Interleaved Trellis Coded Spread-Spectrum for Rician Fading Channels

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Abstract

A direct-sequence spread-spectrum communication system which employs a form of trellis coding is considered. We analyze the performance of this system for a channel model which includes the effects of multiple-access interference, Gaussian noise and Rician fading. Upper and lower bounds on the probability of occurrence of dominant error events are obtained by modelling the continuous probability density functions of noise and interference as discrete probability mass functions. Bit error rates are reported for coded and uncoded systems with equivalent power, bandwidth and transmission rates. It is found that while coded systems without interleaving yield only modest performance improvements over uncoded systems, coded systems with interleaving perform dramatically better.

1. Introduction

Although most work on direct-sequence spread-spectrum (DS/SS) systems assumes the use of binary signalling, there has been a growing interest in the use of nonbinary signalling for these systems. Enge and Sarwate [1] have investigated the performance of orthogonal signalling and concluded that significant performance improvements are possible. Another approach is to combine coding with nonbinary signalling. In [2], we proposed constructing a trellis code over a biorthogonal set of signature sequences. In [3], we showed that these trellis coded DS/SS systems attain significantly lower bit error rates than uncoded systems in the presence of multiple access interference and additive white Gaussian noise. Furthermore, the trellis coded systems require no more bandwidth than an uncoded system.

Since spread-spectrum systems are often used on channels with severe disturbances such as fading, the performance of trellis coded spread-spectrum on fading channels is of interest. If interleaving of sufficient depth is used, trellis coding can provide a form of time diversity which is useful in combating the effects of fading [10]. We expect this effect to carry over into the spread-spectrum environment. In this paper we investigate the performance of trellis coded DS/SS for a Rician fading channel in which interleaving is employed. We conclude that the trellis-coded DS/SS systems perform significantly better than the uncoded system for this environment, with only a modest increase in system complexity.

This paper is organized as follows. In Section 2, we describe our model for a trellis coded DS/SS system. In Section 3, the performance of trellis-coded DS/SS for a Rician fading channel is investigated. We show that the performance analysis can be viewed as an extension of the analysis for a multiple-access interference channel. In Section 4, we extend this performance analysis to the case where we have information on the severity of the fading during each signalling interval. In section 5, we report numerical performance results for a wide range of fading channels.

2. System Model

Our model for a DS/SS system is based on [4]. A more thorough description of a trellis coded DS/SS system is presented in [3]. The transmitter, illustrated in Figure 1, is divided into three sections: a trellis encoder, an interleaver and a modulator. In the first section, the trellis encoder selects a signature sequence $a_i = E(v_i, d_i)$, based on the current state $v_i$ and the current input data $d_i$. The encoder also uses some mapping $V$ to select the next state $v_{i+1} = V(v_i, d_i)$. The encoder then uses some mapping $E$ to select the current signature sequence $a_i = E(v_i, d_i)$, based on the current state $v_i$ and the current input data $d_i$. The encoder also uses some mapping $V$ to select the next state $v_{i+1} = V(v_i, d_i)$. Since spread-spectrum systems are often used on
The signature sequence \( a_i = (a_i, \ldots, a_i, N-1) \) is a sequence of \( N \) binary chips \( a_i \in \{ \pm 1 \} \). Our trellis codes are designed by considering each possible signature sequence as a point in the signal constellation of \( Q \) biorthogonal sequences. If we denote the set of possible signature sequences by \( S = \{ s_1, \ldots, s_Q \} \), where \( s_j \) is an \( N \) chip signature sequence and let \( a_i = s_j \) whenever \( s_j \) is the sequence selected by the mapping \( E(\tau, \delta) \), then we require

\[
N-1 \sum_{n=0}^{N-1} s_{i,n} s_{j,n} = \begin{cases} N, & i = j \\ -N, & i = j \pm Q/2 \\ 0, & \text{otherwise} \end{cases}.
\]

One way of generating a set of \( Q \) nearly biorthogonal signature sequences is by taking \( Q/2 \) different phases of the same \( m \)-sequence along with the negations of those \( Q/2 \) sequences [5]. The results presented in Section 5 are for signal constellations generated with this technique. However, the analysis presented below applies to arbitrary signature sequences.

