Interfacing Synchronous Machines With Power Systems For Fault Analysis: Constructing Symmetrical Components From Machine dqO Quantities

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ABSTRACT

This paper presents a procedure for obtaining time-domain solutions for faulted power systems containing multiple rotating machines. The time-domain solutions are obtained using a combination of both time and steady-state frequency-domain techniques. The transmission network is modeled using steady-state ac techniques, while the rotating machines are represented by sets of coupled differential equations. The interface between the time-domain machine models and the frequency-domain transmission network model transforms the differential equation solutions into the appropriate frequency-domain phasors. The combination of time and frequency-domain analysis allows the total number of calculations required for a solution to be considerably less than the number required for a complete time-domain solution.

INTRODUCTION

The increase in the power of modern computers permits an evaluation of the validity of the assumptions made in traditional fault analysis. As shown in [1], the solutions obtained from traditional symmetrical component analysis may differ significantly from the solutions obtained using a more accurate synchronous machine model of the generators for a simple single-machine system. For simple systems, it is possible to produce solutions calculated entirely in the time-domain. For systems with large transmission networks and multiple rotating machines, however, a complete time-domain solution is still not practical. A major reduction of the computational burden can be obtained by using quasi-ac techniques to model the transmission network, while maintaining time-domain differential equation models for the synchronous machines. There are, however, problems associated with using a steady-state frequency-domain representation of the transmission network.

In general, it is not rigorously correct to use a steady-state representation of the transmission network when the transient response of a power system due to a disturbance is to be determined. Furthermore, the use of a frequency-domain network model requires that frequency-domain techniques must be used to represent all internal network voltages and currents as well as all source terminal voltages and currents. These points will be considered in detail in the following sections describing the network model and the synchronous machine model.

NETWORK MODEL

As previously mentioned, the transmission network is modeled using steady-state ac circuit techniques. In general terms, a relationship between the currents injected into the network from the synchronous machine terminals and the machine terminal voltages is of the form

\[ \tilde{V}_0 = [Z_0] \tilde{I}_0 \] \hspace{1cm} (1a)
\[ \tilde{V}_1 = [Z_1] \tilde{I}_1 \] \hspace{1cm} (1b)
\[ \tilde{V}_2 = [Z_2] \tilde{I}_2 \] \hspace{1cm} (1c)

where 0, 1, 2 indicate sequence values. The equations are single-frequency ac complex phasor-impedance matrix types, and are reduced to include generator port quantities only. Obviously, this type of network representation assumes a constant system frequency. This assumption is valid for fault analysis because the time period of interest is limited to a time interval over which the synchronous machine rotors do not have time to change speed significantly.

If the complex-valued sequence currents of Equation (1) are known, the sequence voltages can be calculated easily using the appropriate sequence impedance matrix. The next section presents the differential equation model for the synchronous machine and briefly describes a solution method for obtaining the currents injected into the network modeled as in Equation (1) at discrete points in time.
**SYNCHRONOUS MACHINE MODEL**

The equivalent circuit for the synchronous machine is shown in Figure 1.

![Synchronous Machine Equivalent Circuit](image)

Figure 1. Synchronous Machine Equivalent Circuit

The corresponding machine equations, written with flux linkages as state variables, are

$$\frac{d}{dt} \tilde{\lambda}_{oqdfdq} = [R] \tilde{i}_{oqdfdq} + \omega [G] \tilde{\lambda}_{oqdfdq} - \tilde{\nu}_{oqdfdq}$$  \hspace{1cm} (2)

where \( \tilde{\lambda}_{oqdfdq}, \tilde{\nu}_{oqdfdq}, \) and \( \tilde{i}_{oqdfdq} \) are 6X1 vectors with

\[
\tilde{\nu}_{oqdfdq} = \begin{bmatrix} v_o & v_d & v_q & v_F & v_D & v_Q \end{bmatrix}^T
\]

\( \tilde{\lambda}_{oqdfdq} \) and \( \tilde{i}_{oqdfdq} \) are similarly defined and

$$\tilde{\lambda}_{oqdfdq} = [L] \tilde{i}_{oqdfdq}$$  \hspace{1cm} (3)

