On Reachability in Acyclic Well-Structured Workflow Nets

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Abstract—Workflow net is a particular class of Petri net for verifying the correctness of workflows. Reachability problem for workflow nets plays an important role in the verification, but the problem is known to be intractable. Limiting our analysis to a subclass of workflow net, called acyclic well-structured workflow net, we gave a necessary and sufficient condition on the reachability problem. Based on this condition, we also constructed a polynomial time procedure for solving the problem.

I. INTRODUCTION

Workflow management systems play a central role among today’s information systems. A workflow management system keeps workflows registered in advance, and assigns the right workers the right work at the right time in accordance with the workflows. An erroneous workflow may lead to extra work, legal problem, angry customers, and managerial problems. As a mathematical tool to verify the correctness of workflows, van der Aalst [1] has proposed an underlying Petri net model, called workflow net, and a correctness criterion, called soundness. Nowadays, workflow net is widely used as one of standard ways for verifying the correctness of workflows [2].

There are two aspects in a workflow: definition and instances. In terms of workflow nets, a workflow definition is represented as the structure of the net, while workflow instances are represented as tokens. The correctness of the workflow definition can be verified by using soundness verification technique. It is known [1] that most actual workflows can be modeled as free-choice workflow nets; and soundness for free-choice workflow net can be verified in polynomial time. In contrast, the correctness of the workflow instances has been considered only in a few researches. The workflow definition may be changed to adapt market growth, legal reform, and so on, even if there exist workflow instances. Those instances may be unable to be stopped because the life-cycle is long. They have to migrate to the new definition. Erroneous migration would cause inconsistency, e.g. loss of the workflow instances, in the new workflow. The correctness of workflow instances can be verified by using reachability verification technique. Unfortunately, it is known that reachability problem for live and safe free-choice Petri nets is NP-complete. Ohta et al. [3] have shown that reachability problem for a subclass of acyclic workflow nets can be solved in polynomial time.

In this paper, limiting our analysis to a subclass of workflow net, called acyclic well-structured workflow net, we give a necessary and sufficient condition on the reachability problem. Note that acyclic well-structured workflow net is a proper superclass of Ohta et al’s workflow net class. Based on the condition, we also construct a polynomial time procedure for solving the problem.

II. WORKFLOW NET AND ITS PROPERTIES

A. Petri Net

A Petri net is a three tuple \(N=(P, T, A)\), where \(P, T,\) and \(A \in (P \times T) \cup (T \times P)\) are finite sets of places, transitions, and arcs, respectively. Let \(x\) be a place or transition. \(x\) and \(\cdot x\) denote \([y, x] \in A\) and \([y, x] \in A\), respectively. They are also extended to a subset \(X\) of \(P\) or \(T\): \(\cdot X = \bigcup_{x \in X} \cdot x\), \(X = \bigcup_{x \in X} x\). A marking \(N\) is a mapping \(M: P \rightarrow \mathbb{N}\). We represent it as a bag over \(P\), which is denoted by \(M=P^\bullet\). A transition \(t\) is said to be firable in a marking \(M\) if \(M(t)\geq 1\). This is denoted by \(M(N, t)\).

A marking \(M\) is said to be reachable from a marking \(M_0\) if there exists a transition sequence \(\sigma = t_1t_2\cdots t_n\) such that \(M_0[N, t_1]M_1[N, t_2]M_2\cdots[N, t_n]M_n\). This is denoted by \(M_0(N, \sigma)M_n\) or simply \(M_0(N, \ast)M_n\). \(\ast\) is called a firing sequence which transforms \(M_0\) to \(M_n\). The set of all possible markings reachable from \(M_0\) in \(N, M_0\) is denoted by \(R(N, M_0)\). The set of all possible firing sequences from \(M_0\) in \(N, M_0\) is denoted by \(L(N, M_0)\).

