Abstract
A three axis tension sensor for a tethered satellite system is modeled and simulated. The design uses an aluminum cross fixed at each end with a rod mounted at the center. Semiconductor strain gage bridges mounted on the cross resolve the tether tension into three orthogonal components. The model incorporates the effects of temperature on the strain gages. The response of the sensor to a range of temperatures and tensions is simulated and a method of thermal compensation proposed.

Introduction
The Tether Dynamics Explorer (TDE) series of experiments involve the deployment of a tethered satellite along the local vertical toward the Earth from an orbiting Delta II rocket. Since the satellite will not carry any thrusters to control its position, stabilization and alignment of the satellite will be accomplished by varying the length of the tether. The magnitude and direction of the tether tension must be measured to control the pay out and retrieval of the tether and, during stationkeeping (constant tether length), sense the effects of any disturbances such as the impact of micro meteoroids.

The measurement of both the magnitude and direction of the tether tension requires a three-axis tensiometer. A wide variety of single-axis tensile force transducers are available commercially. A strain gage based two-axis force transducer is described by Martin. Three-axis force plates are used to study postural stability in quadrupeds. Three-axis force transducers are also used to measure aerodynamic forces on aviation equipment in wind tunnels. However, all these designs are either too bulky or too specialized to be applicable to the TDE series. A prototype tensiometer designed specifically for a tethered satellite has been built by Greene and Kromann. However, this design only measures a single component of tether tension and is subject to a thermal drift of unknown cause.

A three-axis tension sensor has been designed that expands Greene and Kromann's design to three dimensions. This design, shown in Figure 1, uses an aluminum cross fixed at each end with a rod mounted at the center. The tether attaches to the free end of the rod. The normal (to the cross) component of tether tension causes the cross to bow in the direction of the tension. Since the tangential components...
of tether tension are applied at the end of the rod, they produce a moment at the center of each cross piece. This moment results in an "S" shaped flexure. The strain caused by a particular flexure mode is measured by a semiconductor strain gage bridge mounted on the cross. To avoid coupling between strains due to the normal and tangential components of tension on a particular cross piece, the strain gages in the normal component are mounted at the null points of the flexure due to the tangential component and vice versa. The sensor outputs three voltages each proportional to the corresponding component of tether tension which are fed into a digital computer and used to form the magnitude and direction of the tether tension.

A mathematical model of this new tension sensor design has been developed and simulated to reduce the amount of empirical testing and design iterations usually required in building a working prototype. Models for strain gage based force transducers have been developed by Abdullah and Finkelstein. However, these models do not account for the change in strain gage nominal resistance and gage factor with temperature.

The Model

The cross used in the tension sensor is modeled as two independent beams. Each cross piece is modeled as a beam fixed at both ends with a load (due to the normal component of tension) and moment (due to the tangential component) at its center. The basic equations that describe the behavior of the cross pieces come from standard beam theory. Since the stress in a cross piece is well within the proportional limit of aluminum for tensions of 0 - 6 Newtons, the following formula may be used to determine the strain in the cross piece:

\[ \sigma_{max} = \frac{M(x)h}{EI} \]

where,
- \( \sigma_{max} \): Maximum bending stress
- \( E \): Modulus of elasticity
- \( \varepsilon_x \): Strain at point \( x \)
- \( M(x) \): Bending moment at point \( x \)
- \( h \): Cross piece thickness

The bending moment at any point on a cross piece is the sum of the moments due to the normal component and the tangential component. Substituting the bending moment equations for each tension component into equation (1) yields the following relationship:

\[ \varepsilon_x = \frac{h}{4EI} \left[ x \cdot \frac{L}{4} T_n + 2 \left( x - \frac{L}{6} \right) T_t \right] \]

where,
- \( T_n \): Normal tension component
- \( T_t \): Tangential tension component
- \( L \): Length of cross piece
- \( I \): Cross piece moment of inertia

Thus, the strain measured by a particular strain gage bridge is a function of both the applied tension component and the location of the gage on the cross piece. Note that the null point for the normal component is at \( L/4 \) and the null point for the tangential component is at \( L/6 \).

The output of a particular strain gage bridge is a function of both strain and temperature. The nominal resistance of the
strain gage is a function of temperature. The strain gage manufacturer supplies the unbonded nominal resistance of each gage at three different temperatures. Since the resistance of a bonded strain gage is 20 to 30 percent lower than its unbonded value, the three resistance values were each reduced by 25 percent. These bonded values were used to form a quadratic polynomial that gives the nominal bonded resistance as a function of temperature. The resistance of the gage under strain is given by the following equation:

\[ R = R_n (1 \pm (1 - \beta \Delta T)K_\varepsilon) \]  

(3)

where,

- \( R_n \): Strain gage nominal bonded resistance
- \( \beta \): Temperature coefficient of gage factor of each gage
- \( \Delta T \): Change in ambient temperature
- \( K \): Gage factor of the strain gage
- \( \varepsilon \): Measured strain

The change in resistance is positive if the gage is in tension and negative if the gage is in compression.

Even for a matched set of strain gages, the resistance versus temperature characteristic for each gage is slightly different. Consequently, the following bridge output voltage equation is used:

\[ V_o = V_b \left[ \frac{R_2}{R_1+R_2} - \frac{R_3}{R_3+R_4} \right] \]  

(4)

where,

- \( V_o \): Bridge output voltage
- \( V_b \): Bridge excitation voltage
- \( R_i \): Resistance of strain gage i

Since each cross piece is fixed at both ends, thermal expansion of the cross piece will induce strain. However, assuming the cross piece is heated evenly, the thermal expansion strain will affect each gage equally, and the bridge will stay in balance. Consequently, the effects of cross piece thermal expansion are not included in the model.

**Simulation**

The model of the flight quality tension sensor was simulated to determine its behavior under various temperatures and tether tensions. The effects of temperature on the tension sensor simulated by setting all tension components to a constant value and varying the temperature from -55°C to 125°C. Plots of the output voltage versus temperature for tether tensions of 0 - 5 Newton are shown in Figure 2. The variation in output voltage with temperature is due to small differences in the resistance versus temperature characteristic of each strain gages in the bridge.

**Conclusion**

From the simulation results in Figure 2, certain conclusions can be drawn. First, the output voltage of the sensor under zero tension can change as much as 1 percent per degree Celsius in the room temperature range. This phenomenon could cause the thermal drift observed by Greene and Kromann on their tension sensor. In addition, Figure 2 indicates that a change in temperature mainly produces an offset in the bridge output. If a repeatable thermal profile of this offset for a strain gage bridge can be obtained, the bridge could be thermally compensated by sensing the temperature and subtracting the corresponding offset.

Future work on the TDE tension sensor will entail verifying the accuracy of the model through laboratory tests. These tests will concentrate on obtaining a repeatable temperature characteristic of each of the mounted strain gages to compare with the theoretical characteristic used in the model.

**References**


