Coordination of Two-Arm Pushing

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ABSTRACT

Two cooperative manipulators equipped with open-palm end effectors are particularly suited to perform tasks of grasping and transporting large objects by pushing them from two ends. This paper studies dynamic modeling and coordinated control of two-arm pushing. For performance of such task, requires the simultaneous control of the motion trajectory of the grasped object and the interaction force. The control of the interaction force is needed to ensure that the object is not dropped and to avoid excessive pressing. The motion and force control problem is further complicated by the presence of unilateral constraints since the manipulators can only push the object. The control problem is first formulated in the state space. A coordinated control method is then presented which utilizes a state feedback to decouple motion control subsystem and force control subsystem. The unilateral constraints are satisfied during the entire motion period by using a proposed force control planning algorithm. The effectiveness of the control method is verified by simulations.

1 Introduction

Coordinated control of two or multiple manipulators has been studied by many researchers including Nakamura et al. [1], Uchiyama et al. [2], Zheng and Lui [3], Hayati [4], Dauchez and Uchiyama [5], and Tarn et al. [6]. In most work, it is assumed that manipulators rigidly grasp the object so that both pushing and pulling are possible. In terms of modeling, equality constraints are considered only. This assumption requires that the object cannot be grasped by each hand/gripper. The potential of two cooperative manipulators is not fully utilized if they are restricted to manipulate objects grasped by a single hand. Two manipulators can grasp objects which are far beyond the capability of a single hand. For instance, two manipulators can easily transport a large (not necessarily heavy) cardboard box by pushing it from two ends. However, performing tasks by two-arm pushing imposes challenging control problems. Firstly, explicit control of interaction force is essential to avoid dropping the object and pressing it excessively. Secondly, the kinematic constraints are unilateral since manipulators can only push the object. In other words, the normal force applied to the object by the manipulator must be positive.

An excellent work on pushing operation is documented by Mason [7]. The closest to the present effort is the work by Kopf and Yabuta [8] and by Yoshikawa and Zheng [9]. Kopf and Yabuta conducted a comparative study of master/slave and hybrid two arm position/force control through an experiment in which two co-linear arms push an object. In Yoshikawa and Zheng's work, two arms move an object by inserting pins at arm tips into two holes on the object. The arms could pull (or push) the object. Once again, equality constraints are considered only. In this paper, two-arm pushing operations with explicit inequality constraints are studied. Dynamics of two-arm pushing is first represented in the state space. The output of the system consists the object position and the interaction force. A state feedback is then constructed to decouple the motion and force control loops. Finally, the developed control algorithm is verified by simulations.

2 Modeling of Two-Arm Pushing

2.1 Motion Equations

We consider the task of moving an object by two manipulators. Each manipulator has a flat-surface palm as its end effector. The two manipulators grasp and move the object by pushing it from two opposite ends, as depicted in Figure 1. The discussion in this paper is restricted to the one dimensional case for thorough understanding of the problem. The object and palms in this discussion are assumed to be rigid. The one dimensional space under consideration is in the horizontal plane so that gravitational force will not play a role in the motion analysis.

The task for the two manipulators is to move the object, following a desired trajectory. It is a trivial modeling and control problem if forces applied to the object by each individual manipulators, \( F_1 \) and \( F_2 \), are not of concern and if the manipulators are allowed to push and pull the object. The problem of this study is to perform the same task under the following requirement and constraint.
1. **Coordination requirement:** the forces applied to the object by the two manipulators must be coordinated to avoid unnecessary cancellation and to maintain a certain minimum required for grasping the object.

2. **Unilateral constraint:** the two manipulators can push, but cannot pull, the object.