There are well-known two subclasses in Petri nets: free-choice (FC for short) and extended free-choice (EFC for short). \(N\) is said to be FC if \(\forall p_1, p_2 \in P: p_1\cdot p_2\neq \emptyset =\u0001\rightarrow \left|p_1\cdot p_2\right| = 1\). \(N\) is said to be EFC if \(\forall p_1, p_2 \in P: p_1\cdot p_2\neq \emptyset =\u0001\rightarrow \left|p_1\cdot p_2\right| = 1\).

Let \(A_0\) be the incidence matrix of \(N\). A \(|T|\)-dimensional vector \(J\) is called a \(T\)-invariant if \(A_0J=\u0000\). A \(|P|\)-dimensional vector \(J\) is called a \(P\)-invariant if \(J A_0=\u0000\). Two markings \(M\)
WF-net \( N \) has a single source place \( N \), i.e. \( (N, M) \) has a positive \( T \)-invariant; (iii) \( \text{Rank}(\mathbf{A}_N) = |C_N| - 1 \), where \( \mathbf{A}_N \) is the incidence matrix of \( N \), \( C_N \) is the set of clusters \(^1\) in \( N \); and (iv) \( [p_1] \) marks every proper siphon in \( N \). It enables us to solve the soundness problem of EFC WF-nets in polynomial time \([7, 8]\).

The notion of soundness has been variously generalized. One of them is \( k \)-soundness \([9]\), \( k \)-soundness focuses on Condition (i) of soundness, and is parameterized with variable \( k \) which indicates the initial number of tokens in \( p_1 \). Formally speaking, \( N \) is said to be \( k \)-sound if for some \( k \in \mathbb{N} \) all \( T \)-invariants and \( \mathbf{A}_N \)-invariants are \( k \)-sound for any \( k \in \mathbb{N} \).

**B. Workflow Net**

Workflow net (WF-net for short) is a particular class of Petri net, which is used to model and analyze workflows.

**Definition 1:** (WF-net \([1]\) ) A Petri net \( N = (P, T, A) \) is a WF-net iff (i) \( N \) has a single source place \( p_1 \) (\( \bullet p_1 = \emptyset \) and \( \forall \rho \in (P \setminus \{p_1\}) : \bullet \rho = \emptyset \)) and a single sink place \( p_0 \) (\( \rho p_0 = \emptyset \) and \( \forall \rho \in (P \setminus \{p_0\}) : \rho p \neq \emptyset \)); and (ii) every place or transition is on a path from \( p_1 \) to \( p_0 \).

There is a particular subclass in WF-nets: \textit{well-structured} (WS) (for short). A structural characterization of good workflows is that two paths initiated by a transition (a place) should not be joined by a place (a transition). WS is derived from this structural characterization. To give the formal definition of WS, we introduce some notations. Let \( N = (P, T, A) \) be a WF-net. The Petri net obtained by connecting \( p_0 \) with \( p_1 \) via an additional transition \( \tau^* \) is called the \textit{short-circuited net} of \( N \), denoted by \( \overline{N} = (P, T \cup \{\tau^*\}, A \cup \{(p_0, \tau^*), (\tau^*, p_1)\}) \), see Fig. 1.

A path (a circuit) is called to be elementary if no nodes appear more than once in the path (the circuit). The paths (the circuits) appearing in this paper are always elementary. Let \( \rho = x_1 x_2 \cdots x_n \) (\( n \geq 2 \)) be a \textit{handle} \([4, 5]\) of \( \mathbf{c} \) if \( \rho \) shares exactly two nodes, \( x_1 \) and \( x_n \), with \( c \). A handle from a transition to a place is called a \textit{TP-handle}. A handle from a place to a transition is called a \textit{PT-handle}. A WF-net \( N \) is said to be WS if there are neither TP-handles nor PT-handles of any circuit in \( \overline{N} \). We can decide in polynomial time whether a given WF-net is WS by applying a modified version of the max-flow min-cut technique \([6]\).

