Free Choice Simulation of Petri Nets*

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Abstract

Structure theory deals with connections between the behaviour of a system and the structure of the corresponding net. Structure theory is well developed for free choice nets. In this paper it is shown that with a notion of simulation close to bisimulation, it is impossible to simulate all confusion free nets with free choice nets.

1 Introduction and Motivation

Confusion has been pointed out as one of the fundamental situations in concurrent systems that complicate their analysis [12], [15], [4], [11]. For the class of free choice nets confusion is excluded by means of a structural constraint [9]. It might be argued that this is one of the facts that permit to obtain strong results for this class. However, not every confusion free system is free choice [4]. We show here a stronger result. Under a reasonable notion of simulation not all confusion free systems can be simulated by free choice ones.

A concept of simulation, close to bisimulation, has been chosen from [1] because of the preservation of behaviour in both directions (from the original system to simulation and vice versa). In particular, this notion preserves liveness of observable transitions and boundedness. Moreover some properties of a special simulation are also properties of the original net. So, the analysis of a system may be replaced by an analysis of a simulating system, possibly with a simpler structure. An idea of this will be given in section 3. There are many definitions of simulation and equivalence at all ([16], [17] among others). Probably the main result of this paper can be transferred to other notions.

This paper is organised as follows. Section 2 introduces basic concepts that are not necessarily standard. Standard notations and definitions are given in an appendix. In section 3 a notion of simulation is given. Also, section 3 contains a few known results about this notion and a new connection between a system and its strongly connected simulation. Section 4 presents the main result, a sufficient criterion for a system not to have a free choice simulation. In section 5 conclusions and an outlook are presented.

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2 Basic definitions

In this paper the common mathematical notations are used. The basic definitions necessary in Petri net theory are given in this section. They mainly conform with [2] and [10]. Definitions of most other notations, used in this paper, can be found in the appendix.

Definition 2.1 *Petri net, S/T system*

(a) A triple $N = (S, T, F)$ is called *Petri net*, iff

(i) $S \cap T = \emptyset$,

(ii) $F \subseteq (S \times T) \cup (T \times S)$,

where S is a set of *places*, T a set of *transitions*, $X := S \cup T$ the set of all *elements* of N and F the *flow relation*.

(b) A pair $\Sigma = (N, M_0)$ is called *S/T system*, where $N = (S, T, F)$ is a Petri net, and $M_0 : S \rightarrow \mathbb{N}$ is the initial marking.

■ 2.1

Definition 2.2 *Pre/post-set*

For any element $x \in X$ of the net $N = (S, T, F)$ the *pre-set* and the *post-set*, respectively, are defined in the following way:

$$^\bullet x = \{y \in X \mid (y, x) \in F\}$$
$$x^\bullet = \{y \in X \mid (x, y) \in F\},$$

and the extension to a set $Y \subseteq X$:

$$^\bullet Y = \{x \in X \mid \exists y \in Y : (x, y) \in F\}$$
$$Y^\bullet = \{x \in X \mid \exists y \in Y : (y, x) \in F\}.$$

■ 2.2

Definition 2.3 *Subnet*

Let $N = (S, T, F)$ be a Petri net. $N_1 = (S_1, T_1, F_1)$ is called *subnet* of N (denoted by $N_1 \subseteq N$) iff $S_1 \subseteq S$, $T_1 \subseteq T$, $F_1 = F \cap ((S_1 \times T_1) \cup (T_1 \times S_1))$. ■ 2.3

Definition 2.4 *[4]*

Let $N = (S, T, F)$ be a Petri net.
N is called *free choice*, iff
$\forall s_1, s_2 \in S$: if $s_1^\bullet \cap s_2^\bullet \neq \emptyset$, then $s_1^\bullet = s_2^\bullet = \{t\}$ for some $t \in T$. ■ 2.4

Definition 2.5 *Concurrent enabling, conflict*

Let $\Sigma = (S, T, F, M_0)$ be an S/T system and $M \in [M_0\rangle$ a marking.

(a) Two transitions $t_1, t_2 \in T$, $t_1 \neq t_2$ are called *concurrently* enabled under M iff t_1 and t_2 are enabled under M, and the following holds: $\forall s \in {}^\bullet t_1 \cap {}^\bullet t_2 : M(s) > 1$.

(b) Two transitions $t_1, t_2 \in T$, $t_1 \neq t_2$ are called in *conflict* at M iff t_1 and t_2 are enabled under M, but t_1 and t_2 are not concurrently enabled.

■ 2.5

The definition of conflict may be a little more complicated for sets (rather than pairs) of transitions (see e.g. [5]). In this paper it is sufficient to consider the definition above.

Definition 2.6 *Confusion*

Let $\Sigma = (N, M_0)$ be a safe S/T system, $\{t_1, t_2, t_3\} \subseteq T$, and $M \in [M_0\rangle$ such that t_1 and t_3 are concurrently enabled by M.
Let M_σ be the marking defined by $M[\sigma\rangle M_\sigma$.

(a) Σ has *symmetric confusion* iff:
t_2, too, is enabled by M, but neither by M_{t_1} nor by M_{t_3} (see figure 1 (i)).

(b) Σ has *asymmetric confusion* iff:
t_2 is enabled neither by M nor by M_{t_1} nor by $M_{t_3 t_1}$, but by M_{t_3} (see figure 1 (ii)).

(c) Σ has *confusion* iff Σ has symmetric or asymmetric confusion (it may have both, of course).

■ 2.6

The idea is, that the occurence of one (t_3) of two concurrently enabled transitions (t_1, t_3) produces a conflict (between t_1 and t_2) (asymmetric confusion) or solves a conflict (between these two transitions) (symmetric confusion).

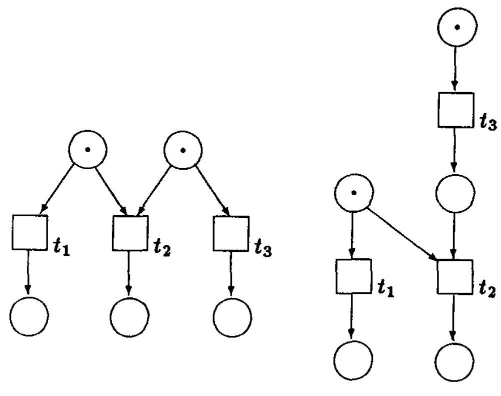
The reason for defining confusion only for safe S/T systems is given in section 5. Problems arise in the definition of confusion for non–safe S/T systems.