We now proceed to model the manipulators-object system. Let \( x_o \) be the position of the mass center of the object and \( m_o \) be the mass of the object. From the Newton's law, the motion equation of the object is

\[
m_o \ddot{x}_o = F_1 + F_2, \quad F_1 \geq 0, \quad F_2 \leq 0 \tag{1}
\]

The two inequalities in the above are from the unilateral constraints. Assuming that both manipulators are one dimensional. Their motion equations can be described as follows

\[
m_1 \ddot{x}_1 + b_1 \dot{x}_1 + c_1 x_1 = r_1 - F_1 \tag{2}
\]
\[
m_2 \ddot{x}_2 + b_2 \dot{x}_2 + c_2 x_2 = r_2 - F_2 \tag{3}
\]

where \( x_1 \) and \( x_2 \) are the position of palms 1 and 2, respectively, \( r_1 \) and \( r_2 \) are the actuator forces, and \( m_1, b_1, \) and \( c_1 \) are the effective mass, damping and spring constants of the manipulators. As long as the manipulators are in contact with the object, we may properly choose the coordinates of palm 1, palm 2, and the object in such a way that

\[
x_1 = x_o \quad \quad \quad x_2 = x_o
\]

It follows that velocities and accelerations of the two palms and the object during contacts are governed by

\[
\dot{x}_1 = \ddot{x}_2 = \ddot{x}_o
\]

\[
\dot{x}_1 = \ddot{x}_2 = \ddot{x}_o
\]

2.2 **Interaction Force**

Since the two manipulators can only push the object, \( F_1 \) is always nonnegative and \( F_2 \) is always nonpositive. Further, to maintain contacts with the object, \( F_1 \) and \( F_2 \) can not be zero. We define *interaction force* as the minimum of magnitudes of \( F_1 \) and \( F_2 \) (see Figure 2),

\[
F_I = \min\{F_1, -F_2\} = \frac{F_1 - F_2 - |F_1 + F_2|}{2} \tag{4}
\]

The interaction force \( F_I \) does not generate motion. It is needed for grasping the object. The amount of the interaction force is determined by the task to be performed. On the one hand, \( F_I \) must be as small as possible to avoid unnecessary cancellation due to coordination requirement. On the other hand, \( F_I \) must be sufficiently large so that the tangential friction force is able to balance the gravity force of the object. The minimal amount of \( F_I \) is then determined by the weight of the object and the coefficient of friction between the object and palms. In this paper, the desired value of \( F_I \), denoted by \( F_I(t) \), is assumed to be given by the task planner. The present problem is to maintain \( F_I \) while the object is in motion, that is, to design a controller which regulates both the motion of the object and the interaction force.

2.3 **State Space Representation**

We are dealing with a system whose inputs are clearly the actuator forces \( r_1 \) and \( r_2 \). To control the motion of the object and the interaction force, the outputs of the system should be related to \( x_o \) and \( F_I \). To completely describe the system, a set of state variables must be selected and state equations must be established.

Since \( x_o = x_1 = x_2 \) during the contact, adding Equations (1), (2), and (3) together to eliminate \( F_1 \) and \( F_2 \), we obtain

\[
m_o \ddot{x}_o + b \dot{x}_o + c x_o = r_1 + r_2 \tag{5}
\]

where \( m = m_1 + m_2 + m_o, \quad b = b_1 + b_2, \) and \( c = c_1 + c_2 \).

Equation (5) will be the basis of the state equation to be established. We now derive a representation for \( F_I \).

Substituting Equation (5) into Equation (2) and collecting terms, we get

\[
F_1 = r_1 - m_1 \ddot{x}_o - b_1 \dot{x}_o - c_1 x_o
= r_1 - \frac{m_1}{m} (r_1 + r_2 - b \dot{x}_o - c x_o) - b_1 \dot{x}_o - c_1 x_o
= (1 - \frac{m_1}{m}) r_1 - \frac{m_1}{m} r_2 + (\frac{m_1 b}{m} - b_1) \dot{x}_o
+ (\frac{m_1 c}{m} - c_1) x_o \tag{6}
\]

A representation for \( F_2 \) can be similarly obtained. We choose the following state variables.

\[
x = [x_1, \quad x_2, \quad x_3, \quad x_4]^T = [x_o, \quad \dot{x}_o, \quad r_1, \quad r_2]^T
\]
The state equation of the system is established by rewriting Equation (5) in terms of the state variables.

