ABSTRACT

In this paper we discuss the use of expert opinion in reliability assessment. We first present an approach for the analysis of life-length data that can be described by a Weibull distribution. Secondly, we address the problem of predicting the number of defects due to fatigue in a railroad track. In both cases, a Bayesian approach is taken and procedures are developed and implemented on a personal computer.

1. INTRODUCTION

The use of expert opinion or informed judgement is becoming prevalent in practical applications of reliability and risk analysis. The issue is particularly aggravated by the fact that modern day components and systems are designed for high reliability and so the availability of even a small amount of failure data is difficult in many situations. In current practice the treatment of expert opinion is undertaken in an ad hoc manner, usually with the experts sitting around the table and reaching a consensus through discussion. Although the results are expressed quantitatively, the approach does not take into account the experts’ biases and differing degrees of uncertainty regarding the numbers given. Also, the method of combining differences of opinions is not formalized. In this paper we consider the use of expert opinion, in a formal manner, in two application areas in reliability. In so doing, we outline principles, and discuss the development of interactive procedures for reliability assessment.

In Section 2, we address the problem of assessing the reliability of items whose life lengths are described by a Weibull distribution. The novel feature of our approach pertains to the incorporation of informed judgement or expert opinion into the analysis, and the provision for incorporating the analyst’s opinions on the expertise of the experts. The approach also uses failure and/or survival data on identical items and provides an interactive and time sequential merging of the modulated expert opinion with the data. We discuss the implementation of the above procedure on a personal computer. A computer program named IPRA was developed for use on a personal computer in an interactive and user friendly manner.

In Section 3, we discuss an interactive PC-based procedure for predicting the number of defects due to fatigue in railroad tracks. The procedure is based on a non-homogeneous Poisson process with a Weibull intensity function and enables incorporation of informed judgement into the analysis. Such informed judgement may be based on opinion of individuals, historical data and/or engineering models based on S-N curves. The original feature of our approach is the use of expert opinion in analysing point processes.

2. AN INTERACTIVE PC-BASED PROCEDURE FOR RELIABILITY ASSESSMENT

The PC-based procedure we present is the implementation of the approach for eliciting, modulating and codifying expert opinion, described in [5]. We will briefly describe the approach and illustrate its implementation on a personal computer using “IPRA”, a computer program developed at the George Washington University by the authors.
Let $X$ denote the time to failure of a fresh unit. The goal is to specify $R(x) = \Pr(X > x)$, the reliability function of the unit. Thus
\[
R(x) = \int \int \Pr(X > x | \alpha, \beta) \pi(\alpha, \beta | \theta) \, d\alpha \, d\beta, \tag{2.1}
\]
where $\Pr(X > x | \alpha, \beta) = \exp(-(x/\alpha)\beta)$, $x \geq 0$, and $\pi(\alpha, \beta | \theta)$ describes the uncertainty about $\alpha$ and $\beta$ given $\theta$, a vector of prior parameters.

A key feature of this procedure is the elicitation of expert opinion to specify $\pi(\alpha, \beta | \theta)$ which describes uncertainty about $\alpha$ and $\beta$, based on expert opinion (and assessment of the expertise of the expert) alone. In the absence of lifetime data on units identical to the unit in question, a plot of $R(x)$ versus $x$ provides the needed answer. A Bayesian credibility interval at uncertainty about $R(x)$ for a obtained. We use
\[
\Pr(R(x) \leq c) = \int \int \pi(\alpha, \beta | \theta) \, d\alpha \, d\beta, \tag{2.2}
\]
where $\{ (\alpha^*, \beta^*); R(x | \alpha^*, \beta^*) \leq c \}$ for $0 < c < 1$.

