Recognition of Circle Form Using Fuzzy Sequential System

Tatsuki Watanabe and Masayuki Matsumoto
Department of Electrical Engineering, Toyo University
Kawagoe-shi, Saitama Pref., 350 JAPAN

Abstract
Regarding fuzzy automata, many studies including the learning systems have been made. However, most of them were in the category of circuit analysis in terms of sequential circuit theory. As for the synthesis of fuzzy sequential circuits, except for that presented by the authors, almost no study has been done so far. This article presents an application of the synchronous fuzzy sequential system to pattern recognition in which fuzzy propositions are used to represent various sizes of circles. The advantages of the proposed system are small amount of required memory elements as well as a fast recognition speed caused by the sequential procedure. Furthermore, we can expect an easy realization of the system with electronic devices because of the simplicity of the fuzzy set operations. The simulation results for some examples are shown.

1 Introduction
Since Zadeh introduced the idea of fuzzy systems, regarding fuzzy automata, many intensive studies such as the maximin automata by Santos [Z] followed by many others including the applications to learning systems have been made. However, as for the synthesis of fuzzy sequential circuits, few articles have appeared so far, except for the method presented by us for synthesizing synchronous fuzzy sequential circuits [5]. The circuit configuration shown as an example in the said article was successfully fabricated using conventional electronic circuit technology and has been presented in one of the Japanese conventions. The same basic circuits can naturally apply to those presented in this paper. We call the system presented here a fuzzy sequential system because we present here only the computer simulation of the system to recognize various sizes of circle forms.

In this sequential system, a figure to be recognized is scanned at every fixed interval and, at each step of the scanning, the similarity grade to a circle form of the scanned part of the drawing is kept as a membership grade. While the scanning is going on, only one set of three consecutive scanned samples are held by turns instead of keeping all the scanned samples.

In the system, circle forms are defined by several fuzzy propositions. The fuzzy sequential systems are characterized by the system descriptions which include fuzzy propositions expressed with histories of inputs. It is noteworthy that, if we constructed the system having the same features as the fuzzy sequential system using the conventional sequential system technology, tremendous number of states in the sequential system to represent different membership grades would be required resulting in large amount of hardware and complexity in the circuit design.

Please note that, as the notations and definitions about the fuzzy systems in this paper accord to those used by Zadeh [3], [4], particular explanations for them are omitted.

2 A Model of Synchronous Fuzzy Sequential Systems
For simplicity, we restrict our model to time-invariant discrete time systems in which the time ranges over integers. Let $u_t$, $y_t$, and $x_t$ denote respectively the input, output and state of a system at time $t$. Such a system is said to be deterministic if it is characterized by state equations of the form:

$$x_{t+1} = f(x_{t}, u_{t}), \quad t = 0, 1, 2, \ldots$$

$$y_t = g(x_t, u_t).$$

The universal sets of $u_t$, $y_t$, and $x_t$ are denoted here as $U$, $Y$, and $X$ respectively. For instance, $x_t$ is expressed as follows:

$$x_t \in X \otimes (\mathbb{R}_{>0})^p; \quad p=0,1, \ldots, P; \quad z=0,1, \ldots, Z; \quad m=0,1, \ldots, M.$$  \hfill (3)

Let $U'$, $Y'$, and $X'$ denote fuzzy sets in $U$, $Y$, and $X$ characterized by the membership functions $\mu(u_t)$, $\mu(y_t)$, and $\mu(x_t)$ respectively. Zadeh introduced the following equations for such a fuzzy system [3]:

$$X'^{t+1} = F_{oc}(X', U')$$

$$Y' = G_{oc}(X', U').$$
If $X'$ and $U'$ are noninteractive fuzzy sets, we can deem the equation (4) the symbolic form of the following equation:

$$\mu(x_{t+1}) = \bigvee_{x_{t}} \bigwedge_{u_{t}} (\mu(x_{t}) \wedge \mu(u_{t}) \wedge \mu(x_{t+1} \mid x_{t}, u_{t}))$$

(6)

where $\mu(x_{t+1} \mid x_{t}, u_{t})$ is the membership function conditioned by $x_{t}$ and $u_{t}$ that are to characterize the fuzzy set $X^{t+1}$.