\[
\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -\frac{1}{m} x_2 - \frac{1}{m} x_3 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}
\]

The above state variables and state equation deserve some explanation. It is noted that we have included \( r_1 \) and \( r_2 \) in the state variables. Since we explicitly control the interaction force, \( F_I \) would be part of the output equations. However, \( F_I \) is algebraically related to \( q \) and \( r_2 \) through \( F_1 \) and \( F_2 \). By enlarging the state space to include \( q \) and \( r_2 \) and adding an integrator on each input channel (+1 \( = u_1 \) and \( +2 = u_2 \)), we are able to formulate the present coordinated control problem as the control problem of an affine system:

\[
\dot{x} = f(z) + g(z)u,
\]

in which the output \( y \) is a function of the state only, instead of a function of both the state and inputs.

As stated earlier, the fulfillment of the task requires simultaneous control of the object motion and the interaction force. Thus, the outputs of the system would consist of \( x \) and \( F_I \), i.e.,

\[
y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_1(x) \\ h_2(x) \end{bmatrix} = \begin{bmatrix} x_2 \\ F_I \end{bmatrix}
\]

It is clear that \( h_1(x) = C_1 x = x_1 \) where \( C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \) and \( h_2(x) \) is obtained by substituting \( F_I \) (Equation (6)) into Equation (4). From Equation (4), \( F_I \) is not differentiable with respect to the state variables. A non-differentiable output function will prevent us from using powerful design techniques such as differential geometric control theory. Specifically for this example of the one-dimensional case, the state equations are linear. The output equations would be nonlinear as well as non-differentiable if \( F_I \) is part of the outputs. An alternative is to control something else while providing the stability of \( F_I \). We will replace \( F_I \) in the output equations by \( F_1 \). To make this possible, we must establish a relationship between errors in \( F_I \) and \( F_1 \), and a planning rule for \( F_1 \) based on the desired values of \( F_I \) which is specified by the task. We defer the discussion on error bounds and force control planning to Subsections 3.2 and 3.3.

With \( F_1 \) replacing \( F_I \) in Equation (8), the output equations become differentiable and linear in state \( x \):

\[
y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_1(x) \\ h_2(x) \end{bmatrix} = \begin{bmatrix} x_2 \\ F_1 \end{bmatrix} = C x
\]

where

\[
C = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{m} - c_1 & 0 & 0 & 0 \\ -\frac{m}{m} - b_1 & \frac{1}{m} - \frac{m}{m} \\ \frac{1}{m} - \frac{m}{m} & \frac{1}{m} - \frac{m}{m} \end{bmatrix}
\]

For the present one dimensional case, both state equations and output equations are linear. For general multi-dimensional case, the system representing two-arm pushing will be nonlinear.

## 3 Coordinated Control of Two-Arm Pushing

In the preceding section, we have characterized two-arm pushing as a dynamic system in the state space. The focus of this section is to design a controller to achieve the task of moving the object in a coordinated fashion.

### 3.1 Feedback Decoupling

As noted earlier, the system representing 1-D two-arm pushing is linear. Nevertheless, the inputs and outputs of the system are coupled. In this subsection, we derive a state feedback which will decouple the force control subsystem from the motion control subsystem.

To construct the feedback for input-output decoupling, we may use Wonham’s geometric approach for linear multivariable systems [10], or differential geometric approach for nonlinear systems [11]. We will use the latter approach since it provides insight into the general nonlinear case of multi-dimensional two-arm coordination. For this purpose, we rewrite state Equation (7) and output Equation (9) together as follows:

\[
\dot{x} = f(x) + g(x)u \quad (10)
\]

\[
y = h(x) \quad (11)
\]

To construct the feedback for input-output decoupling, it is necessary to compute the decoupling matrix [11], which in turn requires the following Lie derivatives.

