Performability, Reliability, and Survivability of Communication Networks: System of Methods and Models for Evaluation

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Abstract

This paper presents a system of methods and models for evaluating the performability, reliability, and survivability of a communication network. All proposed models and methods are shown to be interdependent by way of indexes and parameters of the basic network properties. Indexes for these network properties are found for each message-exchanging terminal pair and for the entire network. Steady-state availability, MTBF (mean time between failures), and MTTR (mean time to repair) are computed for reliability analysis of the network with limited repair. Survivability indexes are found for the network exposed to multiple external (adverse) influences that cause a gradual structural degradation of the network. Models are proposed for the performability evaluation of unreliable virtual circuit network X.25/X.75 with routing adapted to network element failures. Some numerical results of comparative analysis of network performability for adaptive and static routings are presented.

1. Introduction

Digital communication networks are representative of a growing class of distributed real-time systems that present challenging new evaluation problems [1].

Quantitative measurement of a communication network is usually done through performance or reliability analysis. The reliability analysis of a network is generally based on Boolean algebra and probability theory. Traditionally, reliability problems of networks are reduced to terminal-pair, tree, and multiterminal connectivities that depend on the network topology and the probability of failures in nodes and lines. The development of this approach [12, 13], through construction of a composite reliability model based on Boolean algebra, probability, and queueing theory, allows the computation of reliability indexes for a network with limited failed-element repair, which is typical in maintenance practice. A Boolean model for a network with dependent failures has been developed [6].

On the other hand, network survivability is considered as subject to (structural) reconfiguration in the case of an increase in the number of adverse actions upon the network. Methods have been proposed by the author* in [14] for computing survivability indexes—in particular, the probability of communication success—as a function of the action number for some models of multiple external influences with different strategies and destruction ranges of actions. The multiple external influences are treated here as a discrete flow of consecutive actions.

In the past, the performance and reliability of a communication network were usually analyzed separately. But lately some results (see, for example, [1]-[5]) have allowed us to consider a composite performance and reliability measure. The advent of fault-tolerant multiple-processor systems that are subject to reconfiguration and graceful degradation in the case of element failure makes it worthwhile to examine a composite performance and reliability measure known as performability (introduced by Meyer in [2]). Meyer formally defined the performability of a system as the probability that the system will stay above a certain accomplishment level over a utilization interval called the "mission time." It has been proposed to extend this approach, which is based on stochastic process theory, to the communication network [1]. But problems arise because networks are more complicated structurally and functionally than fault-tolerant computer systems.

On the other hand, it has been proposed [4, 5] to use the mean normalized point-to-point *s-t* (or overall) capacity of the unreliable network as a composite performance-reliability measure. This approach, based on Boolean algebra and probability theory, is convenient for capacity-reliability-connectivity analysis relying only on the structural properties of an unreliable network. But it does not take into consideration the performance measures of the network protocols.

The performability indexes of the two latter approaches do not characterize the quality of the user's service by the network. In this paper we use typical grade-of-service (GOS) measures of a packet-switching network for performability evaluation of an unreliable or vulnerable network.

* In papers (12-14) the author's name is given in a different transliteration from Russian into English.

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We propose a bottom-up approach, whereby the lower-level models and methods of hierarchy (see Figure 1) are used for the network element reliability and performability evaluation, the middle-level ones for capacity-survivability-reliability-connectivity evaluation of the network, and the top-level ones for the performability network evaluation. This approach is based on a combination of the methods and models of Boolean algebra and probability and queueing theories. We present a hierarchical system of methods and models interrelated through performance, reliability, and survivability measures. This system includes the models and methods [12, 13, 14] elaborated by the author for reliability and survivability network analysis.