The trellis codes which we have constructed have many of the same properties as more familiar trellis codes for QPSK, and QAM signal sets, including geometric uniformity [6]. The simple trellis code which we will use as an example throughout this paper is illustrated in Figure 2. This code is uniquely defined by one period of its trellis diagram. It is a 4-state trellis code constructed over a 4-ary biorthogonal signal constellation which transmits one bit of data per signalling interval. An uncoded binary system may be thought of as the degenerate case in which the encoder has a single state and the signal constellation consists of two signature sequences. Note that our approach to trellis coding differs significantly from the work of Boudreau et al., who have considered the use of Ungerboeck codes in conjunction with a DS/SS system [7].

Once the signature sequence has been selected, the \( \{ a_i \} \) are interleaved with one another so that a newly ordered succession of signature sequences \( \{ a'_i \} \) is produced.

The transmitter modulates the interleaved signature sequences in the usual manner. Each chip of a signature sequence is modulated by a rectangular pulse \( p_{r_i}(t) \) which takes the value 1 for \( t \in [0, T] \), and is zero otherwise. Since each chip has duration \( T_c \), the signalling interval has duration \( T = NT_c \). Thus the signal transmitted in the \( i \)-th interval is

\[
s(a'_i; t) = \sqrt{2P} \cos(\omega t + \Phi) \sum_{n=0}^{N-1} a'_{i,n} p_{r_n}(t - iT - nT_c),
\]

where \( P \) is the signal power, \( \omega_c \) is the carrier frequency, and \( \Phi \) is a random phase, uniformly distributed on \([0, 2\pi)\). The signals from successive intervals are combined to form the overall signal \( s(t) = \sum_{n=-\infty}^{\infty} s(a'_i; t) \).

The signal \( s(t) \) is passed through a channel where it is delayed and corrupted by noise, fading and multiple-access interference from \( K-1 \) other users. In the absence of fading, the received signal \( r(t) \) may be expressed as

\[
r(t) = n(t) + \sum_{k=1}^{K} s_k(t - \tau_k)
\]

where \( n(t) \) is a white Gaussian process with two-sided power spectral density \( N_0/2 \), \( \{ \tau_k : 1 \leq k \leq K \} \) is a set of random delays, independent and uniformly distributed on \([0, T]\) and \( s_k(t) \) is the signal transmitted by the \( k \)-th user.

A correlation receiver is used. Without loss of generality we consider the receiver for user 1. We assume that synchronization has been obtained so that
we may set \( \tau_1 = \Phi_1 = 0 \), and we take other delays \( \tau_k \) and phases \( \Phi_k \) relative to user 1. During the \( i \)-th interval, the receiver produces a set of \( Q \) decision statistics \( \{ Z_{i,q} \} \), defined by

\[
Z_{i,q} = \int_{T}^{(i+1)T} r(t) \cos(\omega_c t) \sum_{n=0}^{N-1} s_{q,n} \times p_{r,q}(t-nT_e - iT) dt, \quad q = 1, \ldots, Q. \tag{4}
\]

This set of decision statistics is deinterleaved to produce a set of decision statistics \( \{ Z_{i,q} \} \) which may be used as a metric in the Viterbi decoding algorithm.

It is possible to rewrite (4) as

\[
Z_{i,q} = \xi + A + \sum_{k=2}^{K} I_k, \tag{5}
\]

where the first term is due to Gaussian noise, the second term \( A = \sqrt{P/2N}T_e \) is due to the desired signal, and \( I_k \) represents the multiple access interference from user \( k \) at user 1's receiver, which may be written

\[
I_k = \int_{T}^{(i+1)T} s_k(t-\tau_k) \cos(\omega_c t) \sum_{i=0}^{L-1} \sum_{n=0}^{N-1} \frac{1}{2} [s_{p,i,n} - s_{p,i,n}] p_{r,q}(t-nT_e - iT) dt. \tag{6}
\]

The random variable \( \xi \) is Gaussian with mean zero and variance \( N_s T_e N/4 \).

We are interested in a channel model which includes the effects of Rician fading. In this case, the decision statistic in equation (5) becomes

\[
Z_{i,q} = \xi + AF'_{1,q} + \sum_{k=2}^{K} F_{k,i} I_k, \tag{7}
\]

where \( \{ F'_{k,i} : k = 1, \ldots, K, i = -\infty, \ldots, \infty \} \) is a set of independent random variables with the Rician probability density function

\[
p_F(x) = \begin{cases} 2x(1+\gamma) \exp(-\gamma - x^2(1+\gamma)) & x \geq 0, \\ xI_0(2\gamma \sqrt{\gamma(1+\gamma)}) & \text{else.} \end{cases} \tag{8}
\]

The parameter \( \gamma \) represents the ratio of the power in the direct component to the power in the diffuse component. Rayleigh fading \( (\gamma = 0) \) and no fading \( (\gamma = \infty) \) are special cases of this model. This model has been used by other authors, notably Divsalar and Simon [10], to model a channel in which multipath fading is present. We assume that phase coherent reception can be maintained, and that \( F'_{1,i} \) remains constant throughout \( i \)-th signalling interval. The deinterleaved version of \( \{ F'_{k,i} \} \) is \( \{ F_{k,i} \} \). We further assume that the interleaving is of sufficient depth so that adjacent values of \( F_{k,i} \), \( \tau_k \), and \( \theta_k \) will be independent after deinterleaving.

