IMPROVEMENT OF TARGET RESOLVABILITY THROUGH THE APPLICATION OF PHASE-SHIFT KEYING TO QUANTIZED LINEAR FM RADAR/SONAR SIGNALS

J. PATRICK DONOHOE and FRANKLIN M. INGELS

Department of Electrical Engineering, Mississippi State University, Mississippi State, MS 39762

ABSTRACT

The ambiguity function associated with three types of phase encoded quantized linear FM (QLFM-PSK) signals are considered in this paper. The QLFM-PSK signals are classified according to the method of phase encoding. Significant range-Doppler sidelobe magnitude reduction is obtained when phase encoding is applied among the frequency sub-pulses of the QLFM-PSK signals. Application of phase encoding to each of the individual frequency sub-pulses is found to reduce the width of the ambiguity function sidelobe peaks. A combination of the two aforementioned encoding techniques yields reduction in both sidelobe magnitude and width.

INTRODUCTION

A commonly used signal in the design of pulse-compression radar is the 'chirp' waveform. The chirp signal is characterized by a constant amplitude and linearly increasing instantaneous frequency with time. The linear increase in instantaneous frequency describes linear frequency modulation (LFM). A LFM signal can be approximated by increasing the frequency in step intervals to produce a quantized LFM signal (QLFM). The resolvability of multiple targets using QLFM signals can be improved through the application of phase-shift keying (PSK) which yields signals designated as QLFM-PSK.

The target resolvability characteristic of a radar/sonar signal is measured through a two-dimensional correlation function known as the ambiguity function. The ambiguity function \(X(t, u)\) represents the correlation of a signal in terms of delay (t-time shift) and Doppler (u-frequency shift). Thus the ambiguity function yields a measure of target resolvability in range and speed for a given signal transmission. A complete discussion of the ambiguity function and its properties is given by Rihaczek [1]. The ideal ambiguity function is commonly referred to as the "thumbtack" function because of its low level floor region which surrounds a narrow central spike at the origin (t=u=0). In this paper, the characteristics of the QLFM-PSK ambiguity function are analyzed with regard to range-Doppler sidelobe behavior for three distinct encoding techniques. Certain ambiguity function properties of phase-encoded linear FM (LFM-PSK) signals were discussed in a paper by Kochmazov, et al. [2].

ENCODING TECHNIQUES

Examples of the three QLFM-PSK signal types considered in this paper are shown in Figures 2, 3 and 4 along with the corresponding LFM signal (no phase encoding) in Figure 1. The QLFM signal in Figure 1 consists of 6 frequency sub-pulses containing 3, 6, 9, 12 and 15 cycles, respectively. The encoding technique employed by the signal of Figure 2 is designated as a "spanning sequence" where the sequence spans the entire duration of the signal with one phase slot per frequency sub-pulse. A "repeating sequence" is illustrated in Figure 3 where each frequency sub-pulse is encoded with the same sequence. Figure 4 represents an "encoded repeating sequence" in which a repeating sequence is in turn encoded from frequency to frequency. The sequences utilized in the phase-encoded signals of Figures 2, 3 and 4 are Barker codes of length 3 (+,+,-) internal to the frequency sub-pulses and 5 (+,+,+,-,+) from frequency to frequency where "+" denotes a positive phase and "-" denotes a negative phase. The choice of Barker sequences as the method of encoding is dictated by the optimum sidelobe behavior of the Barker code autocorrelation function [3].

THE QLFM-PSK AMBIGUITY FUNCTION

The assumed QLFM-PSK signal consists of \(N_p\) frequency sub-pulses each of duration \(T_p\), which are subdivided into \(N_r\) phase slots each of duration \(T_r\). The complex envelope of the general QLFM-PSK signal \(u(t)\) may be defined as

\[
u(t) = \sum_{n=0}^{N_p-1} c_p(t-nT_r)
\]

where

\[
p_a(t) = \begin{cases} \exp\left[2\pi f_p(t^2/2)\right] & 0 \leq t \leq T_r \\ 0 & \text{otherwise} \end{cases}
\]
and $c_n$ has a value of +1 or -1 dependent on the phase of the nth slot. The frequency of the nth phase slot may be written as

$$f_n = \frac{\theta_m}{T_p}$$  \hspace{1cm} (3)

where $\theta_m$ represents the number of cycles in the mth frequency sub-pulse ($m=1,2,...,N_p$) and the integer $m$ is related to the integer $n$ by

$$\frac{n+1}{N_p} \leq m \leq \frac{n+N_p}{N_p}$$  \hspace{1cm} (4)

The sequence $\theta_1, \theta_2, ..., \theta_{N_p}$ represents the firing order of the transmitted frequencies. Each phase slot is assumed to contain an integral number of cycles. The $\pi/2$ phase factor in equation (2) produces a real component which is sine-dependent ensuring that phase transitions always occur at zero crossings.