\[
L_y h_1 = \frac{\partial h_1}{\partial x} = C_1 g = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
\]

\[
L_y h_2 = \frac{\partial h_2}{\partial x} = C_1 f = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
\]

\[
L_y F_1 = \frac{\partial F_1}{\partial x} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
\]

\[
L_y F_2 = \frac{\partial F_2}{\partial x} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
\]

\[
L_y^2 h_1 = \frac{\partial L_y h_1}{\partial x} = \begin{bmatrix} \frac{1}{m} \end{bmatrix}
\]

\[
L_y^2 h_2 = \frac{\partial L_y h_2}{\partial x} = \begin{bmatrix} \frac{1}{m} \end{bmatrix}
\]

\[
L_y^3 h_1 = \frac{\partial L_y^2 h_1}{\partial x} = \begin{bmatrix} \frac{1}{m} \end{bmatrix}
\]

\[
L_y^3 h_2 = \frac{\partial L_y^2 h_2}{\partial x} = \begin{bmatrix} \frac{1}{m} \end{bmatrix}
\]

\[
L_y^4 h_1 = \frac{\partial L_y^3 h_1}{\partial x} = \begin{bmatrix} \frac{1}{m} \end{bmatrix}
\]

\[
L_y^4 h_2 = \frac{\partial L_y^3 h_2}{\partial x} = \begin{bmatrix} \frac{1}{m} \end{bmatrix}
\]

It follows that the decoupling matrix of the two-arm pushing system is

\[
\Phi(x) = \begin{bmatrix} L_y^4 h_1 \\ L_y^4 h_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{m} & \frac{1}{m} \\ \frac{1}{m} & \frac{1}{m} \end{bmatrix}
\]

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The determinant of the decoupling matrix is
\[
\det(\Phi(x)) = -\frac{1}{m} \neq 0
\]
which implies that the system will globally be decoupled in the entire state space.

Having obtained the above Lie derivatives and the decoupling matrix, the state space transformation and state feedback for input-output decoupling are given as follows [11]. The state transformation is
\[z = [z_1 \ z_2 \ z_3 \ z_4]^T = T(x)\]
\[= [h_1 \ L_f h_1 \ L_f^2 h_1 \ h_2]T = T_* z\] (13)
where the differential \(T_*\) is given by
\[
T_* = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
-\frac{e}{m} & -\frac{e}{m} & \frac{1}{m} & \frac{1}{m} \\
\frac{m e - c_1 m e - b_1}{m} & \frac{m e - b_1}{m} & 1 - \frac{e}{m} & -\frac{e}{m}
\end{bmatrix}
\]
The state feedback is
\[u = \alpha + \beta v\] (14)
where \(\alpha\) and \(\beta\) satisfy the following matrix equations
\[
\Phi \alpha = -\begin{bmatrix} L_f^2 h_1 \\ L_f h_2 \end{bmatrix}
\]
\[
\Phi \beta = I
\]
Since \(\Phi\) is nonsingular and its inverse is
\[
\Phi^{-1} = \begin{bmatrix} \frac{m_1}{m - m_1} & 1 \\
-\frac{1}{m} & -\frac{1}{m} \end{bmatrix}
\]
the state feedback is then given by
\[u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \frac{m_1}{m - m_1} & 1 \\
-\frac{1}{m} & -\frac{1}{m} \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} - \begin{bmatrix} -\frac{e}{m} z_2 + \frac{1}{m} (e z_2 + \frac{1}{m} (z_2 + z_4)) \\
\frac{m e - c_1 m e - b_1}{m} \frac{m e - b_1}{m} (e z_2 + \frac{1}{m} (z_2 + z_4)) \end{bmatrix}
\] (15)
Applying the above state feedback, the system is decoupled into two subsystems in the transformed state space \(z\). The first subsystem is the one which controls the motion of the object and is described by
\[
\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ y_1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ y_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} v_1
\] (16)
\[y_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}
\] (17)
and the second subsystem controls the force \(F_1\) and is described by a first-order system
\[\dot{z}_4 = [0] z_4 + [1] v_2\] (18)
\[y_2 = [1] z_4\] (19)
Now we have two decoupled subsystems. A linear state feedback can be easily designed for each subsystem which stabilizes it by placing the poles at any desired locations.

