Abstract: Induction motor speed control strategies for loss reduction and efficiency enhancement are presented in this study. Induction motor equivalent circuit models for efficiency enhancements were utilized to study the steady-state analysis of speed control of three-phase induction motors. The harmonic effects of non-sinusoidal supply voltages and the non-linear variation of the magnetizing reactance as a function of magnetizing current were also considered in the loss evaluation models.

1. INTRODUCTION

Efficiency enhancement studies of electrical motors, especially induction motors, have become an important topic due to the increasing costs of energy. Therefore the consideration of minimum loss operation of induction motor drives has come up in new designs and control procedures. A drive system should not only satisfy the speed-torque operating point requirements but also satisfy these requirements at minimum power input or at minimum power loss. By controlling the stator voltage as a function of load current, the air-gap flux density can be controlled to yield reduced iron loss, magnetizing current loss, and stator copper loss[1]. Development of the solid-state drives have made the control of the applied voltage and frequency of the induction motors accessible to minimize power losses[2] at the required load-speed operating point. However the solid-state frequency and voltage controllers produce additional harmonic currents that produce extra losses in the stator and rotor[3-4-5]. The overall losses, however, can be minimized with proper control strategies such as excitation[6] or flux[7] control. In PWM inverter fed induction motor drives, the harmonic losses can be reduced by choosing an optimum PWM firing pattern[8].

In this paper, the losses of a three-phase induction motor have been presented under different control options by using the modified equivalent circuit models given in[9]. Including harmonic losses and the effects of variable magnetizing reactance as a function of excitation current, the best equivalent circuit model has been chosen for each control method to give the optimum efficiency operation of the motor.

2. EQUIVALENT CIRCUIT MODELS

The steady-state modified equivalent circuit models of induction motors for high performance applications with ignored core resistance have been given by Yamamura[9]. Similarly the equivalent circuit models including the core resistance are derived from the well-known classical equivalent circuit of figure 1 by representing the magnetizing impedance with series connected resistance \( R_m \) and reactance \( X_m \) instead of parallel connected resistance \( R_m \) and reactance \( X_m \) shown with dotted lines.

In figure 1 the resistances and the reactances are given in Ohms and all the values are referred to the stator side. The subscripts \( s \) and \( r \) indicate the stator and rotor quantities, respectively and \( s \) is the motor slip. The multiplier \( C \) in front of the reactances is called per-unit frequency and given as

\[ C = \frac{f}{f_s} \]

Where:
- \( f \) : Operating frequency (Hz)
- \( f_s \) : Rated fundamental frequency (Hz)

The magnetizing impedance of figure 1 is given as

\[ Z_m = R_m + jX_m \]  

where

\[ X_m = \frac{R_m^2 C X_s}{R_m^2 + C^2 X_s^2} \]

\[ R_m = \frac{R_m C^2 X_s^2}{R_m^2 + C^2 X_s^2} \]
Due to the magnetic non-linearities of the induction machine the magnetizing reactance $x_m$ and the core loss resistance $R_m$ vary as functions of the flux level or the magnetizing current $I_m$. Therefore the magnetizing reactance and the core loss resistance can be represented by the following equations which yield the theoretical curves in figures 2 and 3, respectively.

$$X_m = K_x + K_{x2} I_x^2 + K_{x3} I_x^3$$
$$R_m = K_r + K_{r2} I_x^2 + K_{r3} I_x^3$$

The coefficients of these equations were obtained by curve fitting process over the actual curve.

The voltage equations describing the circuit of figure 1 are written as

$$[V] = [R] [I] + [Z] [I]$$

or simply

$$[V] = [Z] [I]$$

where

$$R = R_s + R_o$$
$$X_s = X_{s2} + X_o$$
$$R_2 = R_2 + R_o$$
$$X_2 = X_{22} + X_o$$

Then the transformation from steady-state generalized T-equivalent circuit to the modified T-equivalent circuit is done by multiplying both sides of equation (7) or (8) by a transformation matrix $T$.

$$[V] T = [Z] T [I]$$

The transformation matrix $T$ must be chosen in such a way that after the multiplication, the input impedance of the generalized T equivalent circuit remains unchanged. Therefore the input voltage and the stator current will also be unchanged. Hence, by choosing $T$ as a real matrix

$$T = \begin{bmatrix} 1 & 0 \\ 0 & a \end{bmatrix}$$

the modified voltage equations of the modified T-equivalent circuit are obtained from equation (10).

$$[V] T = [Z] T [I]$$

The parameters of the modified T-equivalent circuit are then obtained by assigning

$$R_{st} = a R_s$$
$$X_{st} = a X_s$$
$$R_2 = a^2 R_2$$
$$X_2 = a^2 X_2$$

With the proper choice of $a$, the other modified sub-equivalent circuit models are obtained from the modified T-equivalent circuit of figure 4. By taking

$$a = \frac{X_2 - X_{st}}{X_2 - X_{st}} > 1$$

another equivalent circuit without the rotor leakage reactance is obtained. This new circuit, shown in figure 5, is called T-I equivalent circuit, and is used to model the induction motor for field acceleration model (FAM) control which gives efficient high-performance.