Definition 2.7 *Language of an S/T system*

Let $\Sigma = (N, M_0)$ be an S/T system. The set $L(N, M_0) := \{\sigma \in T^* \mid M_0[\sigma\rangle\}$ is called the *language* produced by N under M_0. ■ 2.7

3 A definition of simulation

Given a net system, it is interesting to know whether there exists another one with an equivalent behaviour which is simpler to analyze. In order to adress this problem it is necessary to precise the meaning of equivalent and simpler. In this paper, simpler means free choice. The meaning of equivalent behaviour is precised through the following definition of simulation, taken from [1].



(i) Symmetric (ii) Asymmetric

Figure 1: Representation of confusion

Definition 3.1 Σ' *simulates* Σ

Let $\Sigma = (S, T, F, M_0)$ and $\Sigma' = (S', T', F', M_0')$ be two systems and $f : T \rightarrow T'$ be an injection. Σ' simulates Σ (w.r.t. f) iff there exists a surjection $\beta : [M_0'\rangle \rightarrow [M_0\rangle$, so that the following properties hold:

(i) $M_0 = \beta(M_0')$.

(ii) Let $M_1 = \beta(M_1'), M_1' \in [M_0'\rangle$ and $M_1 \in [M_0\rangle$;

  (a) whenever $M_1[t\rangle M_2$ with $t \in T, M_2 \in [M_0\rangle$
  then $\exists\ M_2' \in \beta^{-1}(M_2),\ \exists\ w \in T'^* :$ $M_1'[w\rangle M_2' \wedge f^{-1}(w) = t$;

  (b) whenever $M_1'[w\rangle M_2'$ with $w \in T'^*, M_2' \in [M_0'\rangle$
  then $M_1\ [f^{-1}(w)\rangle\ \beta(M_2')$.

(iii) $\forall M \in [M_0\rangle : \mid \beta^{-1}(M) \mid < \infty$.

■ 3.1

Compared with other simulation definitions, definition 3.1 is close to bisimulation ([7], [8]).

A system Σ' simulates a system Σ if the behaviour on the set $f(T) \subseteq T'$ of transitions is the same as the behaviour on the original net (therefore the injection f), and some properties are the same (therefore the surjection β). As a consequence, the simulation Σ' has at least as many transitions as the original net Σ. The other transitions $T' \backslash f(T)$ are called τ– or silent transitions. Sometimes the transitions in $f(T)$ are called observable. These transitions simulate the transitions of Σ.

Requirement 3.1 (i) calls for a suitable initial marking of the simulating net. The requirements of 3.1 (ii)

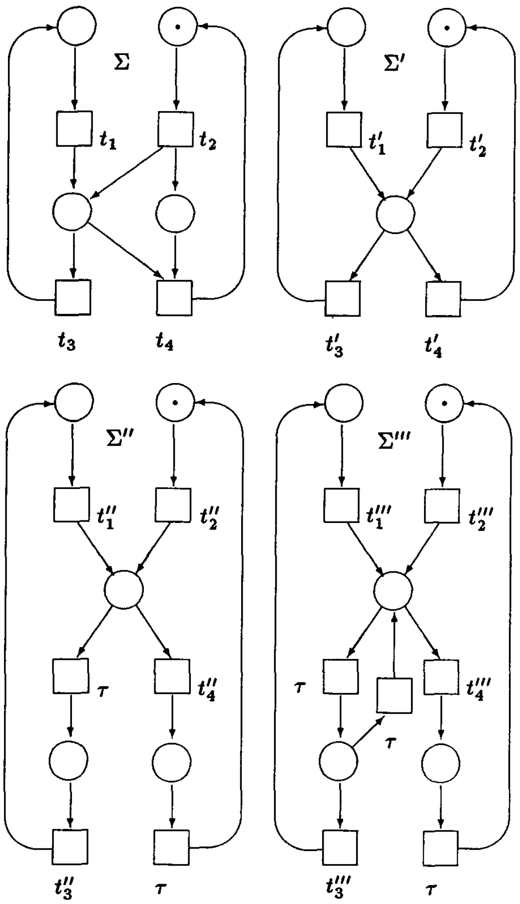


Figure 2: Illustration of simulation

are necessary to guarantee the same behaviour on the set of observable transitions. (a) means that for every firing of a transition of the original net and the resulting marking there is a (relative to f) corresponding firing sequence, and a (relative to β^{-1}) corresponding marking in the simulating net. All intermediate markings have the same image under β. Analogously, (b) means that for every firing sequence and resulting marking of the simulating system there is a (relative to f^{-1}) sequence and (relative to β) a resulting marking in the original system. The last requirement 3.1 (iii) ensures that the simulation of a system with a finite state space has also a finite state space. In other words, bounded systems can be simulated only by bounded systems. In figure 2 a non free choice system Σ is given. The free choice systems Σ' and Σ''' are simulations of Σ, but Σ'' is not because the conflict between t_3 and t_4 in Σ is the conflict between τ and t''_4 in Σ''. This conflict can be solved by the $\tau-$ transition in Σ'' but t''_4 cannot be reactivated via silent transitions after having fired τ.

Remark 3.2 *Simulation is a reflexive and transitive relation*

The definition 3.1 formulates a reflexive and transitive relation on the set of Petri nets.
Simulation is reflexive, since every S/T system is a simulation of itself, and
transitive: if Σ'' simulates Σ' and Σ' simulates Σ, then Σ'' simulates also Σ with $f'' : T \rightarrow T''$ defined according to $\forall t \in T : f''(t) = f'(f(t))$ and $\beta : [M''_0\rangle \rightarrow [M_0\rangle$ defined according to $\forall M \in [M_0\rangle : \beta''^{-1}(M) = \beta'^{-1}(\beta^{-1}(M)))$.

■ 3.2

Fact 3.3 *Simulation preserves boundedness*

Suppose $\Sigma = (S, T, F, M_0)$, $\Sigma' = (S', T', F', M'_0)$, $f : T \rightarrow T'$ *injective and* Σ' *simulates* Σ *with respect to* f.
Then Σ *is bounded iff* Σ' *is bounded.*

Proof: See [1]. ■ 3.3

Fact 3.4 *Simulation preserves liveness of the set of observable transitions*

Suppose $\Sigma = (S, T, F, M_0)$, $\Sigma' = (S', T', F', M'_0)$, $f : T \rightarrow T'$ *injective and* Σ' *simulates* Σ *with respect to* f.
Then Σ *is live iff* $\forall t' \in f(T) : t'$ *is live in* Σ'.