### 3. Performance on Fading Channel with Interleaving

We would like to evaluate the bit error rate of our system for a fading channel. As with all trellis codes, it is a complicated problem to exactly evaluate the probability of bit error \( P_b \). One frequently used method is the union bound. Let \( p \) be the correct path through the trellis and let \( \tilde{p} \) be some incorrect path which separates from path \( p \) beginning at time interval 0 and then remerges at a later interval, and let \( \| p - \tilde{p} \| \) be the Euclidean distance separating the paths \( p \) and \( \tilde{p} \). Let \( \{ d_1, d_2, \ldots \} \) be the set of all distances separating distinct paths through the trellis, such that \( d_1 < d_2 < d_3 < \ldots \). Then we may write

\[
P_b \leq \sum_{i=1}^{\infty} \sum_{\| p - \tilde{p} \| = d_i} W_{p,\tilde{p}} P_{p-\tilde{p}}, \tag{9}
\]

where \( W_{p,\tilde{p}} \) is the number of bit errors which result from choosing path \( \tilde{p} \) instead of \( p \), and \( P_{p-\tilde{p}} \) is the pairwise error probability between paths \( p \) and \( \tilde{p} \). Since paths at smaller Euclidean distances will tend to dominate the error rate, it is reasonable to truncate (9) after a few terms. In [8], we have demonstrated through simulations that for the codes of interest and for moderate signal-to-noise ratio, we may truncate (9) to a single term representing the minimum free Euclidean distance of the code, without seriously affecting accuracy. We will continue that practice in this paper.

Lehnert [9] has developed a technique for obtaining bounds on the probability of error in a binary antipodal system for the case of multiple-access interference. This technique is based on the idea of approximating a complicated continuous probability density function as a discrete probability mass function. In [3], we extended Lehnert's technique to evaluate the pairwise error probability between any two paths \( p \) and \( \tilde{p} \). We now show how this technique can be applied to the present situation. Suppose paths \( p \) and \( \tilde{p} \) diverge for \( L \) signalling intervals, beginning with interval 0. Let \( \{ s_p(0), \ldots, s_p(L-1) \} \) and \( \{ s_{\tilde{p}}(0), \ldots, s_{\tilde{p}}(L-1) \} \) be the signature sequences associated with paths \( p \) and \( \tilde{p} \) respectively during the signalling intervals 0 to \( L - 1 \). Then
we can define the decision statistic $Z_{p-\beta}$ as

$$Z_{p-\beta} = \sum_{i=0}^{L-1} \frac{1}{2}(Z_{p,i} - Z_{\beta,i}).$$

(10)

We express the pairwise error probability as $P_{p-\beta} = \Pr[Z_{p-\beta} < 0]$. We rewrite (10) in the form of equations (5) and (7) as

$$Z_{p-\beta} = \sum_{i=0}^{L-1} \eta_i + \sum_{i=0}^{L-1} F_{1,i} \sqrt{\frac{T_c}{2} N_i T_e} + \sum_{i=0}^{L-1} K_{k,i} I_{k,i},$$

where $\eta_i$ is Gaussian with mean zero and variance $\sigma^2_i = N_c T_e N_i / 4$, $N_i$ is the number of chips in which $s_{p(i)}$ differs from $s_{q(i)}$, and

$$I_{k,i} = \int_{-T}^{(i+1)T} \frac{\partial}{\partial t} \cos(\omega_i t) \sum_{n=0}^{N-1} \frac{1}{2} \left| \hat{s}_{p(n)} - \hat{s}_{q(n)} \right| \|F_{k,i} \sqrt{T_c} (t - nT_e - iT)\| dt,$$

$$k = 2, \ldots, K, \quad i = 0, \ldots, L - 1.$$  

(11)

