Control of a 5-Link Biped Robot for Steady Walking

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ABSTRACT

This paper treats trajectory planning methods and control algorithms for biped robot steady walking. The Model Following plus Variable Structure Systems technique is developed for the control of dynamic walking in the sagittal plane. The Nonlinear Feedback plus Modified α-Computed Torque technique is introduced for walking in the frontal plane. Simulation results are provided for level walking and ascending on staircases. These results indicate that the proposed algorithms could achieve robust trajectory tracking even in the presence of system modeling errors.

1. Introduction

Many algorithms have been developed for the control of steady walking of biped robots [1-4,8]. Nevertheless, most of these algorithms were developed based on linearized models. As a result, biped robots can only walk with a small swing angle. For a large step length, corresponding to a large swing angle, biped robots will fall to the ground before the completion of a walking cycle. In addition, most of these algorithms are designed for static walk, i.e., dynamics are not included in the control model. The only existing control algorithms developed for the dynamic walk of a biped robot was developed by Takanishi, et.al. [8]. However, their approach has the following drawbacks: (1) the control algorithm cannot ensure the stability of the system in the presence of plant perturbations; and (2) for different walking patterns, control parameters must be determined by trial-and-error.

In this paper, we present the Model Following plus Variable Structure System (VSS) technique for control of walking in the sagittal plane, and the Nonlinear Feedback plus Modified α-Computed Torque Technique for control of walking in the frontal plane. Simulation results show that the proposed algorithms can only achieve stable and steady walk, but they also achieve robust trajectory tracking.

2. Mathematic Models of a 5-Link Biped Robot

Consider a 5-link biped robot as shown in Fig. 1(a) and Fig. 1(b), corresponding to the sagittal and frontal planes, respectively. The equation of motion for a 5-link biped robot in the single-leg support phase is

\[ A(\theta) \ddot{\theta} + B(\theta) \dot{\theta} + C(\theta) = DT \]  

where \( A(\theta) \) is the mass matrix, \( B(\theta) \) is the Coriolis matrix, \( C(\theta) \) is the centrifugal matrix, and \( T \) is the torque vector.

and \( \theta \) and \( T \) are defined as shown in the Fig. 1.

Since the time period of the double-leg support phase is very small, the system dynamics can be viewed as a boundary condition of the single-leg support phase. Specifically,

\[ \ddot{\theta}(t) = \dot{\theta}(T) = \theta_M \]

\[ \dot{\theta}(0) = \dot{\theta}(T) = \theta(0) = \theta(T) \]

\[ \ddot{\theta}(0) = \ddot{\theta}(T) \]

where \( \theta_M \) denotes the maximal swing angle of \( \theta \) and \( T \) denotes the period of the single-leg support phase.

The equation of motion for a 5-link biped robot in the frontal plane is

\[ \dot{\theta} + B_\theta \dot{\theta} + C_\theta \theta + E_\theta \cos \theta = D_\theta U \]  

where \( U = [U_\theta, U_\theta]^T \), \( \cos \theta = [\cos \theta_1 \cos \theta_2 \cos \theta_3]^T \), and the boundary conditions are

\[ \delta(0) = \delta(T) = \theta(0) = \theta(T) = 0 \]  

and

\[ \dot{\theta}(0) = -\dot{\theta}(T) = \dot{\theta}(0) = \dot{\theta}(T) = 0 \]  

3. Trajectory Planning and Control in the Sagittal Plane

Desired trajectories of the supporting legs, the torso, and the free swing leg are, generally speaking, non-unique. In this section, we assume that the desired trajectory of torso, denoted by \( \theta(t) \), is zero, i.e., the biped robot is always walking with its torso maintained in an upright position.