Proof: See [1]. ■ 3.4

For free choice simulations, fact 3.4 can be strengthened to extend also to the $\tau-$ transitions of the simulating system, rather than just the observable ones. We need for it the following result:

Proposition 3.5

Let $\Sigma = (S, T, F, M_0)$ *be an S/T system with the following properties:*
(i) Σ *is strongly connected,*
(ii) Σ *is bounded and*
(iii) Σ *is free choice.*

Then Σ *is deadlock free iff* Σ *is live.*

Proof: See [6]. ■ 3.5

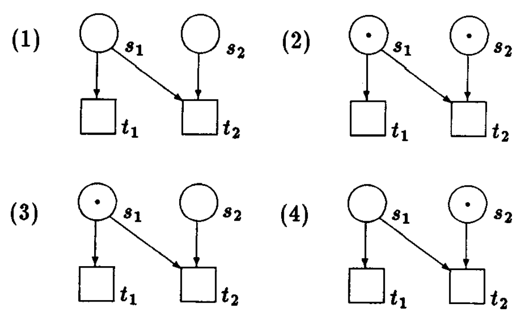


Figure 3: The net N_N with different markings

Theorem 3.6

Let $\Sigma = (S, T, F, M_0)$ be an S/T system and let $\Sigma' = (S', T', F', M'_0)$ be a strongly connected, free choice simulation of Σ.
Σ is live and bounded iff Σ' is live and bounded.

Proof: '$\Rightarrow$' By fact 3.3 Σ' is bounded, since Σ is bounded.

Assumption : Σ' is not live.
Since Σ' is free choice (by assumption), boundedness and strongly connectedness by theorem 3.5 imply that Σ' is not deadlock free. Consequently there exists a marking $M' \in [M_0\rangle : \forall t' \in T' : t'$ is not enabled by M'. Therefore $f(T)$ is not live in contradiction to fact 3.4.

Consequently : Σ' is live.

'$\Leftarrow$' Directly from facts 3.3 and 3.4. ■ 3.6

Knowing a strongly connected free choice simulation of a bounded system Σ by theorem 3.6 a polynomial – time algorithm to decide liveness of Σ exists, since in [3] one is given for bounded free choice systems.

4 Free choice simulation

Figure 3 shows a net $N_N := (\{s_1, s_2\}, \{t_1, t_2\}, \{(s_1, t_1), (s_1, t_2), (s_2, t_2)\})$ with different initial markings.

Proposition 4.1 *Structure of a free choice simulation of N_N*

Let $\Sigma = (S, T, F, M_0)$ be an S/T system, $N_N = (\{s_1, s_2\}, \{t_1, t_2\}, \{(s_1, t_1), (s_1, t_2), (s_2, t_2)\})$ (see figure 3,(1)) a subnet of the underlying net $N = (S, T, F)$.
Let $M_2 \in [M_0\rangle$ be a marking of Σ such that both t_1 and t_2 are enabled in conflict by M_2 (see figure 3,(2)),
and $M_3 \in [M_0\rangle$ be a marking of Σ such that t_1 is enabled by M_3 but t_2 is not (see figure 3,(3)).
Moreover, let $\Sigma' = (S', T', F', M'_0)$ be a free choice system simulating Σ with respect to $f : T \to T'$ with $f(t_1) = t'_1$ and $f(t_2) = t'_2$. Then the following holds for all $i \in \{1, 2\}$:

(i) $\exists s'_i \in {}^\bullet t'_i$ *with* $|\, s'^\bullet_i \,| > 1$.
An input-place of t'_1 as well as an input-place of t'_2 is branched.

(ii) $(\{s' \in {}^\bullet t'_1 \mid |\, s'^\bullet \,| > 1\} =: S'_1) \cap (S'_2 := \{s' \in {}^\bullet t'_2 \mid |\, s'^\bullet \,| > 1\}) = \emptyset$.
The two sets of input-places of t'_1 and t'_2 having more than one transition in its post-sets are disjoint.

(iii) $\forall\, t' \in S'^\bullet_i : |{}^\bullet\, t'\,| = 1$. *Especially:* $|{}^\bullet\, t'_i\,| = 1$ *and* $|\, S'_i \,| = 1$.
Every transition, which has places of S'_1 or S'_2 in its pre-set, has exactly one input-place (especially t'_1 and t'_2).

Proof: (*i*) *(Contradiction to the definition of simulation)*
It will be shown, by contradiction, that at least one place in the pre-set of t'_1 has at least two transitions in its post-set. The proof of the property of t'_2 is analogous.
Since Σ' simulates Σ, there exists a marking $M'_2 \in [M'_0\rangle$ and $\beta(M'_2) = M_2$. Moreover, there exists a (possibly empty) sequence of $\tau-$ transitions enabled by M'_2 that enables t'_1 and a (possibly empty and possibly different) sequence of $\tau-$ transitions enabling t'_2 thereafter.

Assumption: $\forall\, s' \in {}^\bullet t'_1 : |\, s'^\bullet \,| = 1$;
no place of the pre-set of t'_1 has two transitions in its post-set.
Since $\beta\ (M'_2) = M_2$ and $M_2[t_1\rangle$, there exists a sequence $\sigma'_1 \in L(N', M'_2)$ with $\sigma'_1 \in (T' \backslash f(T))^*$ and $M'_2[\sigma'_1\rangle M''_2$ such that t'_1 is enabled under M''_2, and $\beta\ (M''_2) = M_2$.
Moreover, it follows from $M_2[t_2\rangle$ that there exists another sequence $\sigma'_2 \in L(N', M''_2)$ with $\sigma'_2 \in (T' \backslash f(T))^*$ and $M''_2[\sigma'_2\rangle M'''_2$ such that t'_2 is enabled under M'''_2, and $\beta\ (M'''_2) = M_2$.
But by assumption t'_1 is still enabled under M'''_2, because no token of a place of the pre-set of t'_1 could have been removed.
Therefore t'_1 and t'_2 are concurrently enabled by M'''_2 in Σ', but in Σ, t_1 and t_2 are enabled in conflict under M_2.
Consequently, there is in Σ no sequence corresponding to $M'_2[\sigma'_1\rangle M''_2[\sigma'_2\rangle M'''_2[t'_1 t'_2\rangle$ of Σ'. This is the contradiction to definition 3.1 of simulation ((ii)(b)).

