AN IMPROVEMENT TO THE COMPUTATIONAL EFFICIENCY OF
A WIDEBAND FFT Beamformer

Randolph F. Follett       Franklin M. Ingels
Department of Electrical Engineering, Mississippi State University
Mississippi State, Mississippi 39762

Abstract
An improvement to the computational efficiency of a wideband FFT beamformer is presented in this paper. The technique involves the recognition of the fact that the inputs to the beamformer will be real-valued, thereby resulting in symmetric FFT outputs. This fact allows the processing requirements of the beamformer to be reduced considerably.

Introduction
The traditional algorithms used to produce steered beams from sonar arrays are the time delay and phase delay techniques. For large wide frequency band arrays, however, these designs can become prohibitively complex. One alternative scheme involves using the fast Fourier transform (FFT) to implement the phase delay technique in the frequency domain, which presents certain advantages in system design [Refs. 1, 2, and 3]. Perhaps the most important advantages which the utilization of digital signal processing techniques provides are reduction in overall system complexity and a tremendous increase in flexibility.

The overall output of a beamformer is a well known function of time, t, steered beam angle $\theta^y$, where $\theta^y$ is the $y$-th steered beam, $W_i^y$, the sidelobe suppression coefficients and the range focusing phase lags $\phi_x(t)$ as:

$$\tilde{B}(t, \theta^y) = \sum_{i=0}^{N_e-1} W_i^y e^{j\phi_x(t)} e^{-j(kx_i/2d)(t)}$$

(1)

If $W_i^y e^{-j\phi_x(t)}$ is denoted as $\tilde{A}_i$, the complex signal from the $i$-th sensor after sidelobe shading and range focusing, Equation (1) becomes

$$\tilde{B}(t, \theta^y) = \sum_{i=0}^{N_e-1} \tilde{A}_i e^{j(\pi y)}$$

(2)

and has the same form as the Discrete Inverse Fourier Transform, which is

$$F(k) = \sum_{n=0}^{N-1} f(n) e^{-j2\pi nk/N}$$

(3)

Comparison of (2) and (3) shows that the following analogies exist between the two equations:

$$f(n) \leftrightarrow \tilde{A}_i$$

(4)

$$t^y \leftrightarrow 2\pi k N_e$$

and

$$\theta^y = \sin^{-1} \left( \frac{\pi y N_e}{dN_e} \right), \quad y = 0, 1, 2, ..., N_e - 1.$$ 

(5A)

and

$$\theta^y = \sin^{-1} \left( \frac{\pi y N_e}{dN_e} \right), \quad y = (N_e/2)+1, ..., N_e - 1.$$ 

(5B)

A beamformer's array of $N_e$ sensors may be sampled at time $t$ and these $N_e$ samples used as the inputs to a FFT algorithm. The $N_e$ outputs will then be the responses of the $N_e$ "steered" beams.

The system depicted in Figure 1 may be derived by incorporating each of the processing techniques described previously into a Wideband Constant Beamwidth Beamformer. Notice that if a "filled array" of sensors is used, each of the single-frequency beamformers may be implemented using an ideal element spacing of $\lambda/2$ by the appropriate selection of sensor outputs at each frequency. This additional complexity incorporated in the algorithm gives the advantage that the beamwidths are not a function of frequency, but of the number of elements used, so that the beamwidths are constant with respect to frequency.

If the system configuration is shown on a system or block diagram level as in Figure 1, the major system constraints may be determined. These constraints are:

1. Wordlength of the A/D converters
2. Wordlength passed to FFT routines
3. Wordlength used for FFT coefficients and calculations
4. Connection data bus speed requirements
5. Wordlength of the D/A converters
6. Number of data bus connection lines
This paper addresses the constraint of connection data bus speed required.

Reduction of Computational Requirements

If a set of real-valued data is used as the input sequence to an FFT, the values in frequency bins 1 through N/2 - 1 will be the complex conjugates of the values in bins N - 1 down to N/2 + 1. In other words, if X_n is considered to be the output of the FFT of a set of real data, then

\[ X_n = X_{N-n} \quad n = 1, 2, ... \quad N/2 - 1 \]

where N is the size of the FFT. Notice that the X_n term (the 4th term) and X_{N/2} term are not included in this relationship.

Consideration of the flow of information in the FFT beamforming algorithm of Figure 1 and the relationship given in Equation 6 leads us to the conclusion that N/2 - 2 of the N/2 FFT processors in the second tier of FFT's are redundant. Since each of the input lines to the elements in the array of hydrophones is real-valued, the FFT outputs satisfy Equation 6. Thus, the second tier of FFT's needs to perform only the 0 through N/2 frequency bin calculations. The remaining FFT outputs may be formed by taking the complex conjugate of the outputs of the FFT processors numbered 1 through N/2 - 1.

Thus, the effective required bandwidths of the data buses shown in Figure 1 are reduced by almost 50%. (The actual bandwidth requirement is reduced to (N/2 + 1)/N times the original bandwidth requirement.) Considering the fact that the bandwidths required are 1/(N(N/2 - N_n)), this saving can be significant if large values of N and N_n are used. (This may be shown to be the case for a realistic system with reasonable detection requirements. See, e.g., Ref. [1].)

Of course, one must also consider the fact that this symmetry will not hold exactly given the fact that quantization arithmetic will be used in the calculation of each FFT. This fact will, in general, cause some error in the values of the FFT calculation and possibly upset the symmetrical relationship given by Equation 6. A determination of how significant this error is may be made using a comprehensive system simulation as described in [1] (modified to use this improvement). The computational flow of such a simulation is shown in Figure 2. Figures 3 and 4 present the results of such a simulation. Figure 3 presents the response of a system using the "normal" FFT beamforming algorithm and Figure 4 represents the response of a system using only the first (N/2 + 1) values from the second level of FFT's, each for a target lying at 29°. The FFT beamforming computations are performed using 12 bit fixed-point.
Figure 3 FFT Beamformer Response using "Normal" Algorithm

Figure 4 FFT Beamformer Response using Improved Algorithm
wordlengths with six bits to the right of the binary radix. The simulation of the target included a 10 dB signal-to-noise ratio. Clearly, the two beamformer outputs are not identical, but they contain substantially the same information. Both responses identify the target as lying at approximately 29° (beams 15 and 49 correspond to ±27.95°, and beams 17 and 47 correspond to ±32.05°).

The simulations use \( N_s = 256 \) and \( N_e = 64 \), with a sampling period of 1.25 μsec. This yields a \( \tau \) of 256 x 1.25 μsec = 320 μsec. The original bandwidth requirement is then calculated as \( 1/N(N_s N_e) \) or 614.4 MHz (assuming a single data line). By elimination of the redundant FFT calculations, it can be seen that the bandwidth requirement is reduced to \( (N_s/2 + 1)/N_e \) or 309.6 MHz. In addition, it is clear that by using a parallel bus structure, the bandwidth requirement may be further reduced.

Summary

In this paper, a technique for reduction of the computational requirements of a wideband FFT beamformer was presented. By utilizing the inherent symmetry of FFT calculations involving real-valued data, it is possible to reduce the number of FFT processors in the second tier of the beamformer structure by approximately 50%. This allows the bandwidth of the busses connecting the different FFT tiers to use a correspondingly smaller bandwidth.

References