**C. Workflow Net’s Properties**

Soundness is the notion of logical correctness for WF-nets. A WF-net \( N = (P, T, A) \) is said to be sound iff (i) \( \forall M \in R(N, [p_1]): \exists M' \in R(N, M): M' \prec [p_0] \), and (ii) \( \forall M \in R(N, [p_1]): M \geq [p_0] \Rightarrow M = [p_0] \); and (iii) There is no dead transition in \((N, [p_1])\), i.e. \( \forall \tau \in T : \exists \rho \in L(N, [p_1]) : \rho \tau \). The soundness problem of WF-nets is to decide, given a WF-net \( N \), whether \( N \) is sound. \( N \) is sound iff \( (\overline{N}, [p_1]) \) is live and bounded. This implies that the soundness problem of WF-nets is decidable but is intractable. On the other hand, the soundness problem of EFC WF-nets can be solved by using the following necessary and sufficient condition: An EFC WF-net \( N \) is sound iff (i) \( \overline{N} \) has a positive \( P \)-invariant; (ii) \( \overline{N} \) has a positive \( T \)-invariant; (iii) \( \text{Rank}(\mathbf{A}_N) = |C_N| - 1 \), where \( \mathbf{A}_N \) is the incidence matrix of \( N \), \( C_N \) is the set of clusters\(^1\) in \( N \); and (iv) \( [p_1] \) marks every proper siphon in \( \overline{N} \). It enables us to solve the soundness problem of EFC WF-nets in polynomial time \([7, 8]\).

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**III. Reachability for Acyclic Well-Structured Workflow Nets**

**A. Reachability Problem**

The reachability problem of Petri nets is to decide, given a Petri net \( (N, M_0) \) and a marking \( M \) of \( N \), whether \( M \) is reachable from \( M_0 \). Esparza et al. \([10]\) have shown that the reachability problem of live and safe EFC Petri nets is NP-complete. In contrast, Desel et al. \([8]\) have shown that the reachability problem of live, bounded and reversible EFC Petri nets is solvable in polynomial time. This criterion is called the Reachability Theorem (Theorem 9.6 of Ref. \([8]\)). Unfortunately, it is still an open question as to whether the Reachability Theorem holds for live, bounded and non-reversible EFC Petri nets.

We tackle the reachability problem of WF-nets. A WF-net specifies the life-cycle of a workflow instance. The initial state of a workflow instance is represented as the initial marking \((p_1)\). We generalize the initial marking as \([p_k] \) for some \( k \in \mathbb{N} \). This generalization enables us to model two or more workflow instances. The problem to be tackled in this paper is defined as follows.

**Definition 2:** (reachability problem in \((N, [p_k])\) )

Instance: WF-net \((N, [p_k])\), Marking \( M \) of \( N \)

Question: Is \( M \) reachable from \([p_k]\) ?

For any WF-net \( N \), \( p_j \) is a source place, so \((N, [p_k])\) is non-reversible for any \( k \in \mathbb{N} \). Therefore the Reachability Theorem does not hold for WF-nets.

**B. Acyclic Well-Structured Workflow Net**

We focus our analysis on acyclic WS WF-net. This is because acyclic WS WF-net has enough power to model many actual workflows. The Workflow Management Coalition (WfMC for short), an international standardization organization on workflows, has identified four routing constructions:
sequential, parallel, selective, and iterative. Acyclic WS WF-net can model workflows composed of the former three routing constructions. Moreover, acyclic WS WF-net has abundant analysis techniques like an polynomial time algorithm of verifying soundness.

WS WF-net is incomparable with FC WF-net, but acyclic WS WF-net is a subclass of acyclic FC WF-net, as stated in the following property (See Fig. 2).

Property 1: Any acyclic WS WF-net is acyclic FC.

