Partitioned-Charge-Based Modeling: 
A New Approach

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ABSTRACT

By taking moments of the hole (electron) continuity equation, a novel derivation of the partitioned-charge-based (PCB) bipolar junction transistor (BJT) model is presented. The derivation allows for an arbitrary partitioning of the excess charge in the quasi-neutral base (QNB), and thus generalizes the PCB methodology. With this novel approach, an optimal non-quasi-static (NQS) BJT equivalent network representation, within the PCB framework, may be obtained.

I. Introduction

With the continued advancement in processing technology, the speed of modern BJT's has improved and a need for an accurate, and more physical, description of their charge dynamics has become an important issue in the development of compact BJT models for circuit simulation. The Gummel-Poon (GP) [1] BJT model, with its inherent quasi-static limitations, cannot adequately simulate transient events in some advanced BJT designs [2]. Within the past few years several new device modeling methodologies have been presented that circumvent some of the limitations of the quasi-static GP BJT model. The key element in these new approaches, each with its own derivation, is essentially the partitioning of the QNB charge between the emitter-base and collector-base terminals, so as to better simulate the charges which are reclaimable [3] through those terminals during transients. These new methodologies are exemplified by the transient-integral-charge control (TICC) model by Klose and Wieder [4], the inductive (L) model by Chen, Lindholm and Wu [5], a novel AC BJT characterization by Hurkx [6], and the partitioned-charge-based (PCB) model by Fossum and Veeragahaven [2]. In this paper, we provide a novel derivation of a more general PCB model [7], but first we will briefly overview the derivation of recent non-quasi-static BJT models.

In the TICC model, the electron current density equation is integrated with respect to position. A second integration is performed using integration by parts and the electron continuity equation is then inserted to derive Gummel like relations, but these relations do not assume the electron current density to be independent of space and time. In order to evaluate the TICC relations however, the carrier density as a function of position and time must be known. If the quasi-static carrier density is used, the TICC relations may be evaluated analytically, and the result is a partitioning of the excess QNB charge between the emitter-base and collector-base terminals. Chen et. al [5] proved that the TICC relations yield an optimum partitioning for both a uniform and exponentially doped base.

In deriving the L model, the Laplace transform of the electron continuity equation is taken. The resulting differential equation in the complex frequency (s) domain is then solved and a sufficient number of terms from its series solution are kept such that the analytical time domain solution contains non-quasi-static information. The equivalent circuit representation of the L model may be transformed into an equivalent charge control representation. If this is done, then the charge in the QNB is again partitioned between the emitter-base and collector-base junctions.

In the novel AC characterization of a BJT by Hurkx, the time dependent continuity equations are solved by means of a perturbation expansion. This result yields a small signal solution which
retains some non-quasi-static information. In one aspect of the model, an analytic expression (identical to that found in the TICC model, but derived differently) is used to divide the base storage capacitance (Hurkx's terminology) between the emitter-base and collector-base contacts. Thus, the model gives yet another derivation which shows that the excess QNB charge is not entirely reclaimable through the emitter-base terminal.

Fossum and Veeraraghavan were the first to publish a BJT model where the excess minority carrier charge in the QNB was partitioned between the emitter-base and collector-base terminals. The derivation of the PCB model rests on a double spatial integration of the electron continuity equation. Subsequent reorganization of the continuity equation minimizes the error introduced by the quasi-static approximation. However, the partitioning defined by the PCB model is valid only for the case of a uniformly doped base. Both Chen et. al. and Hurkx have shown that the partitioning defined by the PCB model under-estimates the QNB charge that is reclaimable through the collector, when a more realistic exponential doping profile is used for the base. Since an empirical form of the PCB concept has been incorporated into a BJT model that has been coded into a version of SPICE [8], there exists a need to alleviate its shortcoming in regards to its non-optimal charge dynamics. In this paper we present a modification of the PCB method where moments of the hole (electron) continuity equation are taken to yield an arbitrary partitioning of the QNB charge between the emitter-base and collector-base terminals. Thus, an optimal partitioning of the QNB charge, within the PCB methodology, is made possible.