Except for the fuzzy part of the system, the deterministic model of synchronous sequential circuit defined by Mealy [1] is to apply here.

3 Fuzzy Inputs

First, let us present the problem description for which we are to synthesize a fuzzy sequential system to recognize various sizes of circle forms.

Problem 1

Design a synchronous sequential system that meets the following requirements:

"Scan a figure at every scanning interval $\delta$ and indicate, at each cycle of the scanning, the possibility grade for the scanned part of the figure to become an exact circle and, at the end of the scanning, the similarity grade of the scanned figure to a circle. The radiuses of the circles are to range approximately from 46 to 156. The similarity grade is to be expressed as a membership grade in a fuzzy set which signifies the degree to which the scanned figure is close to a circle."

Let the input $u_{t}$ represent a pair $(a_{t}, b_{t})$ where $a_{t}$ and $b_{t}$ are the ordinates of the upper edge and the lower edge of the figure sampled at time $t$ respectively. Fig. 1 shows an example. If there is no drawing to be scanned at time $t$, the values of $a_{t}$ and $b_{t}$ are assumed to be zero.

Three consecutive non-zero input-pairs $(x_{i}, y_{i})$, $i = 0, 1, 2$ which are the memorized values of three consecutive input-pairs $(a_{i}, b_{i})$ are used to generate the various membership grades to be used to calculate the similarity grade to a circle form.

Three consecutive input-pairs $(a_{i}, b_{i})$, $t = j+1, j+2, j+3$ are used as the next ones to $(a_{j}, b_{j})$, $t = j, j+1, j+2$. We use the following differences such as shown in Fig. 2:

$$d_{0} = x_{1} - x_{0},$$
$$d_{1} = x_{2} - x_{1},$$
$$e_{0} = y_{1} - y_{0},$$
$$e_{1} = y_{2} - y_{1}.$$  

(7)  

(8)  

(9)  

(10)

Using these differences, the features of circles are defined here by the following seven fuzzy propositions which are arranged according to the sequence of scanning:

$(P_{1})$ At the beginning of the figure, $d_{0}$ and $e_{0}$ are large.

$(P_{2})$ Next, approximately $n$ sets of three consecutive input-pairs having the following features appear consecutively.

$$d_{0}, d_{1} > 0 \text{ and } e_{0}, e_{1} > 0,$$

$|d_{0}| > |d_{1}|$ and $|e_{0}| > |e_{1}|$

$(P_{3})$ Next, a set of three consecutive input-pairs having the following features appears.

Fig. 1 An example of input data

Fig. 2 Differences between two consecutive samples
$d_0 > 0$ and $d_1 \leq 0$
$e_0 > 0$ and $e_1 \leq 0$

$(P_2)$ Next, approximately $z$ sets of three consecutive input-pairs having the following features appear consecutively, where $z$ is approximately proportional to $p$ with a certain proportional constant.

$d_0 \geq 0$ and $d_1 \leq 0,$
$e_0 \geq 0$ and $e_1 \leq 0.$

During the time when the above conditions for $d_0$, $d_1$, $e_0$ and $e_1$ are met, there is a scanning step wherein the maximum value of $(a_t-b_t)/(2.5)$ becomes approximately equal to $p+2$.

$e_0 > 0$ and $e_1 < 0$.

$(P_3)$ Next, a set of three consecutive input-pairs having the following features appears.

$d_0 \leq 0$ and $d_1 < 0,$
$e_0 \leq 0$ and $e_1 < 0.$

$(P_4)$ Next, approximately $m$ sets of three consecutive input-pairs having the following features appear consecutively.

$m \approx p,$

$d_0, d_1 < 0$ and $e_0, e_1 < 0,$

$|d_0| \leq |d_1|$ and $|e_0| \leq |e_1|.$

$(P_5)$ In the final set of three consecutive input-pairs which precedes the input-pairs $(a_t, b_t)$ wherein $a_t = b_t = 0$, both $d_1$ and $e_1$ are large.