3.1. Trajectory Planning for Level Walking

At first, we divide the period of the single-support phase into five distinct states, each state representing one-fifth of the period \( T \). The reference angles of \( \theta(t) \) (\( t = 1/4, 2/4, 3/4 \)) at time instant \( \frac{K}{4} \) can be obtained. Then a polynomial interpolation is performed. The coefficients of the polynomial are determined by minimizing the squared error between the smooth function \( \theta(t) \) and the desired value at \( t = \frac{K}{4} \). As a result, we obtain the following desired trajectories:

\[ \theta(t) = -1638.44t^6 + 4735.21t^4 - 8599.18t^2 + 1478.34t^3 \]

\[ -29.553t^3 \]

\[ -50.8026t^2 \]

\[ -347.3438t \]

\[ -50.8026t^2 \]

\[ -347.3438t \]

\[ -109.902t^2 \]

\[ -29.553t^3 \]

\[ -50.8026t^2 \]

\[ -347.3438t \]

\[ -50.8026t^2 \]

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\[ -347.3438t \]

\[ -50.8026t^2 \]

\[ -347.3438t \]

\[ -109.902t^2 \]

\[ -29.553t^3 \]
3.2. Trajectory Planning for Ascending a Staircase

Following the same procedure as stated above, we obtain the following desired trajectories:

$$e_1(t) = 730.33t^6 - 844.824t^4 + 549.758t^2 + 662.224t^2$$
$$-2.747t^2 + 3.776t - 45$$

$$e_2(t) = 991.883t^6 - 2514.276t^4 + 2335.177t^2 - 869.049t^2$$
$$-54.875t^2 + 40.761t + 45$$

$$e_3(t) = 8345.355t^6 - 18834.15t^4 + 13259.615t^2 - 2989.138t^2$$
$$-102.441t + 40.761t + 26$$

$$e_4(t) = -627.389t^6 + 1437.117t^4 - 1075.654t^2 + 271.623t^2$$
$$+9.317t^2 + 7.762t + 26$$

corresponding to the five distinct states for ascending as shown in Table 2.

<table>
<thead>
<tr>
<th>$1/T$</th>
<th>0.0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_1^*$</td>
<td>-45°</td>
<td>-40°</td>
<td>-20°</td>
<td>0°</td>
<td>-5°</td>
<td>-25°</td>
</tr>
<tr>
<td>$\delta_2^*$</td>
<td>45°</td>
<td>47°</td>
<td>35°</td>
<td>0°</td>
<td>-10°</td>
<td>-26°</td>
</tr>
<tr>
<td>$\delta_3^*$</td>
<td>26°</td>
<td>28°</td>
<td>31°</td>
<td>36°</td>
<td>40°</td>
<td>45°</td>
</tr>
<tr>
<td>$\delta_4^*$</td>
<td>26°</td>
<td>22°</td>
<td>18°</td>
<td>-20°</td>
<td>-45°</td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Five Distinct States for Ascending a Staircase

3.3. Trajectory Planning for Descending a Staircase

Similarly, corresponding to a particular set of states for a descending staircase, as given by Table 3, we derive the following desired trajectories:

$$e_1(t) = 1475.823t^6 - 3737.711t^4 + 2322.144t^2 - 1033.694t^2$$
$$+3.954t + 15.391 - 3$$

$$e_2(t) = 1830.069t^6 - 4367.897t^4 + 3302.766t^2 - 780.333t^2$$
$$-23.495t + 20.891t + 3$$

$$e_3(t) = 2671.527t^6 - 9981.277t^4 + 8425.765t^2 - 2444.89t^2$$
$$-4.014t + 20.891t + 15$$

$$e_4(t) = -491.906t^6 + 8833.574t^4 - 6029.088t^2 + 1100.074t^2$$
$$+54.968t^2 + 15.391 + 48$$

<table>
<thead>
<tr>
<th>$1/T$</th>
<th>0.0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_1^*$</td>
<td>-3°</td>
<td>-2°</td>
<td>-1°</td>
<td>0°</td>
<td>2°</td>
<td>-15°</td>
</tr>
<tr>
<td>$\delta_2^*$</td>
<td>-3°</td>
<td>-4°</td>
<td>-12°</td>
<td>-18°</td>
<td>-32°</td>
<td>-48°</td>
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<tr>
<td>$\delta_3^*$</td>
<td>48°</td>
<td>55°</td>
<td>53°</td>
<td>48°</td>
<td>30°</td>
<td>3°</td>
</tr>
<tr>
<td>$\delta_4^*$</td>
<td>15°</td>
<td>10°</td>
<td>-4°</td>
<td>-2°</td>
<td>-1°</td>
<td>3°</td>
</tr>
</tbody>
</table>

Table 3. Five Distinct States for Descending a Staircase

3.4. Control of a Biped Robot for Steady Walking

Let $X = [\theta \bar{\theta}]^T$.