Proof: Assume that acyclic WS WF-net is not FC but EFC. N is illustrated in Fig. 3. N includes the key structure of EFC shown in the dotted box in the figure. Since N is an acyclic WS WF-net, there exist a path $p_1 = t_1 \cdots p_1$ and another path $p_2 = t_2 \cdots p_2$ such that $p_1$ shares the subpath from $p_1$ to $x$, i.e. $p_1 \cdots x$, with $p_2$. Similarly, there exist a path $p_3 = l_1 \cdots y_1 \cdots p_3$ and another path $p_4 = l_2 \cdots y_2 \cdots p_4$ such that $p_3$ shares the subpath from $y$ to $p_3$, i.e. $y \cdots p_3$, with $p_4$. If $x$ is a place, there exists a PT-handle from $x$ to $t_1$ ($l_2$) in N. If $y$ is a transition, there exists a PT-handle from $p_1$ to $y$ in N. If $x$ is a transition and $y$ is a place, there exists a TP-handle from $x$ to $y$ in N. These mean that N is not WS. This is inconsistent with the assumption. Thus N is FC. Q.E.D.

The relations among subclasses of WF-net are illustrated as Venn diagrams shown in Fig. 2.

Property 2: For any acyclic WS WF-net N, N is well-formed, i.e. $\overline{N}$ has a live and bounded marking.

Proof: As stated in Theorem 4.2 of Ref. [5], an FC Petri net N is well-formed iff (i) N is strongly connected; (ii) No circuits in N have TP-handles; and (iii) If N has a PT-handle, then each PT-handle has a TP-bridge from the handle to the circuit.

As stated in Property 1, N is an FC WF-net. This means that $\overline{N}$ is a strongly connected FC Petri net. From the definition of WS, there are no circuits in N that have TP-handles and/or PT-handles. Thus $\overline{N}$ is well-formed. Q.E.D.

Any acyclic WS WF-net is sound, as stated in the following property.

Property 3: Any acyclic WS WF-net N is sound.

Proof: As stated in Theorem 6.17 of Ref. [8], an EFC Petri net $(\overline{N}, \overline{M}_0)$ is live and bounded iff (i) N is well-formed; and (ii) $\overline{M}_0$ marks every proper siphon.

As stated in Property 2, $\overline{N}$ is well-formed. Let $\overline{\mathcal{P}}$ denote the set of places of $\overline{N}$. $\overline{\mathcal{P}}\{p_1\}$ includes no siphon. Therefore $[p_1]$ marks every proper siphon. Thus $(\overline{N}, [p_1])$ is live and bounded. This means that N is sound. Q.E.D.

Property 4: For any acyclic WS WF-net N, $(\overline{N}, [p_1])$ is live and bounded for any $k \in \mathbb{N}$.

Proof: As stated in Property 2, $\overline{N}$ is well-formed. The set of places of $\overline{N}$ is denoted by $\overline{\mathcal{P}}$. $\overline{\mathcal{P}}\{p_1\}$ includes no trap. Therefore $[p_1]$ marks every proper trap. From Theorem 8.11 of Ref. [8], $[p_1]$ is a home marking of $(\overline{N}, [p_1])$. This means that $(\overline{N}, [p_1])$ is reversible. Q.E.D.

Any acyclic WS WF-net is generalized sound, as stated in the following property.

Property 5: For any acyclic WS WF-net N, $(\overline{N}, [p_1])$ is reversible for any $k \in \mathbb{N}$.

Proof: Let $k$ be any natural number. $[p_2^k]$ is a home marking of $(\overline{N}, [p_0^k])$. We can prove it in similar ways of Properties 4 and 5. Since $[p_0^k]$, $[p_1^k]$, $[p_2^k]$, $[p_2^k]$ is reachable from $[p_1^k]$ and any marking reachable from $[p_1^k]$. The reachability is preserved even if transition $t^k$ is removed from N. This is because $t^k$ does not exist on a path from $p_1$ to $p_0$. Thus N is $k$-sound.

$k$ is any natural number, so N is generalized sound. Q.E.D.

C. Necessary and Sufficient Condition

We give a necessary and sufficient condition on the reachability problem of acyclic WS WF-nets.