II. Model Development

Figure 1. shows a one-dimensional p-n-p BJT structure. Assuming low injection and neglecting recombination, (which could be accounted for) the one-dimensional hole continuity equation for the QNB region is:

\[
\frac{\partial I_p(x,t)}{\partial x} + \frac{qA \phi p(x,t)}{\partial t} = 0
\]  \hspace{1cm} (1)

To obtain the conventional charge-control relations, one would substitute quasi-static expressions for the current and carrier densities and then integrate (1) over the QNB (from \(x = 0\) to \(x = W_B\)). Because the spatial variation of the time dependent carrier density is not accounted for when the quasi-static approximation is used, the above procedure introduces an error. In an effort to reduce the total error introduced by the quasi-static approximation, one may take an arbitrary moment of equation (1):

\[
\int_0^w f(x) \left( \frac{\partial I_p(x,t)}{\partial x} + \frac{qA \phi p(x,t)}{\partial t} \right) dx = 0
\]  \hspace{1cm} (2)

At this point we will not specify the function \(f(x)\); however, we will require that \(f(0) = 0\). Integrating by parts yields

\[
I_p(W_B,t) = \int_0^w \left[ \frac{df(x)}{dx} I_p(x,t) dx \right] + \int_0^w \frac{f(x)}{f(W_B)} \phi p(x,t) dx
\]  \hspace{1cm} (3)
In equation (3), \( I_p(x,t) \) and \( p(x,t) \) are unknown quantities. However, if we use the quasi-static approximation for \( p(x,t) \) and Gummel's integral charge-control expression for \( I_p(x,t) \), which is independent of \( x \), then we can derive the following:

\[
I_p(W_B,t) = I_{CC}(t) - \frac{dQ_{PC}(t)}{dt} \tag{4}
\]

where \( I_{CC}(t) \) is the collector current defined in the GP model and \( Q_{PC}(t) \) is the integral of a weighted carrier density, the meaning of which will be made clear later. If \( f(x) \) is replaced by \( g(x) \) in equation (2), and \( g(W_B)=0 \) is required, then repeating the procedure which gave (3) will now give

\[
-I_p(0,t) = \int_{x}^{\infty} \frac{dg(x)}{dx} \frac{i_p(x,t)}{g(0)} dx - qA \int_{a}^{\infty} \frac{g(x)}{g(0)} p(x,t) dx \tag{5}
\]

Repeating the derivation of equation (4) yields

\[
-I_p(0,t) = I_{CC}(t) - \frac{dQ_{PE}(t)}{dt} \tag{6}
\]

where \( Q_{PE}(t) \) is again defined as an integral of the weighted carrier density. If we add equations (3) and (5) and require that the sum of \( Q_{PC} \) and \( Q_{PE} \) equal \( Q_p \), the total excess charge in the QNB (\( Q_p = Q_{PC} + Q_{PE} \)), then a relationship between \( f(x) \) and \( g(x) \) can be obtained.

\[
\frac{g(x)}{g(0)} = 1 - \frac{f(x)}{f(W_B)} \tag{7}
\]

Hence, given a function \( f(x) \), \( g(x) \) is specified by (7). Since \( f(x) \) is arbitrary, the QNB charge can be optimally partitioned between the emitter-base and collector base junctions. This is not possible in the PCB model because the weighting functions are fixed.

The time derivatives of the partitioned charges, \( dQ_{PC}/dt \) and \( dQ_{PE}/dt \) as given in equations (4) and (6), represent non-quasi-static charging currents that flow during transients [2]. The PCB equivalent network representation for a forward active BJT is shown in Fig. 2.

**Fig. 2.** Network representation of the PCB model for a forward active p-n-p BJT [2].

### III. Concluding Remarks

As mentioned previously, the original PCB model yields only a fixed partitioning of the QNB charge. If we let \( f(x) = x \) in equation (2), then the
weighting functions which result \((x/W_B)\) and \(1-x/W_B\), as defined in equations (3) and (5). Hence, taking the first moment of the continuity equation yields the PCB model. Since \(f(x)\) is arbitrary, we can obtain an optimum partition, as defined by the TICC, L or Hurkx's model, for an exponentially doped base. In that case \(f(x)\) would be given by the following relation:

\[
f(x) = 1 - \exp(-\eta x/W_B) \tag{8}
\]

where \(\eta\) is the field factor for an exponentially doped base.

Presently, an enhanced version of SPICE [8] uses an empirical formulation of the PCB model where adjustable parameters are used to partition the QNB charge. The generalized PCB model derived in this paper provides a mathematical justification for such an arbitrary partitioning, and it should be useful in defining bounds on those empirical parameters for an improved characterization of the pertinent charge dynamics. In addition, the generalized PCB methodology can be extended for the two-dimensional case and can easily be modified to account for recombination.

In summary, we have shown that by taking arbitrary moments of the electron (hole) continuity equation, a more general PCB methodology can be obtained.

References


