Spherical Maps Visualization

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Abstract. In this work we developed a program to visualize spherical maps (maps on the sphere composed by arcs of circles, not necessarily geodesic ones) allowing the user to modify the position of the observer. Besides, it is possible to insert vertices and edges in a spherical map. The program was implemented using the C++ and the libraries OpenGL and wxWindows.

1. Introduction

A spherical map (maps on the sphere \( S^2 \), that is, the sphere with unit radius centered at the origin of \( \mathbb{R}^3 \)) is a partition of the sphere's surface into three kinds of elements: vertices (points), edges (arcs of circles) and faces (open regions). It’s worth to notice that arcs are not restricted to geodesic ones, but may belong to circles of arbitrary radius.

Shortly, the geometry of a spherical map can be represented using the homogeneous coordinates of the vertices. An edge (i.e. arc) can be represented by the homogeneous coefficients of the plane that defines it – notice that, given an arc \( c \) there is a unique plane \( \alpha \) such that \( c = \alpha \cap S^2 \) [1].

To visualize a spherical map we need to draw its vertices and edges over a 3D sphere surface. Therefore, the program needs to use a perspective projection to simulate the visualization of these 3D objects in the computer’s screen (that is 2D).

2. Visualization methods

The visualization of a vertex is simple since it is enough to supply the coordinates of that point.

To visualize an edge, we use OpenGL function \texttt{gluPartialDisk} that allows us to draw an arc on the plane. So, we use a transformation that takes an arc on the plane and moves it to the appropriate position on the sphere. More precisely, given an edge \( a \) whose extreme points are \( p = [p_x, p_y, p_z, p_w] \) and \( q = [q_x, q_y, q_z, q_w] \) and whose support circle is \( c = ((c_x, c_y, c_z, c_w)) \), the transformation is given by the matrix \( M \), where \( k = c_x^2 + c_y^2 + c_z^2 \).

\[
M = \begin{bmatrix}
\frac{q_x + c_x}{\sqrt{k}} & \frac{p_x + c_x}{\sqrt{k}} & \cos \theta & \frac{r}{\sqrt{k}} \\
\frac{q_y + c_y}{\sqrt{k}} & \frac{p_y + c_y}{\sqrt{k}} & \cos \theta & \frac{r}{\sqrt{k}} \\
\frac{q_z + c_z}{\sqrt{k}} & \frac{p_z + c_z}{\sqrt{k}} & \cos \theta & \frac{r}{\sqrt{k}} \\
0 & 1 & 0 & 1
\end{bmatrix}
\]

3. Results

The figure below exhibits the visualization of the spherical map corresponding to the American continent.

The program also allows the creation/edition of maps, that is, the inclusion of vertices and edges or the modification of their visual features. For example, we can modify the sphere transparency, the light source position, edges color and style (continuous or dashed), sphere color, etc [2].

4. Acknowledgements

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References
