Yield Analysis Of 2D Hexagonal VLSI/WSI Arrays

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Figure 1 also shows the redundant spare PEs that are used when PEs fail. Fifty percent redundancy is used in the structure shown since every two primary PEs share one spare PE. One hundred percent redundancy is achieved when every primary PE shares one spare PE. The following sections present the analysis for determining the yield for various levels of redundancies.

YIELD ANALYSIS

In this section, two methods of analysis are presented. Method A uses Markov's process for analysis and forms a lower bound for yield and Method B uses the assumption that various hexagonal PE clusters are independent to provide an upper bound for yield. In each method both 50% and 100% redundancies are considered.

METHOD A:

An mxn hexagonal array is decomposed into two types of basic structures. Structure I consists of independent sub-units and Structure II consists of independent columns. An independent sub-unit consists of two primary PEs and either one spare PE if 50% redundancy is employed or two spare PEs if 100% redundancy is employed. The total number of such sub-units in the hexagonal array is \( \frac{m(n+1)}{2} \). On the other hand, each independent column consists of \( m \) clusters and each cluster consists of two primary PEs and either one spare PE if 50% redundancy is employed or two spare PEs if 100% redundancy is employed. For the entire array, there are \( n \) such independent columns and the yield due to an independent column can be estimated by using Markov process and thus finding the stochastic transitional probability matrix, which describes the spare assignments.

50% Redundancy: Figure 2 shows two types of structures that are used as basic units for analysis.

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If \( p \) denotes the survival probability factor for both the primary PE and the spare PE and \( q \) denotes the failure probability of both the primary PE and the spare PE, then \( q = (1-p) \).

Let \( Y_{A1}(50\%) \) be the yield of Structure I.

\[
Y_{A1}(50\%) = \sum_{i=0}^{m} \left[ \frac{3C_i}{2} \right] (p)^{m-i} (q)^i
\]

Let \( Y_{A2}(50\%) \) be the yield of Structure II.

\[
Y_{A2}(50\%) = \text{the yield of Structure II}
\]

Where

\[
P_{ij} = \text{1-step transition probability from state } i \text{ to state } j
\]

The spare assignment process is encoded as follows:

State 1: Spare PE is available in the next step.
State 2: Spare PE is not available in the next step.
State 3: Column failure due to inability to replace a failed primary PE.

The initial state probability vector

\[
\begin{bmatrix}
P_1^{(0)} & P_2^{(0)} & P_3^{(0)}
\end{bmatrix} = \begin{bmatrix} p & q & 0 \end{bmatrix}
\]

The stochastic transitional probability matrix,

\[
T = \begin{bmatrix}
P_{11} & P_{12} & P_{13} \\
P_{21} & P_{22} & P_{23} \\
P_{31} & P_{32} & P_{33}
\end{bmatrix}
\]

where

\[
P_{11} = p(p^2 + 2pq), \quad P_{12} = pq^2 + q(2pq + p^2), \quad P_{13} = q^3, \quad P_{21} = p^3, \quad P_{22} = 3pq^2, \quad P_{23} = 3pq^2 + q^3, \quad P_{31} = 0, \quad P_{32} = 0, \quad P_{33} = 1
\]

If the column failure occurs after \( m \) steps, then

\[
Y_{A2}(50\%) = \left[ 1 \cdot P_{3}^{(m)} \right]^2
\]

The overall yield of the hexagonal array with 50% redundancy using Method A is given by

\[
Y_{A}(50\%) = Y_{A1}(50\%) \cdot Y_{A2}(50\%)
\]

100% Redundancy: Figure 3 shows the decomposition of the hexagonal array into two basic structures with 100% redundancy.

![Figure 3. Two Basic Structures with 100% Redundancy](image)

The yield for structure I is given by

\[
Y_{A1}(100\%) = \sum_{i=0}^{2} \left[ \frac{4C_i}{2} \right] (p)^{4-i} (q)^i
\]

In this case, the spare assignment process is encoded in four possible states, which are defined as follows:

State 1: Both spares are available in the next state
State 2: Only one of the spares is available in the next state
State 3: None of the spares are available
State 4: Column failure due to inability to replace a failed primary PE

At Step 0, the initial state probability vector

\[
\begin{bmatrix}
P_1^{(0)} & P_2^{(0)} & P_3^{(0)} & P_4^{(0)}
\end{bmatrix} = \begin{bmatrix} p^2 & 2pq & q^2 & 0 \end{bmatrix}
\]
The stochastic transitional probability matrix

\[
T = \begin{bmatrix}
  P_{11} & P_{12} & P_{13} & P_{14} \\
  P_{21} & P_{22} & P_{23} & P_{24} \\
  P_{31} & P_{32} & P_{33} & P_{34} \\
  P_{41} & P_{42} & P_{43} & P_{44}
\end{bmatrix}
\]

The expressions for \( P_{ij} \) can be derived as in the previous case, where

\[
P_{11} = q(4q^2 + p^2q + 6pq^2 + 3q^3 + 2pq^2 + 4p^2q^2) + p^2(4pq^3)
\]

\[
P_{12} = 2pq(4q^2 + 4pq^2 + 6pq^3 + 2pq^4) + p^2(4pq^3)
\]

\[
P_{13} = q^2(4pq^2 + 4p^2q + 6pq^3 + 2pq^4) + p^2(q^4 + 2pq^4)
\]

\[
P_{14} = q^2(4pq^3 + q^4 + 2pq^4)
\]

\[
P_{21} = p^2(2pq + 2pq^2)
\]

\[
P_{22} = q^2(4pq^2 + 2pq^3 + 2pq^4) + p^2(q^4 + 2pq^4)
\]

\[
P_{23} = q^2(4pq^3 + q^4 + 2pq^4)
\]

\[
P_{24} = q^2(4pq^4 + q^3 + 2pq^4)
\]

\[
P_{31} = p^2(2pq + 2pq^2)
\]

\[
P_{32} = 2pq(4pq^2 + 4pq^3 + 6pq^4 + 2pq^5) + p^2(4pq^3)
\]

\[
P_{33} = q^2(4pq^3 + 2pq^4 + 2pq^5) + q^2(p^2 + 2pq^2)
\]

\[
P_{34} = 2pq(4pq^4 + q^3 + 2pq^4 + q^2 + 2pq^2)
\]

\[
P_{41} = 0
\]

\[
P_{42} = 0
\]

\[
P_{43} = 0
\]

\[
P_{44} = 1
\]