The matrices \([R],[G],\) and \([L]\) are defined as follows:

$$[R] = [P] [R_{oqdfdq}] [P]^T$$  \hspace{1cm} (4)

where \([P]\) is Park's transformation matrix defined as:

$$[P] = \frac{1}{k} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \cos \theta & \cos (\theta - \alpha) & \cos (\theta + \alpha) \\ \sin \theta & \sin (\theta - \alpha) & \sin (\theta + \alpha) \end{bmatrix}$$  \hspace{1cm} (5)

with

$$\theta = \omega t + \beta; \ \alpha = 120 \text{ degrees}; \ k = \sqrt{3}/2; \ \omega = \text{electrical speed in radians per second}.$$

Note that the angle \( \theta \) is measured from the "a" phase magnetic axis on the stator to the rotor D axis (coincident with the "d" axis), CCW positive. The angle \( \beta \) thus locates the rotor at \( t=0 \).

\([G]\) is the "speed voltage" matrix with \( g_{dq} = -1 \) and \( g_{q} = +1 \).

All other entries in \([G]\) are zero. The individual entries in \([L]\) are:

$$[L]_{oqdfdq} = \begin{bmatrix} L_o & 0 & 0 & 0 & 0 & 0 \\ 0 & L_d & 0 & kM_F & kM_D & 0 \\ 0 & 0 & L_q & 0 & 0 & kM_Q \\ 0 & kM_F & 0 & L_F & M_R & 0 \\ 0 & kM_D & 0 & M_R & L_D & 0 \\ 0 & 0 & kM_Q & 0 & 0 & L_Q \end{bmatrix}$$  \hspace{1cm} (6)

The developed electromagnetic torque is given by

$$T_{ew} = \lambda_d i_q - \lambda_q i_d$$  \hspace{1cm} (7)

The abc phase variables are related to the \( 0dq \) variables by

$$\tilde{\nu}_{oqdfdq} = [P] \tilde{\nu}_{abc}$$  \hspace{1cm} (8a)

$$\tilde{\nu}_{abc} = [P]^T \tilde{\nu}_{oqdfdq}$$  \hspace{1cm} (8b)

$$\tilde{i}_{oqdfdq} = [P] \tilde{i}_{abc}$$  \hspace{1cm} (9a)

$$\tilde{i}_{abc} = [P]^T \tilde{i}_{oqdfdq}$$  \hspace{1cm} (9b)

Equations (3) through (9) are used to form the sixth-order state model describing the synchronous machine. Furthermore, these equations provide the means to change from \( 0dq \) variables to \( abc \) variables as necessary. The first step of the solution process involves manipulating these equations into the standard state variable form:
\[
\frac{\text{d} x}{\text{d} t} = [A] x + [B] u \quad (10)
\]

In this form, the state variables are the \(0dqFDQ\) flux linkages, and the input vector consists of \(0dqFDQ\) voltages. The numerically stable trapezoidal method of integration is used to convert the differential equations of equation (10) into the difference equations

\[
\tilde{x}(k+1) = [C] \tilde{x}(k) + [D] \tilde{u}(k) \quad (11)
\]

where the input vector has been held constant over one integration time step. These difference equations, together with the transformation equations, (8) and (9), are used to calculate the instantaneous abc terminal currents injected into the network. The terminal current injections are then used to calculate the terminal voltages which are used as state model inputs for the next calculation.

A phasor representation for the machine terminal phase currents can be obtained from the state model discrete-time outputs. Machine-terminal sequence currents can be calculated using Fortesque's transformation. The sequence voltages at the machine terminals can be calculated from the sequence currents using (1), and subsequently transformed back into phase variables. The phase voltages can be used as inputs into the difference equations of Equation (11). The methods and models of this and the preceding sections can be formally combined as shown in the next section.

**SOLUTION ALGORITHM**

This section presents the important points in the overall solution algorithm which combines the results of the preceding sections. In general, the solution begins with known initial conditions for the flux linkage state variables for each machine. These initial conditions can be calculated for each machine given the prefault machine loading and terminal voltage. In addition, the sequence impedance matrices of Equation (1) are formed. The solution process is as follows, starting at \(k=0\):

**Step 1:** Solve for the machine \(0dqFDQ\) currents from the known flux linkages using Equation (3). Solve for the abc terminal currents using Equation (9b). This step is performed for all machines.