### 3.2 Force Control Planning
In this subsection, we address the problem of force control planning. From the task specification, a desired motion trajectory \(z^d(t)\) of the object as well as a force trajectory \(F_j^d(t)\) of the interaction force will be planned based on factors such as collision avoidance and holding the object while not excessively squeezing it. Due to the difficulty of directly controlling the interaction force, we have argued in section 2.3 to control \(F_1\) instead. The problem of force control planning in this context is to generate a desired trajectory of \(F_1\) based on that of \(F_j\).

From Equation (4), we may obtain the difference between \(F_1\) and \(F_j\)
\[F_1 - F_j = 2F_j + |F_1 + F_j|\]
Replacing \((F_1 + F_j)\) by \(m_e \dot{x}_o\) (from Equation (1)), we have
\[F_1 - F_j = 2F_j + m_e |\dot{x}_o|\]
Now adding the above equation and the motion equation of the object (Equation (1)) and dividing the result by 2, we get
\[F_1 = \frac{1}{2} m_e (\dot{x}_o + |\dot{x}_o|) + F_j\] (20)
Given a desired motion trajectory \(z^d_j(t)\) and a force trajectory \(F_j^d(t)\), Equation (20) in the above provides a dynamic force control planner to calculate the desired trajectory of \(F_1\), i.e.,
\[F_1^d(t) = \frac{1}{2} m_e (\dot{z}_o^d(t) + |\dot{z}_o^d(t)|) + F_j^d(t)\] (21)
This is the planning rule for \(F_1\) in the ideal case. As we will observe in Section 4 of simulations, in the presence of large position errors, this planning rule may command one of the manipulators to pull, which is definitely undesirable. A solution is to replace \(z^d_j(t)\) by the actual motion trajectory.

### 3.3 Error Bounds
In Section 2.3, we have replaced \(F_1\) by \(F_j\) in the output equations to simplify the controller design. To make this replacement valid, we must establish an error bound for \(F_j\). We define the position error as follows
\[\varepsilon_z(t) = z^d_z(t) - z^m_z(t)\] (22)
where \(z^m_z(t)\) is the actual value of \(z_o\). Similarly, the errors in \(F_1\) and \(F_j\) are defined by
\[e_1 = F_1^m - F_1^m\]
\[e_j = F_j^m - F_j^m\]
Since the output equations are composed of \(z_o\) and \(F_j\), \(\varepsilon_z(t)\) and \(e_j(t)\) are directly compensated by the controller, whereas \(e_1\) is left uncompensated. The measured interaction force may be expressed in terms of
the measurement of \( F_1 \) and \( F_2 \), i.e.,
\[
F_1^m = \frac{F_1^m - F_2^m - |F_1^m + F_2^m|}{2}
\]
Using the above equation, the error in the interaction force can be written as
\[
e_1 = F_1^d - F_1^m = F_1^d - F_2^m + F_2^m - |F_1^m + F_2^m|/2
\]
Let \( \Delta m \) be the actual mass of the object. Using Equation (21) and the motion equation of the object, we obtain
\[
e_1 = F_1^d - \frac{1}{2} m_0 (\dot{x}_0^d(t) + |\dot{x}_0^d(t)|)
- \frac{1}{2} \Delta m \dot{z}_0^m(t) - \dot{m}_0 |\dot{z}_0^m(t)|
\]
Taking the absolute value on both sides, we have the following inequality
\[
|e_1(t)| \leq |e_1(t)| + \frac{1}{2} m_0 |\dot{z}_0^d - \dot{z}_0^m(t)|
+ \frac{1}{2} \Delta m |\dot{z}_0^m(t)|
+ \frac{1}{2} \Delta m |\dot{z}_0^m(t)|
\leq |e_1(t)| + m_0 |\dot{e}_1(t)| + |\Delta m| |\dot{z}_0^m(t)|
\]
which establishes a bound on \( e_1 \) in terms of that of \( e_1 \) and \( e_2 \).