When lifetime data $d$ are obtained, the expert opinion is updated as
\[
\pi(\alpha, \beta | d, \theta) \propto L(d | \alpha, \beta) \pi(\alpha, \beta | \theta) \tag{2.3}
\]
where $L(d | \alpha, \beta)$, the likelihood of $\alpha$ and $\beta$, is
\[
\left\{ \prod_{i=1}^{n} \left( \frac{1}{\beta} \right)^{x_i} \beta^{-1} \exp\left( -x_i / \alpha \beta \right) \left\{ \prod_{j=1}^{k} \exp\left( -x_j / \alpha \beta \right) \right\}^{j} \right\}^{1 \over \sum_{i=1}^{n} \left( \frac{1}{\beta} \right)^{x_i} \beta^{-1}} \tag{2.4}
\]
\[d = (x_1, \ldots, x_n, s_1, \ldots, s_k), x_1, \ldots, x_n, s_1, \ldots, s_k \text{ denoting the n(k) observed failure (survival) times.} \]

By analogy with (2.1) and (2.2), we obtain
\[
R(x | d) = \int \int \Pr(X > x | \alpha, \beta) \pi(\alpha, \beta | d, \theta) \, d\alpha \, d\beta \tag{2.5}
\]
and,
\[
\Pr(R(x | d) \leq c) = \int \int \pi(\alpha, \beta | d, \theta) \, d\alpha \, d\beta, \tag{2.6}
\]
where $\alpha^*$ and $\beta^*$ are as defined in (2.2).

2.2 The Elicitation, Modulation and Codification of Expert Opinion

In [5], Singpurwalla considers the elicitation of expert opinion on the median life and the aging characteristics of the item in question, and develops a procedure to construct the joint prior distribution of $\alpha$ and $\beta$, $\pi(\alpha, \beta | \theta)$. The following steps are taken:

1. Construct $\pi(M | \theta)$ through the elicitation and modulation of expert opinion on the median life $M$.

2. Induce $\pi(\alpha | \beta, \theta)$ from $\pi(M | \theta)$.

3. Construct $\pi(\beta | \theta)$ through the elicitation and modulation of expert opinion on the aging characteristics of the item.

4. Obtain $\pi(\alpha, \beta | \theta)$ as $\pi(\alpha | \beta, \theta) \pi(\beta | \theta)$.

2.3 The Elicitation and Modulation of Expert Opinion on $M$

An expert $\mathcal{E}$ is assumed to conceptualize uncertainty about $M$ via some distribution with mean $m$ and standard deviation $s$. In $\mathcal{E}$'s view, 50% of similar units observed until failure will, on average, fail by $m$; $s$ is a measure of $\mathcal{E}$'s conviction in specifying $m$. Suppose that $\mathcal{E}$ gives the numbers $m$ and $s$ to an analyst $\mathcal{A}$. Following Lindley [31], three quantities $a$, $b$ and $\gamma$ are introduced as part of the modulation process to reflect $\mathcal{A}$'s view of $\mathcal{E}$'s biases and precision in declaring $m$ and $s$. The interactive program "IPRA" asks some general questions about $M$, and elicits the quantities $m$ and $s$. Next, the program asks $\mathcal{A}$ questions of a qualitative and quantitative nature, and uses the answers to pin down $a$, $b$, and $\gamma$. Following a theorem in [5], $\mathcal{A}$ has modulated $\mathcal{E}$'s inputs for $M$ to reflect $\mathcal{A}$'s judgement of $\mathcal{E}$'s expertise, and has arrived at
\[
\pi(M | \mu, \sigma) \propto \frac{1}{\sqrt{2\pi} \sigma} \exp\left( -\frac{1}{2} \left( \frac{M - \mu}{\sigma} \right)^2 \right), \quad 0 < M < \infty \tag{2.7}
\]

2.4 The Codification of Expert Opinion on $\beta$

The shape parameter $\beta$ of the Weibull distribution characterizes aging, since for $\beta > ( < ) 1$ its failure rate increases (decreases) in $x$; it is a constant for $\beta = 1$. Hence if expert opinion...
suggests that the item degrades (improves) with age, β is likely to be greater (less) than 1; if neither, β is likely to be in the vicinity of 1. A gamma distribution with density

\[ \pi(\beta|\lambda, p) = \lambda^p \beta^{p-1} e^{-\lambda \beta} / \Gamma(p) \]  

is chosen to model the uncertainty about β. The parameters λ and p are specified by A and are based on B's view and conviction of the aging characteristics of the item. A convenient way for A to specify λ and p is to use the fact that the mean, and variance of (2.8) are given by \( p/\lambda \) and \( p/\lambda^2 \). The program "IPRA" asks B some general questions about β, and elicits responses that help specify λ and p.