If $a$ is chosen as

$$a = \frac{X_2 - X_{st}}{X_2} \leq 1$$

the leakage reactance of the stator is eliminated from the modified T-equivalent circuit to yield the T-II equivalent circuit of figure 6. Finally the L-equivalent circuit model is obtained by removing $Z_{st} = a(R_s + jX_s)$ magnetizing impedance of T-II equivalent circuit to the input terminals as shown in figure 7.

The torque-slip characteristics of the modified circuit models have been given in [9] without the core resistance and harmonic effects. The efficiency enhancement studies of these equivalent circuits are given next.
3. CONTROLLABLE LOSSES

The performance of an induction motor is mainly affected by three variables which are given as the applied voltage, applied frequency and the motor speed. The use of variable voltage and/or variable frequency AC drives changes the performance of the induction motor by affecting the motor parameters. Therefore, the input power to the motor is controlled as a function of the voltage, frequency, slip, and motor parameters to meet the required operating point at optimum "minimum" input power or at minimum power loss. In other words, the motor should produce the required torque at the required speed by drawing the minimum power from the source.

The effects of the variable voltage on core loss resistance and the magnetizing reactance have been considered in the paper since they become functions of the air gap voltage due to the flux change by the variable voltage. The non-sinusoidal supply voltage from the solid-state AC drives causes additional resistive losses in the motor due to the extra harmonic currents. The harmonic resistive losses were considered in the calculations in the case of non-sinusoidal voltage supply. However, the stray losses which are a function of the torque, voltage, frequency, and temperature have been neglected in the loss model. These losses are usually very small compared with the total losses. Since the friction and the windage losses are non-electrical and can be considered constant, they are also neglected if the rotor speed is not affected by the load and treated as constant. These losses can be considered as a part of the load torque[3]. Hence, the controllable loss model components considered in the paper are the following:

- Stator and rotor resistive losses
- Core loss
- Harmonic losses

4. THE LOSS CALCULATIONS

The fundamental model equations to be solved are obtained by using the equivalent circuit models given in the previous sections. The total input impedance of each circuit can be obtained either by using each circuit separately or by assigning the relative value of α in the equations of the modified T-equivalent circuit. However, it should be noted that the total impedance of the L-equivalent circuit must be obtained separately. The equations given below were obtained by using the modified T-equivalent circuit of figure 4. These equations also give the results for T-I and T-I1 equivalent circuits by choosing α according to equations (14) and (15), respectively. Equations of L-equivalent circuit are obtained by using figure 7 in a similar way. Once the total input impedance of the equivalent circuit is found in the form of

\[ Z_{TIN} = Z_{RTIN} + jX_{TIN} \]  

(16)

the stator current, power factor angle, and input power are then calculated as

\[ I_s \cdot \frac{V_N}{Z_{TIN}} \quad \theta \]  

(17)

\[ \theta = \tan^{-1} \left( \frac{X_{TIN}}{R_{TIN}} \right) \]  

(18)

\[ P_{IN} = P_{RTIN} + P_{C} + P_{H} \]  

(19)

where \( m \) = number of phases (usually 3).

Since the input impedance remains unchanged for each equivalent circuit except that of the L-equivalent circuit, the stator current and the total input power also will remain unchanged. However, the rotor impedance, therefore the rotor current and magnetizing current will be different for each modified circuit than those of the generalized circuit. The rotor and the magnetizing currents are found by using the impedances of the circuit. The total controllable power loss consists of three different loss components as mentioned earlier and is given as

\[ P_{c} = m \left( R_{C} I_s^2 + R_{H} I_s^2 + P_{C} + P_{H} \right) \]  

(20)

The first term represents the resistive losses in stator and rotor. The second term is core loss \( P_{C} \) and the third one is the harmonic losses \( P_{H} \). The stator and the rotor copper losses can be obtained by using them as in equation (20). The core and the harmonic losses, however, are calculated as explained below.

Core loss:

\[ P_{C} = m R_{C} I_s^2 \cos \theta_{m} \]  

(21)

where \( \theta_{m} \) = No load power factor angle.

Harmonic losses

As noted before, the non-sinusoidal harmonic voltages cause the harmonic currents, and therefore extra harmonic copper and iron losses. The additional iron losses were assumed negligible in this study. The harmonic currents have the harmonic components in the orders of 5th, 7th, 11th, 13th and so on with respect to the input voltage. For a six step voltage waveform which was considered here, there are only the dominant 5th and 7th components beside the fundamental. Hence the voltage is given as
where $V_h$ is the rms value of the applied harmonic voltage.