We now construct vectors which will model the distribution of the multiple access interference terms $I_{k,i}$. We divide the interval $[-A, A)$ into $2m + 1$ intervals of equal size and let $d^{k,i} = \left( d^{k,i}_m, \ldots, d^{k,i}_m \right)$ be a $2m + 1$ component vector where

$$d^{k,i}_j = \Pr\{ I_{k,i} \in \left[ \frac{A(j - \frac{1}{2})}{m}, \frac{A(j + \frac{1}{2})}{m} \right] \}.$$  

(13)

It is shown in [9] that there exists a closed form expression for each component of $\{d^{k,i} : k = 2, \ldots, K, i = 0, \ldots, L - 1\}$. Thus the vector $d^{k,i}$ approximates the distribution of $I_{k,i}$.

The next step is to find vectors which will model the distribution of the faded multiple access interference $F_{k,i} I_{k,i}$. Note that the random variable $F_{k,i} I_{k,i}$ is not confined to the range $[-A, A]$. However, most of the distribution lies within the interval $[-m, m]$. Thus the vector $d^{k,i}$ approximates the distribution of $I_{k,i}$.

We divide the interval $[-A, A)$ into $2m + 1$ intervals of equal size and let $d^{k,i} = \left( d^{k,i}_m, \ldots, d^{k,i}_m \right)$ be a $2m + 1$ component vector where

$$d^{k,i}_j = \Pr\{ I_{k,i} \in \left[ \frac{A(j - \frac{1}{2})}{m}, \frac{A(j + \frac{1}{2})}{m} \right] \}.$$  

(13)

It is shown in [9] that there exists a closed form expression for each component of $\{d^{k,i} : k = 2, \ldots, K, i = 0, \ldots, L - 1\}$. Thus the vector $d^{k,i}$ approximates the distribution of $I_{k,i}$.

The next step is to find vectors which will model the distribution of the multiple access interference. Let $P_{k,i}^U$ be the probability that the multiple access interference is not contained within some component of the vector $u^{k,i}$. Then $P_{k,i}^U$ is given by

$$P_{k,i}^U = \sum_{j=-m}^{m} d^{k,i}_j \Pr\{ F_{k,i} \geq \frac{m - j + \frac{1}{2}}{m - j - \frac{1}{2}} \},$$

$$k = 2, \ldots, K, \quad i = 0, \ldots, L - 1.$$  

(14)

The vectors $\{u^{k,i}\}$ and $\{v^{k,i}\}$ may be thought of as representing the worst and best case distributions, respectively of the multiple access interference. Let $P_{k,i}^U$ be the probability that the multiple access interference is not contained within some component of the vector $u^{k,i}$. Then $P_{k,i}^U$ is given by

$$P_{k,i}^U = \sum_{j=-m}^{m} d^{k,i}_j \Pr\{ F_{k,i} \geq \frac{m - j + \frac{1}{2}}{m - j - \frac{1}{2}} \},$$

$$k = 2, \ldots, K, \quad i = 0, \ldots, L - 1.$$  

(16)

We now define the vectors $\{u^{k,i} : i = 0, \ldots, L - 1\}$ and $\{v^{k,i} : i = 0, \ldots, L - 1\}$ which will model the distribution of the faded signal component. The components of these vectors are defined to be

$$u^{k,i}_j = v^{k,i}_j,$$

$$= \Pr\{ \frac{A(j - \frac{1}{2})}{m} \leq \frac{\sqrt{\frac{T_c}{2} N_i \frac{T_e}{T_c}}}{m} \leq \frac{A(j + \frac{1}{2})}{m} \},$$

$$i = 0, \ldots, L - 1, \quad j = -m + 1, \ldots, m.$$  

(17)

Since the variables $\{F_{k,i}\}$ and $\{I_{k,i}\}$ are independent of one another, the summation $\sum_{i=0}^{L-1} F_{k,i} \sqrt{\frac{T_c}{2} N_i T_e} + \sum_{i=0}^{L-1} K_{k,i} I_{k,i}$ is the sum of $LK$ independent random variables. The PDF of this summation is the convolution of the PDFs of its components. We define the vectors $u$ and $v$ to be

$$u = u^{1,0} + \cdots + u^{1,L-1} + u^{2,0} + \cdots + u^{K,L-1},$$

$$v = v^{1,0} + \cdots + v^{1,L-1} + v^{2,0} + \cdots + v^{K,L-1}.$$  

(18)

(19)

Once again, the vector $u$ models the worst case distribution and the vector $v$ models the best case distribution.