Assumption: $\forall s' \in {}^\bullet t'_2 :| s'^\bullet | = 1$; no place of the pre–set of t'_2 has two transitions in its post–set.
In this case, too, one can find a sequence of τ– transitions enabling t'_2. Using the same arguments as in the first part of this proof, it can be shown that there is also a τ– sequence enabling t'_1. Now, a marking has been found that enables t'_1 and t'_2 concurrently, and this is the contradiction to definition 3.1 of simulation ((ii)(b)).

(ii) *(Contradiction to the definition of simulation)*

Assumption : $S'_1 \cap S'_2 \neq \emptyset$.
Then there exists a place $s' \in S'_1 \cap S'_2$ with $\{t'_1, t'_2\} \subseteq s'^\bullet$. Because Σ' is free choice, it follows that ${}^\bullet(s'^\bullet) = s'$. Therefore every marking enabling t'_1 enables t'_2, too. But then there is no marking $M'_3 \in [M'_0\rangle$, with $\beta\ (M'_3) = M_3$ (since M_3 enables only t_1 in Σ) in contradiction to definition 3.1, which requires that β is surjective.
Therefore there are two different places s'_i, one in the pre–set of t'_1 and one in the pre–set of t'_2, having at least two transitions in their post–set.

(iii) *(Contradiction to the free choice property of Σ')*
Because of the definition of S'_1 one has for all $s' \in S'_1$: $t'_1 \in s'^\bullet$. According to part (i) of this proposition S'_1 is not empty and therefore there exists at least one $s' \in S'_1$ with $| s'^\bullet | > 1$ and by the free choice property of Σ it follows: ${}^\bullet t'_1 = \{s'\}$. Therefore, $S'_1 = \{s'_1\}$, and thus for all $t' \in s_1'^\bullet$: ${}^\bullet t' = \{s'_1\}$.
Analogously $S'_2 = \{s'_2\}$ can be shown with all consequences.

■ 4.1

Definition 4.2 *Multiset (bag), Parikh-set*

Let X be a set, and $\Sigma = (S, T, F, M_0)$ an S/T system.
A function $B : X \rightarrow \mathbb{N}$ is called *multiset* (or *bag*) of X. For any $x \in X$ $B(x)$ designates the number of occurrences of x in bag B.
Let σ be a firing sequence of Σ. The function $B(\sigma) : T \rightarrow \mathbb{N}$ is called *Parikh-set* of σ, whereby $B(\sigma)(t)$ is the number of occurrences of $t \in T$ in σ. ■ 4.2

For relations and operations on multisets see the appendix.

Definition 4.3 *Pre/post-sets of a firing sequence*

Let $\Sigma = (N, M_0)$ be an S/T system and σ a firing sequence of Σ.
The *pre/post set* of σ is defined (via its bag) by:

$$
\begin{aligned}
{}^\bullet B(\sigma) &= \{s \in S \mid \exists t \in B(\sigma) : s \in {}^\bullet t\}, \\
B(\sigma)^\bullet &= \{s \in S \mid \exists t \in B(\sigma) : s \in t^\bullet\}.
\end{aligned}
$$

■ 4.3

Definition 4.4 *Partial marking*

Let $\Sigma = (S, T, F, M_0)$ be a system, $M \in [M_0\rangle$ a marking, and $S' \subseteq S$ a set of places.
$Proj(M, S')$ defines a marking, not necessarily reachable from M_0, in the following way:

$$
Proj(M, S')(s) = \begin{cases} M(s) & \text{if } s \in S' \\ 0 & \text{if } s \notin S' \end{cases}
$$

■ 4.4

Theorem 4.5 *Some systems containing N_N as subnets are not free choice simulatable*

Let $\Sigma = (S, T, F, M_0)$ be a bounded S/T system, $N_N = (\{s_1, s_2\}, \{t_1, t_2\}, \{(s_1, t_1), (s_1, t_2), (s_2, t_2)\})$ (see figure 3(1)) a subnet of the underlying net $N = (S, T, F)$.
Let $M_2 \in [M_0\rangle$ be a marking of Σ such that t_1 and t_2 are enabled in conflict by M_2 (see figure 3,(2)),
and $M_3 \in [M_0\rangle$ be a marking of Σ such that t_1 is enabled by M_3, but t_2 is not (see figure 3,(3)).
Moreover, let $\Sigma' = (S', T', F', M'_0)$ be a system simulating Σ with respect to $f : T \rightarrow T'$ with $f(t_1) = t'_1$ and $f(t_2) = t'_2$.
Then Σ' is not free choice.

Proof: Contradiction to the simulation behaviour of Σ'

Assumption: Σ' is free choice.
The markings M_2 and M_3, reachable in Σ starting with M_0, are distinct. Therefore, there exist two different markings M'_2 and M'_3 in Σ' reachable from M'_0, with $\beta\ (M'_2) = M_2$ and $\beta\ (M'_3) = M_3$ (since every marking in $[M_0\rangle$ is the image of a marking of $[M'_0\rangle$ under β).
It is sufficient to show that the behaviour of the system Σ' is not the behaviour of a system simulating Σ, starting at a marking $M'_2 \in \beta^{-1}(M_2)$, or at a marking $M'_3 \in \beta^{-1}(M_3)$.
By proposition 4.1 $| {}^\bullet t'_1 | = 1 = | {}^\bullet t'_2 |$ is valid. Let $\{s'_1\} := {}^\bullet t'_1$ and $\{s'_2\} := {}^\bullet t'_2$.
Consider
the sequences $\sigma'_1, \sigma''_1 \in (T' \backslash f(T))^*$ from the markings M'_2 and M'_3 in Σ', with the following properties:
$M'_2[\sigma'_1\rangle M''_2[t'_1\rangle$, and the image of the surjection β of every marking occurring in σ'_1

is M_2 (since σ_1' is a τ- sequence).
$M_3'[\sigma_1''\rangle M_3''[t_1'\rangle$, and the image of the surjection β of every marking occurring in σ_1'' is M_3 (since σ_1'' is a τ- sequence).
Consider now τ– sequences to enable t_2' starting with M_2''. Let $\sigma_2' \in L(N', M_2'')$, $\forall t' \in B(\sigma_2') : t' \in T'\backslash f(T)$ with: $M_2''[\sigma_2'\rangle M_2'''[t_2'\rangle$, of minimal length. This means that there is no other τ– sequence $\sigma_2'' \in L(N', M_2'')$ with $|\sigma_2''| < |\sigma_2'|$ and $M_2''[\sigma_2'' t_2'\rangle$.
Since Σ' is a simulation of Σ, there exists such a sequence.