4 Simulations
The dynamic model and control algorithm developed in the previous sections have been verified through simulations. A challenging task for this two-arm system would be pushing the object back and forth tracking a prespecified trajectory while maintaining a desired level of interaction force. At any instance of time, one manipulator pushes hard to generate the desired acceleration while the other pushes back. Their roles will automatically switch depending on the desired acceleration and errors in both motion trajectory and force trajectory. To simulate this task, the desired motion trajectory of the object is chosen as
\[
x_0^d(t) = \sin(\omega t)
\]
with \( \omega = 2.5 \) and the desired interaction force \( F_I = 5.0 \) N-m. The initial values of \( F_1 \) and \( F_2 \) are assigned to be zero, so is \( F_I \). Figure 3 shows the trajectories of \( F_1 \), \( F_2 \), and \( F_I \). We observe the following:

1. The unilateral constraint is maintained at every instance since \( F_1 \) is always positive and \( F_2 \) is always negative.
2. The two manipulators interchange the roles they play. While one manipulator pushes hard to generate the required motion, the other merely pushes back to maintain the desired interaction force.

3. Even though the original system is linear, its output is nonlinear and non-differentiable, which is the desired result for this task and is achieved by the proper force control planner.
4. \( F_I \) converges to its desired trajectory following a first-order system behavior. Since \( F_I \) is not directly controlled, there is noticeable amount (less than 4%) of errors at the instances when the two manipulators switch their roles.

Figure 4 depicts position and velocity trajectories as well as position and velocity errors. It is clear that both position and velocity trajectories are followed very closely. The tracking errors are virtually zero over the entire period of simulated trajectory.

We also simulated the effect of the force measurement error on the performance of the controller. At the same time, we verified the convergence of the system with initial position and velocity errors. To simulate the force measurement error, a white noise with zero mean and 0.5 variance is added to \( F_I \) when computing force feedback. The plot on the right in Figure 5 shows the actual \( F_I \) and the white noise in the same scale. It indicates that the control algorithm rejects the noise disturbance significantly. In terms of the convergence, if the initial error is small, the control algorithm works well. However, the simulation shows that, with large initial position errors, the second manipulator tends to pull the object in order to catch up with the desired motion trajectory. A solution to this problem is to replace the desired motion trajectory with the actual one in the force control planner. The result in Figures...
5 Conclusion

An approach to the coordinated control of two-arm pushing is presented. Two-arm pushing operations have the potential of grasping and manipulating large objects, such as cardboard boxes, which are not graspable by a single arm/hand. Unlike other two-arm cooperative operations in which the deviation of the interaction force may affect the degree of performance, the success of two-arm pushing operations is critically up to the precise control of the interaction force. Furthermore, the pushing forces must obey a set of unilateral constraints. Those constraints are modeled as inequalities, rather than equalities, which are in general difficult to deal with.

Represented in the state space, one dimensional two-arm pushing is modeled as a standard linear system by properly choosing output equations. A state feedback is constructed which decouples position control and force control. The stability and performance is accomplished by another feedback applied to each individual motion or force control subsystem. A force control planning algorithm is derived which makes it possible to establish the desired force trajectory directly the task specification. An analytic error bound on the interaction force in relation to the system output errors and model parameter error is also derived. Simulations not only confirm the correctness of the control algorithm but also illustrate that the algorithm is robust against the measurement error.

6 Acknowledgement

The author wishes to thank Vijay Kumar and Eric Paljug for valuable discussion and suggestions. This work was in part supported by: Airforce grant AFOSR F49620-85-K-0018, Army/DAAG-29-84-K-0061, NSF/CER/DCR85-19196 Ao2, NASA NAG5-1045, ONR SB-35923-0, NIH grant NS-10939-01, as part of Cerebro Vascular Research Center. NIH 1-RO1-NS-23636-01, NSF INT85-14199, NSF DMC85-17315, ARPA N0014-88-K-0632, NATO grant No. 0224/85, DEC Corp., IBM Corp., and LORD Corp.

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