In this paper we focus primarily on the performability and reliability evaluation of virtual circuit (VC) packet-switching network X.25/X.75. The models for the reliability analysis take into consideration both structural connectivity and performance measures of the network protocols, for example, the mean interruption time caused by switching from a failed VC to a working VC during the data transmission phase. The top-level models for network performability evaluation compute GOS indexes as a function of measures that are in their turn found with the help of detailed models and characterize the procedure of switching from a failed VC to a working VC by an adaptive routing algorithm, the mechanism of frame retransmission in a noisy and unreliable channel, the failure/error detection delay, network connectivity, routing vectors, and so on. The top-level models are developments of the model in [9] and take into consideration the interdependence of VCs, the more general distributions of the frame transmission time and other random variables, and the unreliability of packet-switching nodes.

In section 2, we give the basic notations used in this paper. In section 3, we describe the overall system of interrelated methods and models, elaborated to compute the performability, reliability, and survivability indexes of an unreliable and vulnerable communication network. The indexes for the quality of the user's service by the network are also defined. In section 4, we propose a system of hierarchical models and methods for performability evaluation of the X.25/X.75 network. This section presents descriptions and models of routing adapted to failures, static routing, the procedure of VC switching, flow of data link failures with a given failure detection time, frame transmission procedure in a noisy and unreliable channel, and the total flow of interruptions in the user's data transmission that are caused by VC switching and physical failures. In section 5, we demonstrate the use of our models for numerical performability analysis of a bridge-network with unreliable nodes and links and two message-exchanging node pairs. Section 6 contains conclusions based on the results obtained.

2. Notations

φ_i — the logical function of success for VC_i. If VC_i is working, $\varphi_i = 1$; otherwise $\varphi_i = 0$;

R — the probability function of communication success for a node pair;

τ_{fTAL} — the mean time between failures (MTBF) of the terminal access line (TAL);

τ_{rTAL} — the mean time to repair (MTTR) of the TAL;

A_{TAL} — the steady-state availability of the TAL (the reliability indexes for the TAL of source-user U_s and for the TAL of destination-user U_t can differ);

τ_{fN} — the mean time between interruptions of packet transmission from user U_s caused by element failure in a VC and the subsequent switching from the failed VC to a working VC;

τ_{rN} — the mean time of packet transmission interruption;

A_N — the effective steady-state availability of the network (without the TAL) from the standpoint of user U_s (the fraction of packet transmission phase time within which the network allows packet transmission);

$c_{\Sigma TAL}$ — the overall failure and transmission interruption rate for the TAL of user U_s;

$\tau_{r\Sigma TAL}$ — the overall mean time to restore the TAL of user U_s;

$A_{\Sigma TAL}$ — the overall steady-state availability of the TAL of user U_s;

$\tau_{fDL_j}, \tau_{rDL_j}$ — the MTBF and the MTTR of data link (DL) j;

η_j — the time to detect the failure of DL_j by the retransmission procedure of the data link layer protocol;

ξ_j — the maximum number of damaged frame retransmissions in DL_j by the data link layer protocol procedure (usually ξ_j is the same for all DLs of the network); $\eta_j = \xi_j \tau_j$, where τ_j is the frame transmission time without an error in DL_j;

$\tau_{ftj}(\eta_j), \tau_{ffj}(\eta_j)$ — mean times between detected "true" and "false" failures in DL_j, respectively, provided the time for failure detection is equal to η. A "true" failure is detected when the random time of a DL down state is longer than η. A "false" failure is detected when the

DL_j is working but the random time of frame transmission is longer than η_j;

c_{fj} the total stream rate of all detected failures ("true" and "false") in DL_j;

$\tau_{rrj}(\eta_j)$ the mean down state time of DL_j on condition that failure duration is shorter than η_j;

A_{rDL_j} the availability of DL_j calculated from the parameters of the undetected failure stream in the interval between adjacent detected failures;

$P\{.\}$ the probability of the event described in the brackets.

3. System of Methods and Models

A simplified diagram of interrelations between methods and models, elaborated by the author in [11] for computing the indexes of the basic communication network properties, is presented in Figure 1.