Assumption: $\sigma_2' = \lambda$ (empty sequence).
Hence $M_2''[t_1'\rangle$ and ${}^\bullet t_1' \cap {}^\bullet t_2' = \emptyset$ (proposition 4.1 (ii)) and the assumption (the empty sequence enables t_2' under M_2'') implies that t_1' and t_2' are concurrently enabled under M_2''. This contradicts the fact that Σ' simulates Σ.
Therefore σ_2' contains at least one transition that removes the token of s_1'.

Consequently: $\sigma_2' \neq \lambda$.

It will be shown next that there exists for all transitions t' of $B(\sigma_2')$ in N' an elementary path: $(t' \ldots s_2')$. All transitions occurring in this path are also members of $B(\sigma_2')$, because otherwise the sequence could not be minimal to enable t_2'.

Assumption: There is for some transition $t' \in B(\sigma_2')$ in N' no elementary path to s_2' with the property that all transitions occurring in that path are also members of $B(\sigma_2')$.
Let $T'' \subset B(\sigma_2')$ be defined as the multiset of all these transitions, and $T''' := B(\sigma_2')\backslash T''$. By assumption, T'' is not empty. Moreover, $T''' \neq \emptyset$ since σ_2' contains at least the transition that has s_2' in its post-set (for enabling t_2').
By definition of the multisets, $T''^\bullet \cap {}^\bullet T''' = \emptyset$, since otherwise $t'' \in T''$ and $t''' \in T'''$ with a common place $t''^\bullet \ni s' \in {}^\bullet t'''$ exist, and therefore there exists either an elementary path from t'' to s_2', or none from t''' to s_2'.
Consequently, there is a sequence $\sigma_2'' \in L(N', M_2'')$ with $B(\sigma_2'') = T'''$, hence $\sigma_2' \in L(N', M_2'')$ and $B(\sigma_2') = T'' \cup T'''$ and no transition of T''' needs for firing from M_2'' a token produced by a transition of T'' ($T''^\bullet \cap {}^\bullet T''' = \emptyset$). Therefore, there is a sequence σ_2'' with $B(\sigma_2'') \subset B(\sigma_2')$ enabling t_2' because by definition no transition of T'' is in the pre–set of s_2'. Now a sequence σ_2'' shorter than σ_2' to enable t_2' has been found, contradicting the definition of σ_2'.

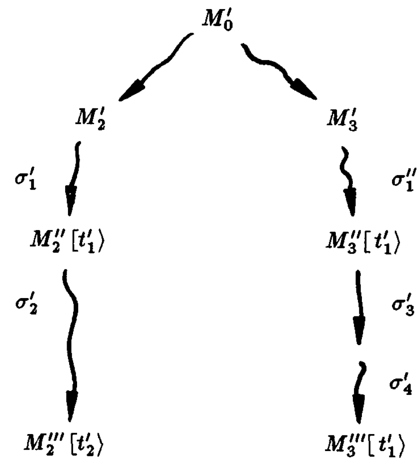


Figure 4: Part of the reachability graph

We define now some firing sequences of Σ' with certain properties. Figure 4 shows a part of the reachability graph of a system (which could be a simulation of the subnet N_N with the two markings, given in theorem 4.5) containing important sequences for this proof. Next to a marking the transitions are indicated, that are enabled by it and which are important.
Define a sequence of τ– transitions $\sigma_3' \in L(N', M_3'') \cap (T'\backslash f(T))^*$ in the following way: $B(\sigma_3')$ has a maximal number of transitions of $B(\sigma_2')$, that is
$\nexists \sigma_3'' \in L(N', M_3'')$, $\sigma_3'' \in (T'\backslash f(T))^*$ with: $|B(\sigma_2') \cap B(\sigma_3')| < |B(\sigma_2') \cap B(\sigma_3'')|$.
Possibly, σ_3' is not unique. Take then from the sequences satisfying this property one of minimal length. This sequence is finite, because by definition of simulation 3.1 (iii) there are always finite τ– sequences for enabling fireable transitions, and σ_2' is finite, too.
Next, it will be shown that σ_3' is not empty.

Assumption: $\sigma_3' = \lambda$.
Since $M_2''(s_1') = M_3''(s_1') = 1$ and $\forall t' \in s_1'^\bullet$: $|{}^\bullet t'| = 1$ (proposition 4.1 (iii)) it follows by the assumption: $\nexists t' \in B(\sigma_2')$: $t' \in s_1'^\bullet$, since otherwise one gets a contradiction to the maximality of σ_3'. By ${}^\bullet t_1' \cap {}^\bullet t_2' = \emptyset$ (proposition 4.1 (ii)) it follows:

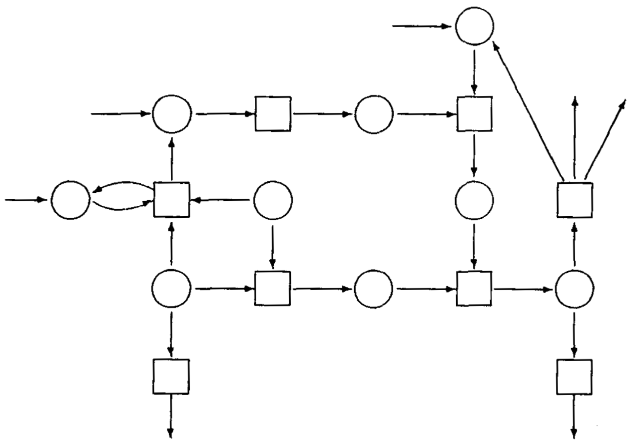


Figure 5: A net, in some sense similar to N_N

$\sigma_2' \in L(N', Proj(M_2'', S'\backslash\{s_1'\}))$, and therefore: $M_2''[t_1'\sigma_2't_2'\rangle$; this means that there is a marking $M' \in [M_2''\rangle$ enabling t_1' and t_2' concurrently. This contradicts that Σ' is a simulation of Σ.

Consequently: $\sigma_3' \neq \lambda$.

