.2).
Suppose that at time \( t_1 \),
\[
|F_{P_1}(x)(t_1)| \geq 120,
\]
\[
|F_{P_2}(y)(t_1)| \geq 150
\]
and
\[
|F_{P_3}(z)(t_1)| \geq 150.
\]
Then robot 1 needs help and \( F_1(x/y/z) \) are specified as
\[
F_{I_1}(x) = \begin{cases} 120 - \frac{1}{3} F^*(x) & \text{if } \frac{1}{3} F^*(x) \geq 120; \\
-120 - \frac{1}{3} F^*(x) & \text{if } \frac{1}{3} F^*(x) \leq -120, \end{cases}
\]
\[
F_{I_1}(y) = \begin{cases} 150 - \frac{1}{3} F^*(y) & \text{if } \frac{1}{3} F^*(y) \geq 150; \\
-150 - \frac{1}{3} F^*(y) & \text{if } \frac{1}{3} F^*(y) \leq -150, \end{cases}
\]
and
\[
F_{I_1}(z) = \begin{cases} 150 - \frac{1}{3} F^*(z) & \text{if } \frac{1}{3} F^*(z) \geq 150; \\
-150 - \frac{1}{3} F^*(z) & \text{if } \frac{1}{3} F^*(z) \leq -150. \end{cases}
\]
To optimally help robot 1, choose \( P \) as \( P = \frac{1}{3} E_3 \). By (3.10) the task can be optimally shared if
\[
F_{I_2} = -\frac{1}{2} F_{I_1} \quad \text{and} \quad F_{I_3} = -\frac{1}{2} F_{I_1},
\]
and the minimum cost function is

\[ J_{optimal} = \frac{1}{16} F_h^T F_h. \]

It can be verified that:

1. the force constraints are satisfied,
2. the null space property holds since

\[ WF_I = F_{t_1} + F_{t_2} + F_{t_3} = F_{t_1} - \frac{1}{2} F_{t_2} - \frac{1}{2} F_{t_3} = 0 \]

and

3. the equivalent control force \( F^* \) is guaranteed because

\[ A^T W (F_p + F_I) = \begin{pmatrix} E_3^T \end{pmatrix} (E_3 E_3) (F_p + F_I) = F_p + F_p + F_p = \frac{1}{3} F^* + \frac{1}{3} F^* + \frac{1}{3} F^* = F^*. \]

The payload's mass \( m \) is set to be \( m = 5kg \). This is not used in the control strategy. Instead, it is estimated via

\[ \dot{m} = -k_1 \int_0^t \Phi_T \Psi_m d\tau - k_2 \Phi_T \Psi_m + \dot{m}(0), \]

with \( k_1 = 0.7, k_2 = 0.5 \), and \( \dot{m}(0) = 0 \). The control parameters are \( \lambda = 1, \beta = 0.4, K = \text{diag}(250,250,250) \). The simulation results show the effectiveness of the proposed control strategy (for more details, see [9]).

5. DISCUSSION AND CONCLUSION

The problem of task distribution among robots is addressed in this paper and the manipulating force/moment constraints are explicitly emphasized. In some applications certain internal forces may be required to provide a desired stress in the object. This imposes a constraint on the internal force of the form,

\[ F_{t_j}(i) \in \Delta_j(i), \tag{5.1} \]

where \( \Delta_j(i) \) describes the region in which the \( i^{th} \) element of the internal force at contact point \( j \) should lie. A similar strategy can be developed to satisfy this requirement. It is natural to ask if one can choose a value for \( F_{t_j}(i) \) \( j \in \Omega_2 \) such that both (3.3) and (5.1) are satisfied. The answer to this question is positive if the constraints imposed in (3.3) and (5.1) do not lead to conflicting choices for \( F_{t_j}(i) \), otherwise, the answer is negative.

REFERENCES