The methods and models for computing the steady-state reliability indexes of the network (see blocks 3-7 and 10 in Figure 1) take into consideration (1) the structure and the functions of the centralized maintenance and repair system, in particular, period τ_j of the maintenance of each periodically maintained network element j, (2) the limited repair of failed network elements due to the interdependence of elements, and (3) the limited capacity of the network, the decrease of which below a given value (due to element failures) causes network failure.

The following steady-state reliability indexes of the network for each pair of end nodes s and t are computed: the availability of network A_ε to perform with given capacity $\varepsilon = C_{st}$; network $MTBF_\varepsilon$ between the decreases of the capacity below ε; network $MTTR_\varepsilon$ of restoring the capacity to the value not lower than ε.

Some methods for reliability analysis are described in [13], and [12] describes a particular case when the working state of the network is defined as terminal connectivity. These methods are also used for computing equilibrium probability that $P\{\varphi_n \bigwedge_{j=1}^{n-1} \bar{\varphi}_j = 1\}$ of path n will be in the working state on the condition that n-1 previous paths have failed. The computation is based on the method of conjunction absorption [11] for the transformation of the communication success logical function into a mixed form of the probability function.

The methods and models for computing the structural survivability indexes of the communication network (blocks 5-9 in Figure 1) are presented in [14]. Some models of multiple external (adverse) influences with different strategies and destruction ranges of actions are proposed. In the case of a passive strategy, the model of external influence is a flow of independent actions. With an active strategy, the model of external influence is a flow of dependent actions: destroyed elements are excluded from the range of subsequent actions. Local (point) and distributed actions are examined. Every local action can disable no more than one element, and every distributed action can disable several elements simultaneously. Survivability indexes are computed, in particular, the probability of communication success $V(m)$ as function of action number m, the mean number of actions causing the network down state (failure), and the network survivability margin against actions upon the network.

The fraction of the network throughput preserved under external (adverse) influence W is used for functional survivability evaluation of the network on the condition that an arrival traffic control system is working. Expressions for upper and lower bounds of functional survivability and performability indexes W_u, W_l, q_u, q_l, T_u, Γ_u, Γ_l (block 2 in Figure 1) are presented in [11].

GOS measures q, T, Γ are used for VC packet-switching network performability evaluation. In dealing with GOS measures, we take into consideration the value of the information in a packet received by the user from the network [9]. Its value depends on the network packet delay. Let $Q(t)$ be the probability that the packet will be delivered through the network in time τ less than t. If t has a negative exponential distribution with mean $1/\nu$, then the probability of timely packet delivery $q(\nu) = \int_0^\infty e^{-\nu t} dQ(t)$. On the other hand, $q(\nu)$ is the Laplace-Stieltjes (LS) transfer of distribution $Q(t)$ with $Re\, \nu > 0$.

The use of $q(\nu)$ instead of $Q(t)$ has some advantages:

- If $t = 1/\nu$, then $q(\nu)$ gives a pessimistic GOS evaluation in comparison with a fairly general class of $Q(t)$ distributions (e.g., when $Q(t)$ is a Γ-distribution) [11];
- It is easier to obtain a mathematical formula for $q(\nu)$;
- From $q(\nu)$ the initial moments of the packet delay can be obtained by differentiation. In particular, the first initial moment is mean packet network delay T.

The other GOS measures used here are T and network throughput Γ, found as the flow rate of the packets with an acceptable network delay ($\leq t$): $\Gamma = \gamma\, q(\nu)$. In addition, the relative network information damage due to the finite network capacity, failures, transmission errors, and/or external (adverse) influence is computed as the fraction of arrival packets untimely delivered to destination $I = (\gamma - \Gamma)/\gamma = 1 - q(\nu)$.

These GOS measures are used for individual terminal and node pairs or for the entire network with the help of the Kleinrock convolution formula [8].