Next, the multiset $T_1' := B(\sigma_2')\backslash B(\sigma_3')$ is defined. T_1' contains all transitions of $B(\sigma_2')$, which could not be fired under σ_3'. By definition: $B(\sigma_2') \subseteq T_1' \cup B(\sigma_3')$.

Assumption: $T_1' = \emptyset$.

Hence $M_2''(s_2') = 0$ $(s_1' \neq s_2')$ and $\sigma_2't_2' \in L(N', M_2'')$; there exists $t' \in B(\sigma_2') : t' \in {}^\bullet s_2'$. By assumption $B(\sigma_2') \subseteq B(\sigma_3')$ and therefore $t' \in B(\sigma_3')$. Since σ_2' is minimal for enabling t_2', $\nexists t'' \in B(\sigma_2') : t'' \in s_2'^\bullet$. Because $\sigma_3' \in L(N', M_3'')$ it follows now: $\sigma_3't_2' \in L(N', M_3'')$ in contradiction to Σ' simulates Σ (Requirement (ii)(b) of definition 3.1).

Consequently: $T_1' \neq \emptyset$.

Let σ_4' be a minimal $\tau-$ sequence with: $M_3''[\sigma_3'\sigma_4't_1'\rangle$. σ_4' is a sequence that enables t_1' again after having fired σ_3' from M_3''. This sequence exists, because Σ' is a simulation of Σ. Let M_3''' be the reached marking. It cannot be excluded that σ_4' is the empty sequence. In the example of figure 5 a subnet of a free choice net that fulfills condition (i) and (ii) but not requirement (iii) of definition 3.1 is shown. The behaviour is similar to that of the system N_N with the two markings of theorem 4.5.

Representing M_3', only the places s_1' and s_3' have a token. Consequently $M_3' = M_3''$ and $\sigma_1' = \lambda$.

Representing M_2' only the places s_1', s_4' and s_6' are marked with one token. Consequently, too, $M_2' = M_2''$ and $\sigma_1'' = \lambda$.

The other sequences are defined like this: $\sigma_2' = \tau_1\tau_3\tau_4\tau_5$, $\sigma_3' = \tau_1\tau_2\tau_3$, $\sigma_4' = \lambda$ and: $T_1' = \{\tau_4, \tau_5\}$.

Next, it will be shown, that the intersection of the post-set of the transitions of the sequence σ_3' with the pre-set of T_1' (those transitions that could not be fired under σ_3') is not empty. Later one will see, that the contradiction to the assumption (Σ' free choice) will be that this set contains unbounded places.

Assumption: $B(\sigma_3')^\bullet \cap {}^\bullet T_1' = \emptyset$.

Since $\sigma_3' \neq \lambda$, it follows: $\exists t' \in B(\sigma_3')$ with $t' \in B(\sigma_2')$, because otherwise there is a contradiction to the minimality of σ_3'. By assumption especially: $\forall t' \in (B(\sigma_3') \cap B(\sigma_2'))$: $t'^\bullet \cap {}^\bullet T_1' = \emptyset$. This contradicts to the minimality of σ_2', because otherwise, there would be at least one $t' \in (B(\sigma_3') \cap B(\sigma_2'))$ without an elementary path to s_2', containing only transitions of $B(\sigma_2')$, since $\nexists t'' \in (B(\sigma_3') \cap B(\sigma_2')) : t'' \in {}^\bullet s_2'$.

Consequently: $B(\sigma_3')^\bullet \cap {}^\bullet T_1' \neq \emptyset$.

Having fired the $\tau-$ sequence $\sigma_3'\sigma_4'$ starting with marking M_3'' the input-place s_1' of t_1' has again one token (at least one transition of $B(\sigma_3)$ lies in the post-set of s_1' removing therefore at a time the token of s_1'). Since every transition of $t' \in s_1'^\bullet$ has only s_1' in its pre-set (proposition 4.1 (iii)) one can find new sequences σ_3'' and σ_4'' with $M_3'''[\sigma_3''\sigma_4''\rangle M_3''''$ defined like the sequences above.

Let $n \in I\!N\backslash\{0\}$ denote the $n-$ th iteration of these 'cycles' yielding a marking M_3^{n+2} after firing the sequence of $\tau-$ transitions: $M_3''[\sigma_3'\sigma_4'\sigma_3''\sigma_4'' \dots \sigma_3^n\sigma_4^n\rangle M_3^{n+2}$.

Again, since $\forall t' \in s_1'^\bullet$: $|\ {}^\bullet t'\ | = 1$ and $M_3^{n+2}(s_1') = 1$, sequences $\sigma_3^{n+1} \in L(N', M_3^{n+2})$ – σ_3^{n+1} contains as many transitions of σ_2' as possible – and σ_4^{n+1} – enabling t_1' again – can be found. Consequently, one can define the multiset $T_1^{n+1} := B(\sigma_2')\backslash B(\sigma_3^{n+1})$ – all transitions of σ_2' that could not fire in σ_3^{n+1}.

Like the proofs done, one can show:

$\sigma_3^{n+1} \neq \lambda$,

$T_1^{n+1} \neq \emptyset$ and

$B(\sigma_3^{n+1})^\bullet \cap {}^\bullet T_1^{n+1} \neq \emptyset$.

It remains to show, that there exists a

non empty set of places and an integer $n_0 \in \mathbb{N}$ so that after having fired a 'cycle' $\sigma_3^{n_0+n}\sigma_4^{n_0+n}$, $n \in \mathbb{N}\backslash\{0\}$ this set contains more token than before. If this is done, a contradiction to definition 3.1 (iii) arises, since every 'cycle' is only composed of τ–transitions.

But first we will prove an inclusion property of the multisets T_1^n: $T_1^n \subseteq T_1^{n+1}$ for every positive integer n. This means that if a transition $t' \in B(\sigma_2')$ could not be fired under $\sigma_3^n\sigma_4^n$ it cannot fire under any sequence $\sigma_3^{n+m}\sigma_4^{n+m}$. Therefore it must be shown that $\forall t' \in T' : T_1^n(t') \leq T_1^{n+1}(t')$ (see appendix).

Let $t' \in T_1^n$ and let $m \in \mathbb{N}\backslash\{0\}$ be the positive integer with $T_1^n(t') = m$.

With $B(\sigma_2') \subseteq B(\sigma_3^n) \cup T_1^n$ follows $B(\sigma_2')(t') = B(\sigma_3^n)(t') + T_1^n(t')$. Otherwise there would be a contradiction to definition of T_1^n. Moreover, we have by definition $B(\sigma_2')(t') \leq B(\sigma_3^{n+1})(t') + T_1^{n+1}(t')$.