Eq. (1) can be written as

$$\dot{X}(t) = F X + G T$$

where

$$F = \begin{bmatrix} -A^{-1} \text{C} \theta(t) & -A^{-1} \text{B} \bar{\theta}(t) \end{bmatrix}$$
$$G = \begin{bmatrix} 0 \\ A^{-1} \text{D} \end{bmatrix}$$

Define $Y = \theta \| 0 \| X$.

Also let the reference model be given by

$$\dot{X}_d(t) = F_d(t) X_d(t) + G_d(t) T_d(t)$$
$$Y_d(t) = \bar{\theta}(t)$$

Let the control input to the biped system be given by

$$T = T_d + T$$

where $T_d$ is the feedforward compensation signal of the control input and $T_d$ is the output of an adaptation mechanism, obtained by a modified VSS control algorithm which shall be described later. Let

$$T_d = -K \dot{X} + K_d T_d$$

Let $e = X_d - X$. From (7) and (10), we have

$$e(t) = X_d(t) + (F_d(t) - F_d(t)) X_d(t) + G_d(t) T_d(t)$$

Let

$$G_d = K_d T_d$$

Now the problem becomes that of the selection of an adaptive law for $T_d$ such that $\lim_{t \rightarrow \infty} \dot{e}(t) = 0$.

Define a time-varying sliding hypersurface

$$S(e(t),l) = L \cdot e(t)$$

where $L = \begin{bmatrix} l_1 & 0 & \cdots & 0 \\ 0 & l_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & l_n \end{bmatrix}$.

Let the global sliding condition be

$$\dot{S}(e(t),l) = -\varepsilon |e|$$

where $\varepsilon$ is a positive function of $S$. It had been shown [5] that, for any initial conditions, a switching control can be developed so that the error vector $e(t)$ will approach zero along the sliding hypersurface (14). In general, control laws which satisfy the sliding condition (15) are discontinuous across the surface $S(e(t))$, leading to control chattering. We can remedy this situation by smoothing out the control discontinuity in a thin boundary layer neighboring the switching surface. The boundary layer $B_i$ of $i^{th}$ subsystem can be defined as

$$B_i = \{ e_i = 0, S_i(e_i) > 0 \}$$

where $S_i(e_i) = S_i(e_i) - l_i E_i$

and $E_i$ is the desired tracking precision of the $i^{th}$ subsystem. Unlike [5] which uses a saturation function to smooth the control discontinuity in a thin boundary layer, we use a PI controller. Following a similar procedure as in [5], we derive the following VSS control law.

$$T_d = \left( K_d^+ + K_d^- \right) \varepsilon + \left( K_d^+ + K_d^- \right) X_d + \left( K_d^+ + K_d^- \right) T_d$$

$$\begin{bmatrix} K_d^+ + K_d^- \\ K_d^+ + K_d^- \end{bmatrix} X_d + \begin{bmatrix} K_d^+ + K_d^- \\ K_d^- + K_d^+ \end{bmatrix} T_d$$

where

$$Q = \left[ K_d \int_{0}^{1} e(t) \right]$$

500
and

\[ M(x) = 1, |x| \leq 1 \]
\[ = \frac{1}{K_1 + K_2 \int (y) \, dt}, \quad |x| > 1 \]

The complete block diagram of the Model Reference + VSS control algorithm is shown in Fig. 2.

4. Trajectory Planning and Control in the Frontal Plane

4.1. Planning the trajectory of the supporting leg and the swing leg

Let the desired trajectory of the supporting leg be

\[ \theta(t) = C_1 \sin h \frac{\pi t}{2} + C_2 \cos h \frac{\pi t}{2} \]

and let the desired trajectory of the swing leg be

\[ \theta(t) = D_1 \sin h \frac{\pi t}{2} + D_2 \cos h \frac{\pi t}{2} \]

where \( \theta(t) \) and \( \theta(t) \) satisfy the boundary conditions (5).