Some models for the performability evaluation of an unreliable VC network X.25/X.75 with static routing and routing adapted to failures (block 1 in Figure 1) are presented in section 4.

4. Model for Performability Evaluation of VC Network X.25/X.75

4.1 Routing algorithms

We shall now examine VC packet-switching network X.25/X.75. An example of the network part used by a pair of packet-switching nodes PSN_s and PSN_t is shown in Figure 2. There are M VCs between PSN_s and PSN_t according to the network routing. Users U_s and U_t are attached to PSN_s and PSN_t, respectively, by packet-mode terminals and terminal access lines (TALs). Switching nodes are connected to each other by trunk lines (TLs). User U_s generates a packet flow with arrival rate γ_u, and all users attached to PSN_s produce a total packet flow of rate γ. The rate of the total packet flow through the jth network element (PSN or TL) produced by all user pairs in the network is denoted by λ_j. The GOS measures are found from Figure 2 by formulas for the independence assumption by L. Kleinrock [8]:

$$q(v) = q_s(v)\, q_N(v)\, q_t(v) \qquad (1)$$

$$T = T_s + T_N + T_t \qquad (2)$$

Let us consider the static routing and the routing adapted to failures. Routing vector $\overline{\sigma} = (\sigma_i)$ for each message-exchanging node pair (PSN_s, PSN_t) is calculated by an algorithm in [11] that takes into consideration the traffic requirement matrix for all node pairs (PSN_s ,PSN_t), the reliability of nodes and lines, the throughputs of nodes and the capacities of lines, and the set of possible routes.

With static routing, the virtual packet-switched connection is established only once—at the beginning of the session between the users in a pair—and cannot be changed, whatever the technical state of the network elements that compose the path between users. Before forwarding a call-request packet, PSN_s chooses an outgoing line from the ones available in a random way. The final probability that the ith path will be chosen to establish a virtual connection between the users of the pair attached to PSN_s and PSN_t is equal to σ_i.

Routing adapted to failures takes into consideration the current technical state of the network elements during the establishment of a connection and the data transmission phase. Multiple switchings from a failed VC to the next working VC (in the routing table) may occur during a session. The process of VC switching causes interruptions in data transmission between users during the session. Now σ_i is the probability that VC π_i will be chosen first as the VC from U_s to U_t. This VC π_i will be used only if it is working; otherwise the next working VC in the routing table of PSN_s will be formed during the connection phase.

Let an ordered list of VCs between PSN_s and PSN_t be given. (A criterion for ordering VCs can be the composite capacity-reliability index [11].) The probability of choosing the ith VC to establish a virtual connection from PSN_s to PSN_t is

$$P_i = \sum_{j=1}^{M} \sigma_j P\left\{\varphi_{j_n} \bigwedge_{m=1}^{n-1} \overline{\varphi}_{j_m} = 1\right\}, \quad j_1 = j, j_n = i \qquad (3)$$

where j_n is equal to number i of VC π_i that occupies the nth position after the VC with number j_1 in the ordered list.

4.2 The basic model: $M/G/\vec{\vec{1}}$

The $M/G/\vec{\vec{1}}$ queue with an unreliable server, first-come-first-served service discipline, and an unlimited buffer for waiting customers is used below as (a) the model for the performance of a DL with an unreliable and noisy channel; and (b) the model for packet transmission through the TAL of user U_s in accordance with the DL layer and the network layer protocols.