**Step 2:** Form the positive, negative, and zero sequence current injections at the terminals of each machine.

**Step 3:** Solve Equation (1) for the machine-terminal sequence voltages. Solve for the abc machine-terminal voltages using Fortesque’s transformation.

**Step 4:** Set \(k=k+1\) and solve the difference equations over the next time step.

Repeat steps 1 through 4 over the entire solution time.

**DISCUSSION OF ASSUMPTIONS AND APPROXIMATIONS**

In each of the previous sections, assumptions and approximations have been made to allow the development of a straightforward solution algorithm. While many of the assumptions have been mentioned briefly already, this section will reexamine the development of the component models in particular with a special emphasis on these simplifying measures. We begin with the transmission network model.

As mentioned previously, the use of steady-state ac circuit theory to develop the network model assumes a constant system frequency. This assumption is reasonably valid for fault analysis because the forces acting on the machine rotors immediately following a network fault can not overcome the inertia of the rotors during the short time period of interest (usually one or two cycles of the fundamental frequency current). Unfortunately, the assumption regarding the balanced nature of the network is not generally valid.

While many advances in steady-state network modeling have been reported in the recent literature, the most appropriate technique to use in combination with the proposed solution algorithm to obtain maximum simulation accuracy is unclear at this point. Our choice to represent the transmission network in terms of its symmetrical components is simply one of convenience, in that this is the form in which the system data is normally available. Furthermore, the use of zero, positive, and negative sequence quantities to represent the transmission network does offer advantages when the approach for constructing the complex-valued machine terminal currents injected into the network is considered.

The model for the synchronous machine lies at the heart of the entire simulation and therefore must be modeled accurately during the time period of interest. In particular, recall that the equations for the machine model, Equations (3) through (9), ignore the effects of speed and excitation control.
systems. The effects of these subsystems can be removed for fault analysis purposes because these systems simply cannot respond to a disturbance during the short time period of interest. Finally, machine magnetic saturation is ignored in this analysis. We now consider results for a simple system with one synchronous machine, presented in the following section.

**EXAMPLE RESULTS**

The solution algorithm of the previous section has been applied to investigate the transient response of the single-source system in Figure 2. The positive and zero sequence transmission line data is shown in Tables 1 and 2, respectively. The positive and zero sequence transformer data is shown in Table 3. The standard sequence models for the transmission line and transformer shown in [2] are used. Table 4 gives the synchronous machine data. A pre-fault converged power flow solution has been used to initialize the state vector for the synchronous machine. The converged power flow solution showing bus voltages is shown in Table 5. The scheduled generations and loads for the system is shown in Table 6.

A balanced three-phase fault is applied to the system at bus 5 at \( t=0.05 \) seconds. The fault resistance is 0.1 pu. The machine-terminal transient voltages and currents are shown in Figures 3 and 4. The results can be easily verified using the standard symmetrical component fault analysis procedure.

![Figure 2. Example Single-Machine System](image-url)
Table 6. Power Flow Solution: Generation and Load

<table>
<thead>
<tr>
<th>Bus</th>
<th>P_{gen}</th>
<th>Q_{gen}</th>
<th>P_{load}</th>
<th>Q_{load}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.00</td>
<td>0.00</td>
<td>0.50</td>
<td>0.20</td>
</tr>
<tr>
<td>4</td>
<td>0.00</td>
<td>0.00</td>
<td>0.50</td>
<td>0.20</td>
</tr>
<tr>
<td>6</td>
<td>1.01</td>
<td>0.37</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

The algorithm presented in this paper allows the simulation of a faulted power system including detailed models for the synchronous machine sources. The method uses a combination of time and frequency-domain analysis techniques to provide a fast, accurate prediction of the fault currents in the network. The use of the sixth order synchronous machine model allows many of the standard fault analysis assumptions to be investigated. The results presented in this paper agree with the preliminary results in [1] in that we have shown that discrepancies exist between answers obtained using detailed machine models and answers obtained using standard fault analysis procedures.

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REFERENCES


CONCLUSIONS

The algorithm presented in this paper allows the simulation of a faulted power system including detailed models for the synchronous machine sources. The method uses a combination of time and frequency-domain analysis techniques to provide a fast, accurate prediction of the fault currents in the network. The use of the sixth order synchronous machine model allows many of the standard fault analysis assumptions to be