The similarity grade of the scanned figure to a circle is given as the intersection, i.e., "MIN" of the truth values of the fuzzy propositions above.

The fuzzy set labeled "large" to be applied to the propositions $(P_3)$ and $(P_2)$ is represented as Fig. 3. This fuzzy set is characterized by the membership functions $w_{d_1}$ and $w_{e_1}$ for upper edge and lower edge of the figure respectively. The fuzzy sets labeled "plus", "zero" and "minus" are shown in Fig. 4 and Fig. 5 which are applied to the propositions $(P_2)$ through $(P_5)$ where Fig. 4 is used to represent degrees of "positive", "zero" and "negative" for values of $d_0, d_1, e_0$ and $e_1$, and Fig. 5 for $|d_0| - |d_1|$ and $|e_0| - |e_1|.$

The following membership functions are used:

$P_0 = P_{d_0} \land P_{e_0},$ \hspace{1cm} (11)
$P_1 = P_{d_1} \land P_{e_1},$ \hspace{1cm} (12)
$pls = plsd \land plse,$ \hspace{1cm} (13)
$zd = zsd \land zse,$ \hspace{1cm} (14)
$zd = zsd \land zse,$ \hspace{1cm} (15)
$zr = zsr \land zse,$ \hspace{1cm} (16)
$n0 = n_0 \land n_0,$ \hspace{1cm} (17)
$n1 = n_1 \land n_1,$ \hspace{1cm} (18)
$nm = nmse \land nmse.$ \hspace{1cm} (19)

Please note that, in Figs. 4 and 5, the membership functions which recognize the fuzzy sets for example, labeled "plus" over the universal sets of $d_0, d_1, |d_0| - |d_1|$ and etc. are denoted here as $P_{d_0}, P_{d_1},$ $plsd$ and etc. respectively. The similar notations are used for the other membership functions as shown in Figs. 4 and 5. The following membership functions are used:

$P_0 = P_{d_0} \land P_{e_0},$ \hspace{1cm} (11)
$P_1 = P_{d_1} \land P_{e_1},$ \hspace{1cm} (12)
$pls = plsd \land plse,$ \hspace{1cm} (13)
$zd = zsd \land zse,$ \hspace{1cm} (14)
$zd = zsd \land zse,$ \hspace{1cm} (15)
$zr = zsr \land zse,$ \hspace{1cm} (16)
$n0 = n_0 \land n_0,$ \hspace{1cm} (17)
$n1 = n_1 \land n_1,$ \hspace{1cm} (18)
$nm = nmse \land nmse.$ \hspace{1cm} (19)
once it becomes smaller than \( R \) because only MAX and MIN operations are included in the equation (6).

4 Construction of State Transition Diagram

When we draw a state transition diagram in accordance with the given problem that includes fuzzy statements, we can use the same idea for reduction of the diagram as in the case of deterministic sequential circuit.

First, we are to define the initial state which corresponds to the instant when the system is put into operation. The second step is to define the next states to the initial state followed by the others. In the case of problem 1, it is enough for us to have the value of \( \mu(x_{t+1}, x_t, u_t) \) fixed to the maximum value \( R \) if we define the states following the initial state so that they are distinguished each other simply by the occurrence times of either of three kinds of the inputs \( p, z \) and \( m \), the meanings of which will be explained later.

In this case, (6) reduces to:

\[
\mu(x_{t+1}) = \bigvee_{x_t, u_t} (\mu(x_t) \land \mu(u_t)).
\]  

As stated above, the inputs to the system are converted to the membership grades expressed by eqs. (11) through (19). The state transition diagram which include only such states that may have non-zero values as the similarity grades to a circle, is shown in Fig. 6. Referring to eqs. (11) through (19), the meanings of indices to state \( q_{pzm} \) in the diagram are as follows:

- \( p \) : times of state transitions caused by fuzzy input-pair \( (p_0, p_1) \)
- \( z \) : times of state transitions caused by either of fuzzy input-pair \( (p_0, z_0) \) or \( (z_0, p_1) \)
- \( m \) : times of state transitions caused by either of fuzzy input-pair \( (z_0, m) \) or \( (m, z_0) \).