We now compute upper and lower bounds on the pairwise error probability $P_{p-\beta}$ to be

$$P_{p-\beta} \leq \sum_{j=0}^{K L m + m} u_j$$

$$Q \left\{ \frac{2 P_{T_e} \sum_{i=0}^{L-1} N_i (2j - L K)}{N_0} \right\}.$$  

(14)
noise ratio and model the PDF of the Gaussian noise as a vector. We use convolutions to evaluate the pdf of \( \eta_i + \sum_{k=2}^{K} F_{k,i} I_{k,i} \). Second, we must evaluate the effects of the desired signal component and the weighting factor \( F_{k,i} \) in a single mapping operation, similar to (14) and (15). Finally, we perform an additional \( L - 1 \) convolutions to combine the PDFs from each of the \( L \) signalling intervals. There are several complicating factors. To keep the computation tractable, the value of \( m \) must be changed several times throughout the calculation, making the appropriate optimistic or pessimistic assumptions each time. Computing an upper bound on the probability that the interference is out of the range modeled by our vectors also becomes more complicated. For the sake of brevity, we omit these details, noting only that the algorithm is still able to operate in \( O(m^2) \) time.

4. Performance with Channel State Information

We now turn to the case where channel state information is available at the receiver. That is, we assume that the value of the fading \( \{F_{k,i}\} \) experienced by the desired signal for each interval is known to the receiver. There are several methods for obtaining such information, such as the use of pilot tones. In the absence of multiple access interference, an optimal receiver will weight each decision variable by the value of the fading variable \( \{F_{k,i}\} \). Thus, the Viterbi decoder now operates on the metric \( \{F_{k,i} Z_{k,i}\} \). Although this is not necessarily optimal for the case when multiple access interference is present, it is reasonable to use this receiver in the same sense that it is reasonable to use a correlation receiver for demodulating DS/SS in multiple access interference.

To analyze this receiver, we must find \( P_{p-\phi} = \Pr[Z_{p-\phi} < 0] \) where \( Z_{p-\phi} \) is now

\[
P_{p-\phi} = \sum_{i=0}^{L-1} \sum_{k=2}^{K} p_{k,i}^{\phi} + \sum_{i=0}^{L-1} \sum_{k=2}^{K} p_{k,i}^{\phi} I_{k,i} + \sum_{j=-K}^{K} \sum_{i=0}^{L-1} v_j \frac{\sqrt{2 PT_c N_i + 2j + LK}}{N_0} \]

where \( Q(\cdot) \) is the standard Q-function. Note that this bound becomes tighter as the number of components \( m \) increases, and becomes looser as the product \( KL \) increases. The most computationally intensive operation is the mapping of equations (14) and (15), so this algorithm requires \( O(m^2) \) computational steps. As we previously indicated, \( p_{k,i}^{\phi} \) becomes vanishingly small for \( m > 3 \).

5. Numerical Results

We have evaluated bit error rate for an uncoded system and for coded systems under a variety of assumptions. The signal constellations were constructed out of \( m \)-sequences of length \( N = 31 \). The number of users ranged from 1 to 5, and the direct to diffuse power coefficient \( \gamma \) of the Rician distribution was allowed to range from 0.01 through 100.0. In each case, comparisons are made on the basis of equivalent bandwidth, signal-to-noise ratio (SNR) and transmission rate.

Figure 3 plots upper and lower bounds on \( P_b \) as a function of SNR for \( \gamma = 0.0 \) (Rayleigh fading) and \( K = 1 \). We have included several cases. The baseline performance of an uncoded system is the first set of bounds plotted. A coded system without interleaving gains several dB over the uncoded system, but the dependence of \( P_b \) on SNR still exhibits the inverse linear relationship which is characteristic of Rayleigh fading channels. When interleaving is added, dramatic performance improvements are obtained. \( P_b \) as a function of SNR now decreases at a rate which is faster than linear. When channel state information is available, further modest improvements are possible. Upper and lower bounds on coded performance with interleaving and channel state information are shown as points.

For the case of pure Rayleigh fading and a single user only, it is possible to verify the channel state information results, using the exact analytic method of Cavers and Ho [11]. This exact curve is plotted as a solid line and agrees well with our bounds.

Figures 4, 5, and 6 take a different approach. In these figures, SNR is fixed at 20 dB, and \( P_b \) is plotted as a function of \( \gamma \). Figures 4, 5, and 6 present the
Acknowledgement

This research was supported by the Unisys Corporation and by the National Science Foundation under contract number ECS-8451266.

References