Let c be the rate of the server failure stream that is approximated by the Poisson process; λ the rate of the Poisson arrival stream of customers (packets); $B(t)$ the service time distribution; $H(t)$ the distribution of the service time, including the interruption time to restore the failed server; $W(t)$ the distribution of the customer waiting time for the service to begin; $D(t)$ the restoration time distribution of the failed server; and $\beta(v), h(v), \omega(v), \delta(v)$ the Laplace-Stieltjes transfers of distribution functions $B(t)$, $H(t)$, $W(t)$, $D(t)$ respectively. Then the performance indexes of the $M/G/\vec{\vec{1}}$ queue, in accordance with [15], are as follows:

$$\begin{cases} q(\upsilon) = \omega(\upsilon)h(\upsilon), \qquad h(\upsilon) = \beta(\upsilon + c - c\delta(\upsilon)) \\ \omega(\upsilon) = \dfrac{\left(1 - \lambda h^{(1)}\right)\left(\upsilon + c(1 - \delta(\upsilon))\right)A}{\upsilon - \lambda(1 - h(\upsilon))} \end{cases} \qquad (4)$$

Here A is the steady-state availability of the server.

4.3 Procedure of VC switching

The procedure of switching from a failed VC to a working VC by adapted routing during the data transmission phase is shown in Figure 3. One of the network elements in the current VC fails at moment *b*. During interval η the procedure of the DL layer detects the failure of the element after the packet (frame) has been transmitted ξ times. The interruption of the data transmission is started at moment *a* because terminal U_s cannot receive acknowledgment ACK for all packets transmitted within the sending window after moment *a*. After interval η, at moment *c* control packet "RESET VC" is sent to PSN_s and PSN_t. At moment *d*, PSN_s receives this packet. During the interval between moments *d* and *e*, PSN_s establishes a new VC. Then at moment *e*, PSN_s issues an instruction for terminal U_s to repeat unconfirmed packets. At moment *f*, the pause in data transmission is over. (The models described below can be used for the optimization of frame retransmission number ξ).

From the diagram in Figure 3 we can derive a formula for the interruption interval when network element *j* has failed. If the lengths of the control and confirmation packets are the same, this interval is

$$T_p^{(j)} = T_d + T_{cc}^{(j)} + \eta^{(j)} + T_{rc} \quad , \tag{5}$$

where T_d - is the mean time for the data packet delivery from U_s to U_t; T_{rc} - is the mean return delay (two-way delivery) of the control packet in reestablishing a new VC between PSN_s and PSN_t; $T_{cc}^{(j)}$ - the total mean delay for the confirmation from U_t to failed element *j* and for the control packet from element *j* to U_s.

Mean interruption time $T_p^{(j)}$ is calculated by data and control packet delays in the M/D/1 queue network on the condition that there are no failures detected in the VC (see Figure 2).

4.4 Model for packet transmission in the TAL of user U_s

For user U_s, interruption interval $T_p^{(j)}$ is seen as the restoration time of the TAL. Overall interruption flow rate $c_{\Sigma TAL}$ for user U_s is the sum of the TAL failures (rate $1/\tau_{fTAL}$) and the interruptions caused by VC switching (rate $1/\tau_{fN}$). If transmission interruptions caused by VC switching and physical failures in the TAL of user U_s are independent of each other, then

$$\begin{cases} c_{\Sigma TAL} = \dfrac{1}{\tau_{fTAL}} + \dfrac{1}{\tau_{fN}}, \quad A_{\Sigma TAL} = A_{TAL} \cdot A_N \\ \tau_{r\Sigma TAL} = \dfrac{1 - A_{\Sigma TAL}}{A_{\Sigma TAL} \cdot c_{\Sigma TAL}}, \quad A_N = \tau_{fN} / (\tau_{fN} + \tau_{rN}) \\ A_{TAL} = \tau_{fTAL} / (\tau_{fTAL} + \tau_{rTAL}) \cdot \end{cases} \tag{6}$$

The VC failure rate, on condition that PSN_s and PSN_t are connected, is given by

$$\begin{cases} \dfrac{1}{\tau_{fN}} = c_N = \sum_{i=1}^{M} P_i \, c_{\pi_i} / R \\ R = \sum_{i=1}^{M} P_i \ , \quad c_{\pi_i} = \sum_{i=1}^{M} c_{fj} \end{cases} \tag{7}$$

The mean time of packet transmission interruption caused by element failures in the VC and switchings from the failed VC to a new VC is

$$\tau_{rN} = \frac{\tau_{fN}}{R} [\sum_{i=1}^{M} P_i \sum_{j \in \pi_i} T_{pi}^{(j)} c_{fj} + 1 - R], \tag{8}$$

where $T_{pi}^{(j)}$ is the mean time of interruption in data transmission through VC π_i when network element *j* has failed. The value of $T_{pi}^{(j)}$ is computed by formula (5).