From eqns. (20) and (21), we have

\[ C_1 = \theta_s \left( -D_2 \sin h \frac{\pi}{2} \right) \]
\[ D_1 = -C_1 \sin h \frac{\pi}{2} - \theta_s \]

4.2. Control of the biped for steady walking in the frontal plane

Let the nonlinear feedback be

\[ \tilde{u}(t) = -D_1 \sin h \frac{\pi t}{2} + C_2 \cos h \frac{\pi t}{2} \]

where the feedback \( \theta_s \) and \( \theta_s \) represent the estimated values. Substituting (22) into (20) yields

\[ \tilde{\theta}(t) = C_1 (\theta_s - \theta) + \tilde{C}_1 \theta_s \]

Note that \( \tilde{\theta}(t) \) is generated via the Modified Computed Torque method to improve the robust property against modeling errors.

In the presence of modeling errors, we seek to add a compensating control signal \( \delta \tilde{S}_d(t) \) to the commanded acceleration as

\[ \tilde{S}_d(t) = S_d(t) + \delta \tilde{S}_d(t) \]

We can design \( \delta \tilde{S}_d(t) \) as

\[ \delta \tilde{S}_d(t) = S_d(t) - \delta \tilde{S}_d(t) \]

Unfortunately, (38) is not implementable because \( S_d(t) \) is unknown. In this research, \( S_d(t) \) is generated via the Modified Computed Torque method as shown in Fig. 3. The command acceleration signal \( S_d(t) \) is given by

\[ S_d(t) = S_d(t) + (\alpha^2 - 1) [S_d(t) - \tilde{S}(t)] \]

The compensating control signal \( \delta \tilde{S}_d(t) \) is

\[ \delta \tilde{S}_d(t) = (\alpha^2 - 1) [S_d(t) - \tilde{S}(t)] \]

For \( S_d(t) \) given by eq. (38), the closed-loop dynamics becomes

\[ \tilde{\theta}(t) = S_d(t) - \frac{1}{\alpha^2} W(t) \]

The error equation becomes

\[ \delta \tilde{\theta}(t) + q_1 \delta \tilde{\theta}(t) + q_2 \delta \tilde{\theta}(t) = \frac{1}{\alpha^2} W_d(t) \]

The control law now becomes

\[ U(t) = D \tilde{\theta} \]

Thus, applying the Modified \( \alpha \)-Computed Torque control algorithm, we observe the following:

1. As the feedforward gain \( \alpha^2 \) is increased, the nonlinear modeling error vector \( W(t) \) is decreased by a factor of \( \alpha^2 \).
2. As the feedforward gain \( \alpha^2 \) is increased, eq. (38) approaches the ideal linear system eq. (24).
3. The effect of modeling error \( W(t) \) on the system dynamics is only \( \frac{1}{\alpha^2} \) of that corresponding to the \( \alpha \)-Computed
Torque method. Consequently, the tracking performance of our modified approach is better than that of the α-Computed Torque method.

5. Simulation Results

Consider a 5-link biped robot as shown in Fig. 1. Assume 10% modeling error exists. Fig. 4.1 shows simulation results for level walking and ascending in the sagittal plane when the Model Following + VSS control algorithm is employed. From these results, we conclude that the Model Following + VSS method could accurately track the desired trajectory. Fig. 5.2 shows simulation results for trajectories tracking in the frontal plane when the Nonlinear Feedback and the Modified α-Computed Torque methods are employed. It turns out that the proposed controller schemes can effectively achieve trajectory tracking in the presence of modeling errors.

6. Conclusions

Specific control algorithms have been presented for level walking, ascending and descending a staircase by a 5-link biped robot. Simulation results show that, even in the presence of disturbances, gait stability can be achieved within two steps after the start of walking. In addition, the biped robot can walk with prespecified patterns. These algorithms are being implemented.

Acknowledgements

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References


Figure 1a. The torques, forces, angles are shown in Sagittal Plane

Figure 2. Model Reference + VSS Control for a 5-link Biped Robot in the Sagittal Plane
Figure 1.b Frontal Plane

Level walking and Ascending a staircase when the Model Following + VSS Control algorithm is employed.

Figure 3. Block Diagram of Biped Locomotion by Using Modified Computed Torque Technique
Fig. 4.2 Trajectories tracking results when the Nonlinear Feedback and Modified a-Computed Torque method is employed.

Fig. 4.3 Stick diagram for ascending staircases in the Sagittal plane.