Assumption: $T_1^{n+1}(t') < m = T_1^n(t')$.

Then we have $B(\sigma_3^{n+1})(t') > B(\sigma_3^n)(t')$, since $B(\sigma_3^n)(t') + T_1^n(t') \leq B(\sigma_3^{n+1})(t') + T_1^{n+1}(t')$. It follows with $B(\sigma_2')(t') > B(\sigma_3^n)(t')$ the valuation:

$| \; B(\sigma_2') \cap B(\sigma_3^n) \; | < | \; B(\sigma_2') \cap B(\sigma_3^n\sigma_4^n\sigma_3^{n+1}) \; |$

in contradiction to maximality of σ_3^n with respect to the number of transitions of σ_2', since we have: $\sigma_3^n\sigma_4^n\sigma_3^{n+1} \in L(N', M_3^{n+1})$.

Consequently: $T_1^{n+1}(t') \geq m = T_1^n(t')$.

Therefore we have $\forall t' \in T', \forall n \in \mathbb{N}\backslash\{0\} : T_1^n(t') \leq T_1^{n+1}(t')$ and consequently $T_1^n \subseteq T_1^{n+1}$.

Let denote $n_0 \in \mathbb{N}\backslash\{0\}$ the least integer so that the multisets T_1^n of transitions of every iterationstep with n greater than n_0 are equal: $\forall n \in \mathbb{N} : T_1^{n_0} = T_1^{n_0+n}$. This integer exists because every defined and used sequence is finite and because of the inclusion property of the multisets.

In the folowing will be shown, that the pre–set of $T_1^{n_0}$ contains unbounded places. These places are also contained in the post–set of the sequences $\sigma_3^{n_0+n}$. For that purpose, it is necessary to prove, that the post–set of every 'cycle' $\sigma_3^{n_0+n}\sigma_4^{n_0+n}$ contains a place that is in the pre–set of a transition t' of $T_1^{n_0}$ with more than one input–place. Therefore, we have for every $n \in \mathbb{N}\backslash\{0\}$ the

assumption: $\forall t' \in T_1^{n_0}$ with

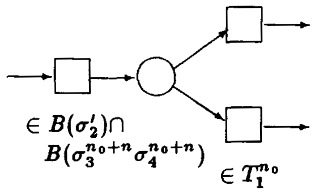


Figure 6: Impossible subnet of a possible simulation

$B(\sigma_3^{n_0+n}\sigma_4^{n_0+n})^\bullet \cap {}^\bullet t' \neq \emptyset$: $| {}^\bullet t' | = 1$.

Since for any $m \in \mathbb{N}$: $B(\sigma_3^{n_0+m}\sigma_4^{n_0+m})^\bullet \cap {}^\bullet T_1^{n_0} \neq \emptyset$, we can assume that there exists at least one such t'.

For any of these transitions there is no transition $t'' \in B(\sigma_2')$ in the pre–set of its input–place, because t'' fires in every sequence $\sigma_3^{n_0+n+m}\sigma_4^{n_0+n+m}$ ($T_1^{n_0} = T_1^{n_0+m}$) and there would be a contradiction to maximality of $\sigma_3^{n_0+n}\sigma_4^{n_0+n}$ (see figure 6).

Consider the subsequence of σ_2' with maximal length that can fire under $M_3^{n_0+n+1}$ and call it $\sigma_2^{n_0}$. Note that $\sigma_2^{n_0} \subset \sigma_2'$, since no τ– sequence enables t_2' from $M_3^{n_0+n+1}$. Hence $M_3^{n_0+n+1}(s_1') = 1$ this sequence is not empty.

We have $\forall t''' \in B(\sigma_2^{n_0}) : t'''^\bullet \cap {}^\bullet T_1^{n_0} = \emptyset$ by assumption and the fact above (otherwise the contradiction to maximality of $\sigma_3^{n_0+n}\sigma_4^{n_0+n}$).

But then for all transitions of $B(\sigma_2^{n_0})$ no elementary path to s_2' exists such that all transitions occurring in this path are members of $B(\sigma_2')$ contradicting the existence of these paths for all transitions of $B(\sigma_2')$.

Consequently: $\exists t' \in T_1^{n_0}$ with $B(\sigma_3^{n_0+n}\sigma_4^{n_0+n})^\bullet \cap {}^\bullet t' \neq \emptyset$ such that $| {}^\bullet t' | > 1$.

This transition t' cannot fire in any sequence $\sigma_3^{n_0+n+m}\sigma_4^{n_0+n+m}$. Because Σ' is free choice, the token in the pre–set of t' produced of $\sigma_3^{n_0+n}\sigma_4^{n_0+n}$ could not be removed by any later 'cycle'.

This implies: $\forall n \in \mathbb{N}\backslash\{0\} \exists s' \in {}^\bullet T_1^{n_0}$: $M_3^{n_0+n+1}(s') < M_3^{n_0+n+2}(s')$, and we have: $\forall n \in \mathbb{N}\backslash\{0\} : M_3^{n_0+n+1} \neq M_3^{n_0+n+2}$.

Since $M_3' \in [M_0'\rangle$ and $\sigma_3'\sigma_4'\sigma_3''\sigma_4''\sigma_3'''\sigma_4''' \ldots \in L(N', M_3')$ is a τ–sequence, the image of every intermedi-

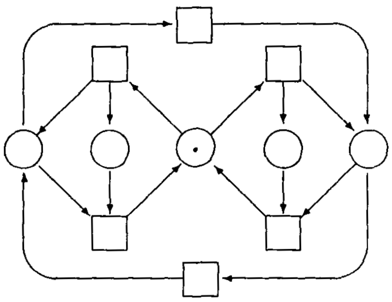


Figure 7: A safe system, which is not free choice but confusion free

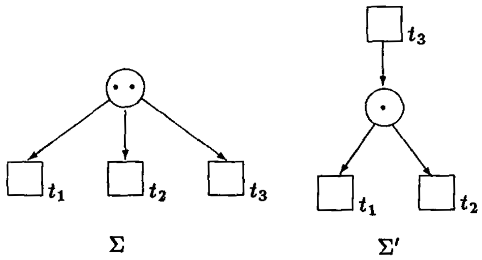


Figure 8: S/T systems with confusion?

ate marking M_3^n of β is M_3. Therefore $| \beta^{-1}(M_3) |= \infty$ in contradiction to requirement (iii) of the definition 3.1 of simulation.