If the column failure occurs after \( m \) steps, then

\[
Y_{A2}(100\%) = \left[ 1 - T^m \right]_{P_{44}} = 0
\]

where

\[
T^m = \begin{bmatrix}
  P_{11}^m & P_{12}^m & P_{13}^m & P_{14}^m \\
  P_{21}^m & P_{22}^m & P_{23}^m & P_{24}^m \\
  P_{31}^m & P_{32}^m & P_{33}^m & P_{34}^m \\
  P_{41}^m & P_{42}^m & P_{43}^m & P_{44}^m
\end{bmatrix}
\]

The overall yield of the hexagonal array with 100% redundancy using Method A is given by

\[
Y_{A}(100\%) = Y_{A1}(100\%) \cdot Y_{A2}(100\%)
\]

The reconfiguration of PEs in Structure I is straightforward when the sub-units are independent. The reconfiguration of PEs in Structure II takes place when a primary PE in a column fails. When this happens, the spare PEs are assigned to the failed primary PEs in the column, starting at the top. If this spare is unavailable because of failure, or because it has been already assigned, then the spare below the primary PEs is used.

Figures 4a and 4b show the plots of yield versus \( p \) for 50% and 100% redundancy and various array sizes.

**Figure 4a. Array Yield \( YA \) versus \( p \) with 50% Redundancy**

**Figure 4b. Array Yield \( YA \) versus \( p \) with 100% Redundancy**

**METHOD B:**

If the hexagonal PE clusters in an \( m \times n \) array are assumed to be independent, then an upper bound for yield is obtained. Figure 5 shows how the array is decomposed into two structures. Structure I consists of independent hexagonal PE clusters as shown in Figure 6 and Structure II consists of independent peripheral sub-units as shown in Figure 7.
If \( Y_B(50\%) \) and \( Y_B(100\%) \) denote the overall array yield for 50% and 100% respectively, then

\[
Y_B(50\%) = \sum_{i=0}^{3} C_i (p^{3-i} (q)^i) + \sum_{i=0}^{1} 3C_i (p^{3-i} (q)^i) 
\]

\[
Y_B(100\%) = \sum_{i=0}^{6} 12C_i (p^{12-i} (q)^i) + \sum_{i=0}^{2} 4C_i (p^{4-i} (q)^i) 
\]

The reconfiguration of PEs in Structures I and II are local to a cluster. When a PE in a cluster fails, it is replaced by the spare PE connected interstitially to that cluster only, and not by other spare PEs in the array. Hence the reconfiguration scheme requires extra hardware overhead associated with each structure. Figures 8a and 8b show the plot of yield versus \( p \) for 50% and 100% redundancy and various array sizes.

**CHIP AREA UTILIZATION**

Chip Area Utilization Factor (CAUF) is defined as follows:

\[
CAUF = \frac{\text{no. of processors in computational array} \times \text{array yield}}{\text{expected no. of good processors on each chip}}
\]
This factor measures the fraction of failure free processors that are utilized on the average in implementing an array on silicon. Clearly a design that maximizes this fraction will maximize chip area utilization.

For a 2D hexagonal array with R percent (50 or 100 only) interstitial redundancy,

$$\text{CAUF} = \frac{\frac{y}{R/100}}{(1 + \frac{R}{100})^p}$$

Figure 9a shows a plot for CAUF versus processor survival probability $p$ for two levels of redundancy ($R=50\%$ and $R=100\%$) for an 8x8 2D hexagonal array whose yield is estimated using Equations 7 and 8.

From the plot we can infer that for an 8x8 2D hexagonal array, if an individual PE has a probability of survival factor greater than 0.93, then a lesser amount of redundancy ($R=50\%$ rather than $R=100\%$) can be employed for greater CAUF, thus maximizing the use of the silicon area and reducing the extra area-overhead caused due to higher level of redundancy.

Figure 9b shows a plot for CAUF versus $p'$ for a variety of array sizes employing the same redundancy ($R=100\%$). For smaller values of processor survival probability ($p=0.7$ to 0.85), it is observed that smaller array sizes yield a much higher chip area utilization.

CONCLUSIONS

In this paper two methods for determining the yield of hexagonal arrays have been proposed. The first method models spare PE assignment process as a discrete parameter Markov-chain with a one-step stochastic transition probability matrix, while the second method partitions the array into independent clusters. Using the first approach, the results show that a lower bound on the yield is obtained, while the second approach provides an upper bound. Using these results along with the effect of array size and the level of redundancy used, the chip area utilization factor was determined. For a wide range of array sizes and processor survival probabilities, it is observed that at most 50%-65% utilization of failure free processors on the chip can be achieved. Also the chip utilization factor was higher when smaller array sizes were considered. It was also shown that as the processor survival probability increased beyond a certain value, a higher chip area utilization factor can be achieved by using a lower level of redundancy.

REFERENCES


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