2.5 The Distribution Induced by Expert Opinion on the Weibull Parameters

If we make the simplifying assumption that β is independent of M, then \( \pi(M, \beta|\mu, \sigma, \lambda, p) \), the joint density of β and M, is given by the product of (2.7) and (2.8). As a result, the quantity α has a distribution induced from (2.7) as

\[ \pi(\alpha|\beta, \mu, \sigma) \propto \frac{1}{\sqrt{2\pi \sigma}} \exp \left( -\frac{1}{2} \left( \frac{\alpha - \beta \mu}{\sigma} \right)^2 \right) \]  

\[ 0 \leq \alpha < \infty \]  

(2.9)

Multiplying (2.8) by (2.9), we obtain \( \pi(\alpha, \beta|\theta) \), the joint prior density of α and β.

During the execution of "IPRA", upon the completion of the elicitation and modulation procedure, a computational subprogram is loaded in which the user is asked to input failure and/or survival data. The computational program provides the user with prior and posterior reliability functions for a specified time interval and with the prior and posterior distributions of reliability for specified mission times. The program provides tabular as well as graphical displays of the results. (See Figures 1 - 4 and [1] for more details). In the light of new lifetime data, "IPRA" allows the user to upgrade the results from the previous analysis and therefore perform the analysis of the reliability of an item in a time sequential manner.
3. A PC-BASED SYSTEM FOR PREDICTING NUMBER OF DEFECTS IN RAILROAD TRACKS

In what follows, we describe a procedure which uses expert opinion for prediction of number of defects due to fatigue in railroad tracks. The procedure is implemented on PC and being used in the railroad industry.

3.1 Preliminaries

In [4] a nonhomogeneous Poisson process (NHPP) with a Weibull intensity function is proposed as a model for predicting the number of defects in railroad tracks. This model is also known as Weibull process.

Let $N(t)$ denote the number of defects due to fatigue in a rail of length $L$ which has been subjected to a cumulative load of $t$ million gross tons (mgt's). The probability of observing $k$ failures is given by

$$ P(N(t)=k | L, \alpha, \beta) = e^{-LM(t)} \frac{[LM(t)]^k}{k!}, $$

$$ k=0, 1, \ldots, \text{ (3.1)} $$

where $M(t)=\frac{t}{\alpha^\beta}$ is the expected number of defects, $\alpha$ and $\beta$ are unknown quantities and $L$ is specified. Let the prior density $\pi(\alpha, \beta | \theta)$ describe uncertainty about $\alpha$ and $\beta$ given $\theta$. Prior to any data, probability statements about number of defects in the mgt interval $(t, t+s)$, for some $s>0$, is made via

$$ P(N(t,t+s)=k | L) = \int \int P(N(t,t+s)=k | L, \alpha, \beta, \theta) \pi(\alpha, \beta | \theta) \, d\alpha \, d\beta. \text{ (3.2)} $$

Given data, $d$, on number of defects in $h$ disjoint mgt intervals, uncertainty about $\alpha$ and $\beta$ is updated via (2.3) and $P(N(t,t+s)=k | d,J)$ is obtained from (3.2) by replacing the prior with the joint posterior density $\pi(\alpha, \beta | \theta, d)$. The likelihood $L(d | \alpha, \beta)$ is given by

$$ \prod_{j=1}^{h} e^{-\hat{M}_j} \frac{[\hat{M}_j]^n_j}{n_j!}, $$

where $n_j$ is the observed number of defectives in the mgt interval $(t_{1j}, t_{2j})$ and

$$ \hat{M}_j = \frac{t_{2j}}{\alpha_j^\beta} - \frac{t_{1j}}{\alpha_j^\beta}, \quad j=1, 2, \ldots, h. \text{ (3.3)} $$

As in Section 2, the important issue is the specification of the prior distribution $\pi(\alpha, \beta | \theta)$ based on expert opinion and this will be discussed next.