Consequently Σ has no free choice simulation. ■ 4.5

Remark 4.6 *Necessity for the contradiction to definition 3.1 (iii)*

Figure 5 shows an unbounded free choice system with similar behaviour to the system N_N of figure 3 with respect to the markings (2) and (3) in 3. In figure 5 one can find a marking (only one token on each s_1' and s_3') corresponding to marking (2) and a marking (only one token on s_1', one on s_4' and one on s_6') corresponding to (3).

If a system Σ fulfills the assumptions of theorem 4.5 it may be, that $M_2 \notin [M_3\rangle$ and $M_3 \notin [M_2\rangle$. Therefore, if one replaces every occurance of N_N in Σ by the system of figure 5, one gets a free choice system Σ' failing the simulation property only by requirement (iii) of definition 3.1. Droping this requirement implies the wrongness of theorem 4.5. ■ 4.6

5 Concluding Remarks and Outlook

The system of figure 7 taken from [4] is safe and not free choice. Moreover it is confusion free. By theorem 4.5 it cannot be simulated by a free choice system because it contains N_N as a subnet $(\{s_1, s_2\}, \{t_1, t_2\}, \{(s_1, t_1), (s_1, t_2), (s_2, t_2)\})$, and the markings ((one token on s_1 and one on s_2) and (one token on s_1 but none on s_2)) are reachable.

Corollary 5.1

Not every confusion free system Σ has a free choice simulation Σ'. ■ 5.1

In section 2 the notion of confusion for safe S/T systems was given. Normaly, confusion is defined for elementary net systems. Every safe S/T system without side conditions (places s with ${}^\bullet s \cap s^\bullet \neq \emptyset$) can be viewed as an elementary net system (the occurence rules coincide in this case). Our definition of confusion becomes for these systems the definition of [14].

But the notion of confusion is much more complicated for non-safe S/T systems. In fact, no satisfactory final definition has been proposed so far. Consider figure 8. The system Σ has, which one would not expect, asymmetric confusion, since firing of t_3 produces a conflict between t_1 and t_2. On the other hand, Σ' has symmetric confusion, because firing of t_3 may solve the conflict between t_1 and t_2.

This paper is a first work about simulating systems with free choice systems. Many questions still remain open, including the following ones.

First, are there systems with free choice-like behaviour in the sense that conflicts appear only in free choice subnets of the system, without a free choice simulation? An example is given in figure 9 which, probably, cannot be simulated by a free choice net.

Which is a necessary and sufficient criterion for the question whether a free choice simulation of a system exists or not?

What about other notions of simulation? Do the results still hold when the simulation notion is weakened?

In remark 4.6 the reason about the contradiction to definition 3.1 (iii) constructed in the proof of theorem 4.5 is given. If one strengthened the assumptions in that theorem and requires $M_2 \in [M_3\rangle$ or $M_3 \in [M_2\rangle$, would it be possible to get a contradiction to restriction (ii) of the simulation definition?

6 Appendix

A definition of a net and a system is given in definition 2.1. A marking is a function $M: S \rightarrow \mathbb{N}$. A system is strongly connected iff $\forall x, y \in S \cup T : (x, y) \in F^*$. If $(s, t) \in F, s \in S, t \in T$ then s is called input-place of t.

A marking M enables a transition $t \in T$ iff $\forall s \in {}^\bullet t$: $M(s) \geq 1$. The enabling of t is denoted by $M[t\rangle$. An enabled transition can occur, yielding a new marking M' defined by the rule: $M'(s) = M(s) - 1$ for $s \in {}^\bullet t \setminus t^\bullet$, $M'(s) = M(s) + 1$ for $s \in t^\bullet \setminus {}^\bullet t$, and

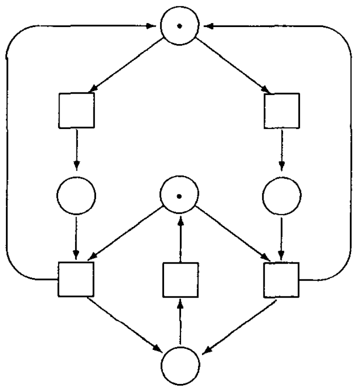


Figure 9: A non free choice simulatable system?

$M'(s) = M(s)$ otherwise. The occurrence of t is denoted by $M[t\rangle M'$.

An occurrence sequence is a sequence $\sigma = M[t_1\rangle M'[t_2\rangle M'' \ldots M^n$. It is said that σ starts with M and leads to M^n. Sometimes the intervening markings are omitted, since they are determined by M and the sequence of transitions. The sequence is then called firing sequence. The empty sequence is denoted by $\sigma = \lambda$. The marking M enables σ (denoted by $M[\sigma\rangle$) iff there are intermediate markings such that σ is an occurrence sequence starting with M. The set $[M\rangle$ is defined as the set of all markings M' such that some occurrence sequence leads from M to M'.

A system (N, M_0) with $N = (S, T, F)$ is live iff for every $t \in T$ and for every $M \in [M_0\rangle$ there is some $M' \in [M\rangle$ such that $M'[t\rangle$. The system is bounded iff for every place $s \in S$ there is a number $k \in \mathbb{N}$ such that all markings $M \in [M_0\rangle$ satisfy $M(s) \leq k$. It is 1-bounded or safe iff the number k can be taken as 1 for every place. The system is deadlock free iff $\forall M \in [M_0\rangle \exists t \in T : t$ is enabled by M.

The known relations and operations on sets are defined on bags (or Parikh sets) in the following way (A, B, C bags, X set): $x \in X$ is an element of B, ($x \in B$) iff $B(x) > 0$. (For $B(x) = 0$ the notation $x \notin B$ has been chosen.) The complexity of B is the number of the occurrence of all elements of X in B: $|B| = \sum_{x \in X} B(x)$. A is called subbag of B, ($A \subseteq B$) iff $\forall x \in X : A(x) \leq B(x)$. A is called equal to B iff $\forall x \in X : A(x) = B(x)$.
Union: $A \cup B = C$ iff $C(x) = max(A(x), B(x))$.
Intersection: $A \cap B = C$ iff $C(x) = min(A(x), B(x))$.
Difference: $A \backslash B = C$ iff $C(x) = 0$, if $A(x) < B(x)$; $C(x) = A(x) - B(x)$ otherwise.

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