The $M/G/\vec{1}$ can be employed for modeling packet transmission in the TAL of user U_s. Formula (4) is to be used for computing the probability $q_s(v)$ that the packet delay in the TAL will be no longer than the exponentially distributed time with mean $1/v$ after replacement of $q(v)$ by $q_s(v)$, c by $c_{\Sigma TAL}$, A by $A_{\Sigma TAL}$, and of λ by γ_u. Distribution $B(t)$ with LS transfer $\beta(v)$ and the mean packet delay in the TAL of user U_s $T_s = q_s^{(1)}$ are found by formulas (17) below, with the above-mentioned replacement of symbols and the substitution of $\tau_{r\Sigma TAL}$ for $d^{(1)}$.

4.5 Model for data link failures

The failures of a VC network element and of the TAL are detected by the data link protocol. If a noise burst on the line destroys a frame with a packet, then the second-layer software in the source machine retransmits the frame once or more.

Let us examine a DL of the network. The number of the DL will not be shown in the formulas below to simplify the notation. Let η be the time used by the frame retransmission procedure of the DL layer protocol to detect a DL failure. We will distinguish between "true" and "false" failures. A "true" failure is detected when the random time of a DL down state t_d is longer than η, ($t_d > \eta$). A "false" failure is detected when the DL is working but the random time of frame transmission t_{fr} is longer than η, ($t_{fr} > \eta$).

The mean number of DL failures during the time of observation $\tau >> \tau_{fDL} + \tau_{rDL}$ is $n = \tau / (\tau_{fDL} + \tau_{rDL})$. The mean number of detected "true" failures is $n_t = n\, P\{t_d > \eta\} = n\,(1 - D_{DL}(\eta))$, where $D_{DL}(t)$ is the repair (down state) time distribution for the DL. After each detected failure the DL is idle during $\tau_{rDL} = d^{(1)}$. The mean time between detected "true" failures for repair time distribution $D_{DL}(t)$ is therefore calculated by this formula:

$$\tau_{ft}(\eta) = \tau / n_t - \tau_{rDL} = \frac{\tau_{fDL} + \tau_{rDL}}{1 - D_{DL}(\eta)} - \tau_{rDL} \cdot \qquad (9)$$

Let λ be the rate of frame arrivals at the DL. The mean number of the frames transmitted in the DL during the time of observation $\tau >> 1/\lambda$ is $n = \lambda\tau$. The mean number of detected "false" failures is $n_f = n\, P\{t_{fr} > \eta\}$. The time distribution of detected "false" failures coincides with the total time distribution of frame transmission and retransmissions $B(t)$. Let $b^{(1)}$, $b^{(2)}$ be the first- and second-order initial moments for distribution $B(t)$. Then after a "false" failure is detected, the mean remaining time of self-restoration is equal to $b^{(2)}/2b^{(1)}$ in accordance with the restoration theory. Hence the mean time between detected "false" failures is

$$\tau_{ff}(\eta) = \frac{\tau}{n_f} - \frac{b^{(2)}}{2b^{(1)}} = \frac{1}{\lambda(1 - B(\eta))} - \frac{b^{(2)}}{2b^{(1)}} \cdot \qquad (10)$$

According to (9) and (10), for $0 < \eta < \infty$ we get these bounds:

$$\tau_{fDL}(\eta) = \tau_{ft}(0) < \tau_{ft}(\eta) < \tau_{ft}(\infty) = \infty \;, \qquad (11)$$

$$\frac{1}{\lambda} - \frac{b^{(2)}}{2b^{(1)}} = \tau_{ff}(0) < \tau_{ff}(\eta) < \tau_{ff}(\infty) = \infty \cdot \qquad (12)$$

Thus, for $\eta = 0$ all "true" failures are detected and each frame transmission is detected as a failure, too. But for $\eta = \infty$ the DL failures are not detected. Hence, computing η is an important task for choosing the parameters of the DL layer protocol.