For simplicity, the fuzzy inputs in Fig. 6 are indicated using the above notations \( p, z \) and \( m \).

---

Fig.6 State transition diagram of problem 1

---
In the problem 1, the membership function $\mu(u_i)$ is defined as follows:

$$\mu(u_i) \subset \{(p_0, p_1), (p_2, p_3), (z_0, z_1), (m_0, m_1), (\varepsilon_0, \varepsilon_1)\} \quad (21)$$

The states such as those in this problem may have membership functions other than $\mu(x_i)$ included in (6). Such membership functions are the ones that are defined by the given problem whereas the membership function $\mu(x_i)$ represents the grade of occurrence of state $x_i$.

In the case of this problem, they are such membership functions as $\mu_4(p, z)$ and $\mu_6(p, z)$ shown in Tables 1 and 2 respectively. The membership function $\mu_4(p, z)$ is to characterize the fuzzy set labeled "$z$ is approximately proportional to $p^2$" that is included in the proposition (6) above. The membership function $\mu_6(p, m)$ is to characterize "$m \equiv p^2$" included in the proposition (9) above.

The initial state is $q_0000$ in Fig. 6. Considering the largest size of the circle to be treated in the problem 1, it is assumed for the maximum values of $p$, $z$, and $m$ to be 1, 2, and $M$ up to which the state transition diagram in Fig. 6 is to be extended. Except for the state $q_0000$, if the input pair $(a_i, b_0) = (0,0)$ appears in the state $q_0000$, the state shifts to the next state $q_00m$ as shown in Fig. 6 (b) where the final similarity grade to a circle is generated as the output $ccl$ indicated in Fig. 8.

5 State Assignment

It is possible for more than one states in a state transition diagram of a fuzzy sequential system to have non-zero values of the membership functions $\mu(x_i)$ at time $t$. Let us call such a group of states that may take non-zero values of $\mu(x_i)$ at a time a coexisting-state group abbreviated to CE group [5]. The groups of states in Fig. 6 denoted as $G_0$, $k = 0, 1, \ldots$ are such CE groups.

In case of a deterministic sequential circuit, each state $q$ in the state transition diagram is assigned with a binary code vector of the form $(b_0, b_1, \ldots, b_m)$ where $b_i$ is a binary value taken by each state variable $y_i$ to distinguish the states.

Let $b_0, b_1, \ldots, b_m$ denote the state variables in the fuzzy sequential system to distinguish the CE groups each other. As each state $q_{mem}$ in the state transition diagram has its own grade of membership denoted as $\mu(q_{mem})$, every state has to be associated with a variable to keep its own value of $\mu(q_{mem})$ which is distinguishable in the CE group. Let us call such a variable a grade-of-membership variable. Thus, a code expressed as $(b_0, b_1, \ldots, b_m, E_{00}, \ldots, E_{mm})$ is to be assigned to each state in the state transition diagram, where the code $(b_0, b_1, \ldots, b_m)$ is to discriminate a CE group and $E_{mm}$ to hold a membership grade of $\mu(q_{mem})$. If we did not employ such state variables $b_i$, every state would have to have its own grade-of-membership variable resulting in an increase of memory elements in the system.

CE groups in a state transition diagram can be treated in the same manner as states in a deterministic sequential circuit [5]. In the case of problem 1, the transition diagram for the CE groups can be drawn as Fig. 7.

6 Configuration of the System

The configuration of the system to realize the state transition diagram in Fig. 6 is shown in Fig. 8.