Theorem 1: Let $(\overline{N}, [p_1^k])$ be an acyclic WS WF-net for some $k \in \mathbb{N}$. A marking $M$ is reachable from $[p_1^k]$ in N, i.e. $[p_1^k]\{N, \ast\}M$, iff (i) let $\varphi_x$ be the incidence matrix of $\overline{N}$, the equation $\varphi_x \cdot X = \overline{\mathcal{M}}\{p_1^k\}$ has some rational-valued solution for $X$; and (ii) $M$ marks every proper trap of $\overline{N}$.

Proof: This proof is based on the Reachability Theorem, i.e. let $(\overline{N}, M_0)$ be a live, bounded and reversible EFC Petri net, a marking M is reachable from $M_0$ in N, i.e. $M_0\{N, \ast\}M$, iff M and $M_0$ agree on all P-invariants and M marks every proper trap of N.

Property 1 implies that $\overline{N}$ is an FC Petri net. Property 4 implies that $(\overline{N}, [p_1])$ is live and bounded. Property 5 implies that $(\overline{N}, [p_1])$ is reversible. Therefore $(\overline{N}, [p_1])$ is a live, bounded and reversible FC Petri net. It is known from
Theorem 2.34 of Ref. [8] that $M$ and $[p_i^k]$ agree on all P-invariants iff the equation $A \neg \mathbf{X} = M - [p_i^k]$ has some rational-valued solution for $\mathbf{X}$.

By combining the Reachability Theorem and these results, we obtain that $M$ is reachable from $[p_i^k]$ in $\overline{N}$, i.e. $[p_i^k][\overline{N}, \star]M$, iff the equation has some rational-valued solution; and $M$ marks every proper trap of $\overline{N}$.

For reference, we prove only the “if” part. If $M$ and $[p_i^k]$ agree on all P-invariants then there is a marking $L$ such that $L \in R(\overline{N}, M) \cap R(\overline{N}, [p_i^k])$ (Theorem 9.5 of Ref. [8]). Since $\overline{N}$ is a well-formed EFC Petri net, $\overline{N}$ is structurally bounded. We have $M[\overline{N}, \star)L[\overline{N}, \star][p_i^k]$. Since $\overline{N}, [p_i^k]$ is live, $(\overline{N}, M)$ is also live. Therefore $(\overline{N}, M)$ is live and bounded. Since $M$ marks every proper trap of $\overline{N}$, $M$ is a home marking of $(\overline{N}, M)$. Therefore $L(\overline{N}, \star)M$ holds. Thus $[p_i^k][\overline{N}, \star)L$, we have $[p_i^k][\overline{N}, \star]M$. The reachability is preserved even if transition $t'$ is removed from $\overline{N}$. This is because $t'$ does not exist on a path from $p_1$ to $p_0$. Thus $M$ is reachable from $[p_i^k]$, i.e. $[p_i^k][\overline{N}, \star]M$.

**Q.E.D.**

On the basis of this theorem, we construct a procedure of solving the reachability problem of acyclic WS WF-nets.

Input: Acyclic WS WF-net $(N, [p_i^k])$ for some $k \in \mathbb{N}$, Marking $M$ of $N$  

Output: Is $M$ reachable from $[p_i^k]$ in $N$?

1° $\Rightarrow$ Check Condition (i) of Theorem 1  
If the equation $A \neg \mathbf{X} = M - [p_i^k]$ has no rational-valued solution for $\mathbf{X}$, output no and stop.

2° $\Rightarrow$ Check Condition (ii) of Theorem 1  
Let $\overline{R}$ be the set of places of $\overline{N}$ which are not marked by $M$. Compute the maximal trap $\overline{Q}$ in $\overline{R}$ by using the following sub algorithm [8]:

1) $\overline{Q} \leftarrow \overline{R}$  
2) While there exists $p \in \overline{Q}$ and $t \in p^\bullet$ such that $t \notin \overline{Q}$,
   $\overline{Q} \leftarrow \overline{Q} \cup \{p\}$  
3) Output $\overline{Q}$ and stop.