The harmonic current components are obtained as

$$I_{hn} = \sum_{h=1,3,5,7} \frac{V_h}{Z_h} (V_{1h} + V_{3h} + V_{5h} + V_{7h})$$

(22)

Total harmonic current of the rotor is also obtained as that of the stator given by equation (23). The total copper losses due to the harmonics are then calculated as

$$P_{Cu'h} = \sum_{h=1,3,5,7} R_h I_{hn}^2$$

(23)

5. LOSS MINIMIZATION

The controllable losses of the induction motor are given by equation (20). The current components of that equation are functions of applied voltage and motor parameters. Since the motor parameters vary with the applied frequency and the motor speed, the loss equation is said to be a function of applied voltage, frequency, and speed or slip, and it is simplified as

$$P_{Cu'h} = f(V, F, s)$$

(25)

To obtain the optimal "minimum" conditions, the partial derivations of this equation are set to zero as

$$\frac{\partial P_{Cu'h}(V, F, s)}{\partial V} = 0$$

$$\frac{\partial P_{Cu'h}(V, F, s)}{\partial F} = 0$$

$$\frac{\partial P_{Cu'h}(V, F, s)}{\partial s} = 0$$

(26)

the optimal settings of input voltage, frequency and slip are obtained, respectively, for minimum power loss. A minimum power input search scheme is given in reference [10]. The torque versus speed, input power versus speed and power loss versus speed curves of the induction motor using different equivalent circuit models are given in the next section for varying voltage and frequencies. The power losses with or without the harmonic content are given in figure 8. As it can be seen from the figures the losses due to the harmonics do not have significant effects on the optimum settings because the harmonic currents are usually small when compared with the fundamental current.

Figure 8. Power-versus-speed curves of induction motor for classical equivalent circuit.

6. RESULTS

A 1 HP three-phase induction motor with six poles were used in the analysis. The motor has 0.3Ω stator and 0.15Ω rotor resistances, and 0.5Ω and 0.2Ω motor and rotor leakage resistances at 60 Hz rated frequency, respectively. Four control methods were applied over four types of equivalent circuit models. The results are given in figures 9, 10, 11 and 12. The graph numbers from 1 to 8 in these figures represent the variation of the applied frequency from 20 to 90 Hz with 10 Hz steps and the letters from a to h represent the variation of the applied voltage from 60 to 270 Volts with 30 V. steps. The following control methods were used in four cases.

Case 1-Variable frequency-constant voltage operation
Case 2-Variable voltage-constant frequency operation
Case 3-Variable voltage-variable frequency operation
Case 4-Constant Volt/Hertz operation

The voltage in case 1 and the frequency in case 2 were kept constant at their rated values. In case 3, while the frequency was being increased, the voltage was also increased as proportional to the increase in frequency. That was done by multiplying the rated voltage by the per-unit frequency $C$. The applied voltage of the motor was adjusted each time with the changing frequency in case 4 in order to obtain a constant Volt/Hertz operation. The excitation voltage $E_{dc}$ was set to a value each time by adjusting the input voltage $V_I$ and therefore the excitation current $I_{exc}$ was kept constant. In order to adjust the input voltage in this case, a closed loop control system is required.

7. CONCLUSIONS

A comparison of the results using the four equivalent circuits, classical, T-I, T-II, and L are given. In case 1, the classical equivalent circuit gives higher starting and breakdown torque with higher power input and losses than the other circuit models at low frequencies. However, above the rated frequency the other circuits give better torque response with lower power input and losses. The torque performance at the frequencies above the rated value is improved in the circuit order of classical, T-I, T-II, and L. The power input and losses of the circuits except the L-circuit model are almost the same for case 2, but T-II circuit model gives better torque response than classical, and classical gives better response than T-I. In case 2, the L circuit model has higher torque performance than the others with lower power input and losses.

In case 3, the circuit models L, T-II, and T-I have better torque with lower power input and losses at the frequencies below the rated frequency. They also give higher torque than the classical equivalent circuit above the rated frequency but with higher power input and losses.

All of the circuit models give the same breakdown but different starting torque curves in case 4. Below the rated frequency, the classical circuit has higher starting torque with higher power input and losses. At the frequencies under the rated value, the power input and losses are high for starting torque of all the circuits. In case 4, the excitation current, therefore the airgap flux is maintained constant by applying a constant V/f ratio control. Due to the resistances in the circuit and the changes in the supply voltage, the airgap flux fluctuates and causes different maximum torque points at each frequency. However, employing a closed loop control scheme, the input voltage is controlled to yield a constant excitation current and keep the airgap flux unchanged.

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REFERENCES


Figure 9. Results of variable frequency - constant voltage operation (Case 1).

Figure 10. Results of variable voltage - constant frequency operation (Case 2).
Figure 11. Results of variable voltage - variable frequency operation (Case 3).

Figure 12. Results of constant Volt/Frequency operation (Case 4).