The ambiguity function of a time signal $u(t)$ is defined by

$$X(\tau,\nu) = \frac{1}{2E} \int \int u^*(\sigma)u(\sigma-\tau)e^{j2\pi\nu\sigma}d\sigma$$  \hspace{1cm} (5)

where $E$ represents the total signal energy. The QLFM-PSK ambiguity function is obtained by inserting (1) into (5) and evaluating the resulting integral. The derivation follows that for the QLFM (frequency-hopped) signal given by Costas [4]. The ambiguity function exhibits a symmetry relationship defined by

$$|X(\tau,\nu)| = |X(\tau,\nu)|$$  \hspace{1cm} (6)
so that only nonnegative values of \( \tau \) are needed to compute the ambiguity function. The value of the time delay may be expressed in terms of the number of phase slots contained therein to yield

\[
\tau = kT_p + \delta
\]

(7)

where \( 0 \leq \delta \leq T_p \) and \( k=0,1,2,\ldots,N_p-1 \). Using (7) and (1) in (5) yields

\[
X(\tau,\upsilon) = \sum_{r=0}^{N_p-1} A + \sum_{r=0}^{N_p-1} B
\]

(8)

where

\[
A = \frac{C_{x_{\sigma_{\nu}}} (T_p-\delta) \text{sinc}[\beta(T_p-\delta)]}{N_p T_p}
\]

\[
\times \exp[j\pi(2k(T_p+2rT_p+T_p+\delta)-2\beta,\delta)],
\]

(9)

\[
B = \frac{C_{x_{\sigma_{\nu+1}}} (\delta) \text{sinc}(\gamma\delta)}{N_p T_p}
\]

\[
\times \exp[j\pi(\gamma(2kT_p+2rT_p+2T_p+\delta)-2\beta,\delta)],
\]

(10)

\[\beta = f_x - f_{x_{\nu}} + \upsilon\]

(11)

\[\gamma = f_x - f_{x_{\nu+1}} + \upsilon\]

(12)

In the next section, the magnitude of the ambiguity function is plotted for each of the previously defined QLFM-PSK signals. The vertical scale on all ambiguity function magnitude plots given in this paper are linear and range from 0 to 1 with the peak value of one located at the origin (\( \tau=\upsilon=0 \)).

**ANALYSIS OF AMBIGUITY FUNCTION PLOTS**

The magnitude of the ambiguity function for the QLFM-5 signal with firing order 3,6,9,12,15 (Figure 1) is shown in Figure 5. The triangular-shaped ridge which lies along the \( \tau=\upsilon \) line illustrates the undesirable ambiguity properties of the standard QLFM signal. Application of the spanning sequence to the QLFM signal (Figure 2) produces the ambiguity function magnitude shown in Figure 6. The sidelobes of the spanning sequence ambiguity function show a significant decrease in magnitude when compared with those of the signal with no phase encoding.

When phase encoding is applied internally to each frequency of a QLFM signal as with the repeating sequence (Figure 3), the general shape of the resulting ambiguity function closely resembles that of the standard QLFM signal as shown in Figure 7. A close comparison of Figure 7 with Figure 5 shows that the width of the ambiguity function sidelobes is decreased...
when a repeating sequence is utilized. By combining frequency sub-pulse encoding with frequency to frequency encoding as illustrated by the encoded repeating sequence (Figure 4), the ambiguity function sidelobes experience a reduction in both magnitude and width as shown in Figure 8. A comparison of the four ambiguity function plots of Figures 5, 6, 7 and 8 shows that the encoded repeating sequence yields the best range-Doppler sidelobe behavior in terms of approximating the ideal "thumbtack" ambiguity function. The sidelobe levels of the encoded repeating sequence ambiguity function are lower than those of the spanning sequence due to the sharper sidelobe contributions which lead to less interaction between adjacent sidelobes.

CONCLUSIONS

Three distinct phase encoding techniques have been applied to the standard QLFM signal and the resulting effects on the ambiguity function range-Doppler sidelobes have been noted. When phase encoding is applied from frequency to frequency with no internal encoding, the ambiguity function sidelobes are decreased in magnitude. The sidelobe widths of the QLFM signal ambiguity function are decreased by applying phase encoding internal to each frequency sub-pulse. By encoding each frequency sub-pulse of the QLFM signal while also encoding from frequency to frequency, both the magnitude and width of the ambiguity function sidelobes are decreased.

Similar encoding techniques have been applied to frequency-hopped signals with firing orders generated from Costas arrays. These results will be reported in the near future.

REFERENCES