If $\overline{Q} \neq \emptyset$, output no and stop.

3° Output yes and stop.

This procedure runs in polynomial time, so we can immediately obtain the following theorem.

**Theorem 2:** The following problem can be solved in polynomial time: Given $(N, [p_i^k])$ be an acyclic WS WF-net for some $k \in \mathbb{N}$, to decide if a given marking $M$ is reachable from $[p_i^k]$.

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**IV. Example**

A workflow definition may be changed to adapt market growth, legal reform, and so on, even if there exist workflow instances. Such a phenomenon is called *dynamic workflow change*. In terms of WF-nets, dynamic workflow change is to replace a WF-net $(N_{old}, M_{old})$ by another WF-net $(N_{new}, M_{new})$. If $N_{old}$ has $k$ workflow instances, all the instances have to be migrated to $N_{new}$ appropriately. If not, “dynamic bugs” would occur. For example, some instance is lost or becomes impossible to terminate normally. Formally speaking, if $M_{old}$ is reachable from $[(p_i^{old,k})]_{N_{old}}$, i.e. $[(p_i^{old,k})][N_{old}, \star]M_{old}$, then $M_{new}$ must be reachable from $[(p_i^{new,k})]_{N_{new}}$, i.e. $[(p_i^{new,k})][N_{new}, \star]M_{new}$. This is why
reachability verification technique is very important to verify the correctness of workflow instances.

Let us consider an example of dynamic workflow change. Figure 4 (a) shows a WF-net $N_1$, which represents a workflow of a web shop. Assume that the initial marking of $N_1$ is $\{(p^1_1)^4\}$. This means that there are four workflow instances. Let $M_1$ be $\{p^1_1, p^2_1, p^3_1, p^4_1\}$. Since $\{(p^1_1)^4\} \\subseteq \{N_1, t^1_1 t^2_1 t^3_1 t^4_1 t^5_1 t^6_1 t^7_1 t^8_1 t^9_1 t^10_1\} M_1$, $M_1$ is reachable from $\{(p^1_1)^4\}$. By using this fact, we explain how our procedure runs. In Step 1, we check Condition (i) of Theorem 1. The incidence matrix of $\overline{N}_1$ is given as follows:

$$
\begin{align*}
A_{\overline{N}_1} = & \begin{bmatrix}
-t^1_1 -t^2_1 -t^3_1 & t^4_1 t^5_1 t^6_1 t^7_1 & t^1_2 t^2_2 & t^3_2 t^4_2 t^5_2 t^6_2 t^7_2 t^8_2 t^9_2 t^{10}_2 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0
\end{bmatrix} \\
\text{Let } \Delta M_1 = M_1 - \{(p^1_1)^4\}. \text{ The equation } A_{\overline{N}_1} X = \Delta M_1 = (-4, 1, 0, 1, 0, 1, 0, 1, 0, 0) \text{ has a rational-valued solution: }
\end{align*}
$$

$$
X = \begin{bmatrix}
4 & 3 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

In Step 2, we check Condition (ii) of Theorem 1. Let $\overline{R}_1$ be the set of places of $\overline{N}_1$ which are not marked by $M_1$, $\overline{R}_1$ is $\{p^1_1, p^2_1, p^3_1, p^4_1\}$. We compute the maximal trap $\overline{Q}_1$ in $\overline{R}_1$. $\overline{Q}_1$ is initialized as $\overline{R}_1$. Let us first choose $p^1_1 \in \overline{R}_1$. $p^1_1 \overline{Q}_1 = \{t^1_1\}$, but $t^1_1 \notin \overline{Q}_1$. Therefore, $p^1_1 \in \overline{Q}_1$. $\overline{R}_1$, $\overline{p}^1_1$, $\overline{p}^2_1$, and $\overline{p}^3_1$ are respectively removed from $\overline{Q}_1$ because of a similar reason. Therefore we have $\overline{Q}_1 = \emptyset$. In step 3, the procedure outputs yes. This means that $M_1$ is reachable from $\{(p^1_1)^4\}$ in $N_1$.