The main function of the fuzzy logic circuit in Fig. 8 is to produce all the values of the state variables $f_{E00}, \ldots, f_{Emm}$ and the grade of membership variables $f_{E00}, \ldots, f_{Emm}$ for the next states using the input at time $t$. Each state is represented by the combination of the vector $(b_0, \ldots, b_m)$ which holds the present CE group number and the vector $(E_{00}, \ldots, E_{mm})$ each member of which holds the grade of membership of the state $q_{mem}$ in the present CE group. For simplicity, each grade of membership variable $E_{mm}$ in this system is defined so as to keep the similarity grade to a circle form to be attached to the state $q_{mem}$. According to the combinations of the values of $p$, $z$, and $m$, the grades of membership $f_{Emm}$ of the next states are calculated as listed below. In the followings, the listed cases are arranged in the sequence of scanning stages of a circle: 

(1) $p = 0$, $z = 0$, $m = 0$

$$f_{E00} = p_0 \land p_1 \land p_3 \land w_i \quad (22)$$

where $w_i$ is the membership function which represents the fuzzy set "$d_0A_0$ is large" and is expressed as $w_d \land w_a$, using $w_d$ and $w_a$ shown in Fig. 5.

(2) $p > 0$, $z = 0$, $m = 0$

(2-1) The case in which the same case as (2-1) is to follow

$$f_{E00} = w_{ae} \land p_0 \land p_3 \land w_i \quad (23)$$

where $w_{ae}$ is the membership function which represents the grade of membership of the preceding state.

(2-2) The case in which case (4) below is to follow
\[ f_{EXM} = w_{EXM} \Lambda P_0 \Lambda z_1 \Lambda P_1 \]  \hfill (24)

(2-3) The case in which case (3) below is to follow
\[ f_{EXM} = w_{EXM} \Lambda P_0 \Lambda P_1 \Lambda \mu_x(p, z) \Lambda \mu_x(p, r) \]  \hfill (25)

where \( \mu_x(p, z) \) is the membership function indicated in Table 1 and \( \mu_x(p, r) \) is the one introduced from the fuzzy set "\( p \approx r \)" that is, "\( p \) is approximately equal to \( r \)" where \( r = (a_t - b_t)/(26) \). Table 2 is used for this as well as for \( \mu_x(p, m) \) which appears in the case (6) below.

(3) \( p > 0, z = 0, m > 0 \)
\[ f_{EXM} = w_{EXM} \Lambda P_0 \Lambda m_1 \]  \hfill (26)

(4) \( p > 0, z > 0, m = 0 \)
(4-1) The case in which the same case (4-1) is to follow
\[ f_{EXM} = w_{EXM} \Lambda z_0 \Lambda z_1 \]  \hfill (27)

(4-2) The case in which case (5) below is to follow
\[ f_{EXM} = w_{EXM} \Lambda z_0 \Lambda m_1 \Lambda m_2 \Lambda \mu_x(p, z) \Lambda \mu_x(p, r) \]  \hfill (28)

(5) \( p > 0, z > 0, m > 0 \)
\[ f_{EXM} = w_{EXM} \Lambda m_1 \Lambda m_3 \]  \hfill (29)

---

**Fig. 8 Circuit configuration of the fuzzy sequential system**

---

Note
1. :memory element
2. :gate circuit
3. arrangement of clock-pulses
4. memory

---

90
If the input pair \((a_1, b_1) = (0, 0)\) appears, the final grade of membership of similarity to a circle form \(c_{cl}\) is generated using the following procedure.

\[
c_{cl} = C_{w} \land w_{r},
\]

where

\[
C_{w} = \bigvee_{x_{2}} f_{c_{1}} = \mu_{w}(p, m),
\]

\(w_{r}\): the membership function which represents the fuzzy set "\(d_1, \lambda_0\) is large" and is expressed as \(w_{a1}, \lambda w_{a1}, w_{a2}, \) and \(w_{a3}\), shown in Table 2 which represents "\(p \notin m\)"

### Simulation Results

Some examples of the circle-recognitions are shown in Figs. 9 and 10 indicating the similarity grades to circles, using \(R = 32\) as the maximum value. They were produced by computer simulations according to the description above. From Fig. 10, we can recognize that the system can work normally in the designated range.
8 Conclusion

It has been shown that the fuzzy sequential system can conveniently apply to the actual problems such as figure recognitions and have the ability to express the results of recognitions as the grade values which represent our actual feelings. Its advantages are the fast operation speed as well as the easy implementation of the system due to the small amount of memory elements and the simplicity of fuzzy set operations.

References