3.2 Elicitation and Modulation of Expert Opinion

Elicitation of prior distributions on the parameters in a Weibull process is discussed in [6]. Let $M_i=\frac{t_i}{\alpha_i^\beta}$ for $i=1, 2$. Suppose that an expert $\mathcal{E}$ conceptualizes his/her uncertainty about $M_i$ via some distribution and declares an analyst $\mathcal{A}$ two numbers $m_i$ and $s_i$, which are measures of location and scale respectively.

The analyst views the $m_i$'s and $s_i$'s as realizations of two random variables $\mathcal{M}_i$ and $\mathcal{S}_i$, $i=1, 2$, and models the distributions of $\mathcal{M}_i$ and $\mathcal{S}_i$ based on his/her opinion of the expertise of $\mathcal{E}$ and also his/her perceived correlation between $\mathcal{M}_1$ and $\mathcal{M}_2$ and $\mathcal{S}_1$ and $\mathcal{S}_2$.

Under the assumptions introduced in [6], it can be shown that the joint distribution of $M_1$ and $M_2$ given the expert input $m_i$, $s_i$, $m_2$, and $s_2$ is given by
\[ x(M_i, M_2 | m_1, m_2, \alpha, \beta) \propto \]
\[
\exp \left\{ -\frac{1}{2} \left( \frac{m_2 - (a_1 + b_1 M_i)}{\gamma_1 \delta_1^2} \right) \right\} \\
\left\{ 1 - N \left( \frac{m_2 - (a_1 + b_1 M_i)}{\gamma_1 \delta_1} \right) \right\}^{1/2} \gamma_1 \delta_1 \]
\[
\exp \left\{ -\frac{1}{2} \left( \frac{m_1 - (a_2 + b_2 M_2)}{\gamma_2 \delta_2^2} \right) \right\} \\
\left\{ 1 - N \left( \frac{m_1 - (a_2 + b_2 M_2)}{\gamma_2 \delta_2} \right) \right\}^{1/2} \gamma_2 \delta_2 \]
\[
\exp \left\{ -\frac{1}{2} \left( \frac{M_2 - M_1}{\gamma_3} \right)^2 \right\} \\
\left\{ 1 - N \left( \frac{M_2 - M_1}{\gamma_3} \right) \right\}^{1/2} \gamma_3 \delta_3 \]
\[
\frac{1}{M_2 - M_1} \exp \left\{ -\frac{1}{2} \frac{M_2 - M_1}{M_2 - M_1} \right\}, \]
\[
0 < M_1 < \infty, \quad M_1 < M_2 < \infty \tag{3.4}
\]

where \( N(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp \left\{ -\frac{t^2}{2} \right\} dt \).

As before, the quantities \( a_i, b_i, \) and \( \gamma_i \) reflect \( A \)'s view of \( S \)'s biases and precision in declaring \( M_i \) and \( s_i \), for \( i = 1, 2 \). We note that once (3.4) is obtained, the joint distribution of \( \alpha \) and \( \beta \) can be induced using the relationship \( M_i = \{1/\alpha\}^\beta, \) \( i = 1, 2 \).

A computer package was developed to implement the above approach (see [2]). It elicits and modulates expert opinion about the number of defects in two management intervals selected by the expert. The modulated expert input is then used to assess the distribution of number of defectives in other load intervals. If data on number of defects is available from a particular railroad site, then the computer conducts a posterior analysis and updates the distribution of the number of defects on the basis of data. The computer provides graphical displays of the prior and posterior distributions of the number of defects for selected load intervals. It also provides graphical feedback to the expert via displays of prior and posterior distributions of \( M_i \) and \( M_2 \).

REFERENCES


FOOTNOTE

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