Next, this web shop decides to change the workflow definition so as to process two sequential activities, Billing and Shipping, concurrently. The new workflow can be modeled as a WF-net $N_2$, which is shown in Fig. 4 (b). Note that $N_2$ is not in Ohta et al.’s subclass of acyclic WS WF-net. All the workflow instances in $N_2$ have to be migrated to $N_1$. Let us consider the following migration rule: Migrate each token to $p^1_1$, $p^3_1$, $p^5_1$, and $p^7_1$ are respectively migrated to $\{p^1_1, p^3_1, p^5_1, p^7_1\}$.

1. The incidence matrix of $\overline{N}_2$ is given as follows:

$$
\begin{align*}
A_{\overline{N}_2} = & \begin{bmatrix}
-t^2_1 -t^2_2 -t^2_3 & t^2_4 t^2_5 t^2_6 t^2_7 t^2_8 t^2_9 t^2_{10} \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0
\end{bmatrix} \\
\text{Let } \Delta M_2 = M_2 - \{(p^2_1)^4\}. \text{ The equation } A_{\overline{N}_2} X = \Delta M_2 = (-4, 1, 0, 1, 0, 1, 0, 0, 1, 0) \text{ has no rational-valued solution. In fact, } \text{Rank}(A_{\overline{N}_2}) = 8 \neq \text{Rank}(A_{\overline{N}_1}) = 9. \text{ Therefore, the procedure outputs no. This means that } M_2 \text{ is not reachable from } \{(p^2_1)^4\} \text{ in } N_2. \text{ Thus we can say that the migrated workflow instances are correct.}
\end{align*}
$$

Some researchers [11], [12], [13] have proposed migration rules which avoid dynamic bugs. Sadiq et al.’s rule regards the activities performed in the old workflow as ones done in the new workflow. For the detail of the rule, refer to Ref. [14]. Using this rule, $M_1$ of $N_1$ is migrated to $M_3 = \{p^1_2, p^2_2, p^3_2, p^4_2\}$ of $N_2$ (See Fig. 5). Let us check whether $M_3$ is correct. In Step 1, we check Condition (i) of Theorem 1. Let $\Delta M_3 = M_3 - \{(p^2_1)^4\}$. The equation $A_{\overline{N}_2} X = \Delta M_3 = (-4, 1, 0, 1, 0, 1, 0, 1, 0, 1) \text{ has a rational-valued solution: }

$$
X = \begin{bmatrix}
4 & 3 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

In Step 2, we check Condition (ii) of Theorem 1. Let $\overline{R}_3$ be the set of places of $\overline{N}_2$ which are not marked by $M_3$. $\overline{R}_3$ is $\{p^1_2, p^3_2, p^5_2, p^7_2\}$. The maximal trap in $\overline{R}_3$ is empty. In step 3, the procedure outputs yes. This means that $M_3$ is reachable from $\{(p^2_1)^4\}$ in $N_2$. Thus we can say that the workflow instances migrated according to Sadiq et al.’s rule are correct.

V. Conclusion

In this paper, we have tackled the reachability problem of acyclic WS WF-net. We have given a necessary and sufficient condition on the reachability problem. Based on the condition, we have also constructed a polynomial time procedure for solving the problem. Finally we have illustrated the procedure with an example of dynamic workflow change.

As future works, we plan to give a necessary and sufficient condition for a class larger than acyclic WS WF-net, e.g. sound acyclic EFC WF-net or sound cyclic WS WF-net. We would also like to tackle polynomial time solvability of subproblems of the reachability problem, such as the legal firing sequence problem and its related problems, which were successfully
investigated by Watanabe [15], by restricting the domain to WF-net.

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References