Summary. The QRS portion of the electrocardiogram of the complexes of patients were sampled at the rate of 1024 samples per second. The power spectral densities were calculated using the fast Fourier transform on a quarter of a second portion of an FM recorded ECG, centered on the peak of the R wave. It was discovered that in a typical QRS complex of normal duration, virtually all of the power is contained in frequencies below 30 Hz with peak power occurring in the range of 4 to 12 Hz. It should be noted that the frequency corresponding to the location of the peak is the heart rate of the patient. Furthermore, most noticeably, premature ventricular contractions typically contain less high frequency power than normal beats, with virtually all the power contained in frequencies below 12 Hz with peak power located at about 4 Hz. Finally, notches in some QRS complexes was associated with a broadened distribution of power in the power spectral density even though the tail from 30-100 Hz contained relatively less power than that contained in the frequency band below 30 Hz.

Introduction

Time series such as electrocardiograms (ECG), electroencephalograms (EEG), etc. can be analyzed both in the domain of time and also in the equivalent frequency domain. The domain methods are usually filtering procedures and some of the appropriate ones are the autoregressive or distributed lag models, the moving average models or combinations of autoregressive and moving average models called ARMA models. These latter models are used to predict interactively the duration of lapsed times between occurrences of consecutive "events," such as the R-R interval measured from peak to peak of the R waves. Because these methods were discussed extensively in (1) by Box and Jenkins these procedures are known in the literature as the Box-Jenkins approach.

The spectrum is the principal function of interest in the frequency domain analysis. It is essentially a harmonic decomposition of variance and is, therefore, equivalent to the corresponding analysis of variance in the time domain of the phenomenon where the total variance is decomposed or distributed to the various contributing sources. Spectrum analysis has been shown to be statistically equivalent to averaging over the periodogram or empirical spectrum.(2) but the time-consuming periodogram computation, even on high-speed digital computers, discouraged averaging in the frequency domain in favor of the time domain techniques mentioned above. In 1965, Cooley and Tukey (3) described an algorithm, the fast Fourier transform (FFT) technique used in this paper, that significantly reduced the computer time for spectral analysis. This refinement of spectrum analysis can be used to improve methods of recording data, to develop models that adequately account for the observed harmonic content (power or variance) and to assist in the removal of unwanted harmonic components. (4,5)

Methods

Let V(1), V(2),...V(N) denote the N sampled ordinates of the observed electrocardiogram. They can be considered as an observed realization of size N from an underlying discrete, real, stationary process denoted by {V(j)}j=1,2,...

Define
\[ \rho(n) = E(V(k)V(n+k)) \] (1)

\( \rho(n) \) is called the autocovariance of lag n and the graph of \( \rho(n) \) plotted against n is called the covariance-o-gram (n=0,1,2,3,...N-1). Normalizing the \( V(n) \) such that \( E(V(n)) = 0 \) and \( Var(V(n)) = 1 \), \( \rho(n) \) becomes the autocorrelation and the covariance-o-gram will be the correlogram

\[ \rho(n) \] has the representation given by
\[ \rho(n) = \int_{-\pi}^{\pi} \sin \lambda d\Phi(\lambda) \] (2)

where \( F(\lambda) = \int f(\lambda) d\lambda \) is called the power spectrum and its derivative \( f(\lambda) \) is called the power spectral density (p.s.d). Inverting the Fourier representation given by (2), the p.s.d can be formally written as
\[ f(\lambda) = \frac{1}{2\pi} \sum_{-\infty}^{\infty} \rho(n) = \frac{1}{2\pi} \sum_{-\infty}^{\infty} \cos n\lambda \rho(n). \] (3)
\( f(\lambda) \) given by (3) is the p.s.d represented as a weighted average of the correlogram.

To estimate \( f(\lambda) \) we used the Cooley-Tukey FFT algorithm as described in the BMDP programs of the Health Sciences Computing Facility at UCLA. (6)

The power spectral densities were computed for the QRS complexes of the electrocardiograms (ECGs) of patients in the Coronary Care Unit, Los Angeles County--University of Southern California Medical Center who had suffered recent myocardial infarctions. The PSD's of various QRS complexes were compared in the same patient and between patients. PSD's were calculated (estimated) using fast Fourier transform on a 0.25 second portion of an FM recorded ECG which was centered on the peak of the R wave. Data were sampled at 1024 samples per second and PSD's calculated at four hertz intervals.

**Results**

Important results are as follows:

(1) In a typical QRS of normal duration, virtually all of the power is contained in frequencies below 30 Hz with peak power occurring in the range of 4 to 12 Hz.

(2) Premature ventricular contractions (PVC's) typically contain less high frequency power than normal beats, with virtually all the power contained in frequencies below 12 Hz with peak power at about 4 Hz.

(3) Notches in the QRS are accompanied by a definite broadening of the p.s.d with significant power present in the range 30-100 Hz, although the additional power is still much smaller than that contained in the band below 30 Hz.

**Discussion**

High frequency notches in the QRS have been shown to have a positive correlation with coronary heart disease. (7) Because of the consequent broadening of the distribution of power in the frequency domain of the patient's p.s.d, spectral analysis of ECG signals and in particular that of the QRS complex could provide a useful tool for assessing the significance of QRS notches in individual patients and determining the relationship to other clinical parameters.

Similarly, assessing the power spectrum of ectopic beats may be useful in determining clinical significance. It is well known that body surface potentials of small voltages may be difficult to record, however, new techniques for acquiring such signals non-invasively, as well as invasively, may enhance the value of this technique and will provide new areas for clinical investigation.

**Bibliography**


Figure 1.

Plot of QRS complex at a high sampling rate is shown on the left at high gain. Next to the QRS plot is a plot of its frequency plot with most of the power below 30 Hz. On the right, the lower frequencies have been removed by digital filtering and the plot shows residual power between the lower cut-off of 60 and 124 Hz, and higher frequencies at 92 and between 308 and 328 Hz. (Each small bar represents data at 4 Hz intervals). By successively raising the lower cut-off and plotting the residual data on a time base, the relationship between notching and high frequencies can be established.