The total stream rate of all detected failures ("true" and "false") in the DL is

$$c_f = 1/\tau_{ft} + 1/\tau_{ff} \,. \qquad (13)$$

Formula (13) is used in (7) and (8) to calculate failure rates c_{fj} for all DLs in the network.

Formula (11) shows that, for the given $\eta > 0$ only the failures with durations $t_d > \eta$ are detected. This means that there is a remaining stream of undetected failures with $t_d \leq \eta$ in the DL. In the interval between detected failures, the mean time between undetected failures coincides with the MTBF of the DL τ_{fDL}. The mean duration of an undetected failure is shorter than τ_{rDL} and depends on η. Then, for DL down state time distribution $D_{DL}(t)$, on the condition that failure duration $t_d \leq \eta$, the mean down state time is found by the following expression:

$$\tau_{rr}(\eta) = \int_0^{\eta} x \frac{dD_{DL}(x)}{D_{DL}(\eta)} = \eta - \frac{1}{D_{DL}(\eta)} \int_0^{\eta} D_{DL}(x)dx \cdot \qquad (14)$$

The availability of the DL in the interval between detected failures is

$$A_{rDL} = \tau_{fDL} / (\tau_{fDL} + \tau_{rr}) \,. \qquad (15)$$

Thus, indexes A_{rDL}, τ_{fDL}, and τ_{rr} show the so-called virtual reliability of the DL exposed to undetected failure actions. They are used to calculate q_N and T_N in (1) and (2).

4.6 Model for frame transmission procedure in an unreliable, noisy channel

The total time of the frame transmission and retransmissions in a noisy channel is a random variable with distribution function $B(t)$ and mean $b^{(1)}$. Relying on experimental results for a frame with a constant length [11], we approximate the total transmission time distribution by

$$B(t) = \begin{cases} 0 & ,\ t < \tau \\ 1 - \alpha e^{-\frac{\alpha}{\Delta}(t-\tau)} & ,\ t \geq \tau \end{cases}$$
$$\Delta = b^{(1)} - \tau \;, \qquad (16)$$

where τ is the frame transmission time without errors in the DL and α is the probability that the total frame transmission time will be longer than τ. This probability coincides with the frame error probability, which can be computed for a case when errors occur in individual bits or for a case when errors tend to come in bunches [7].

A well-known model (e.g., [10]) can be used for computing frame delay $b^{(1)}$ in a noisy channel. The $M/G/\vec{\vec{1}}$ queue described by formula (4) is used for modeling the performance of a DL with an unreliable and noisy channel. For distribution *B(t)* in (16) we get

$$\begin{cases} \beta(\upsilon)=e^{-\upsilon t}\dfrac{\alpha+\upsilon\Delta(1-\alpha)}{\alpha+\upsilon\Delta},\ b^{(2)}=\tau(\tau+\Delta/\alpha)+2\Delta^2/\alpha \\ n^{(1)}=b^{(1)}/A,\ h^{(2)}=b^{(1)}cd^{(2)}+\dfrac{\tau^2\alpha-2\Delta^2}{A^2\alpha} \\ \omega^{(1)}=\dfrac{A}{2}\left(cd^{(2)}+\dfrac{\lambda h^{(2)}}{A(1-\lambda h^{(1)})}\right) \\ q^{(1)}=h^{(1)}+\omega^{(1)},\ A=\dfrac{d^{(1)}}{1/c+d^{(1)}}, \end{cases} \qquad (17)$$

where $b^{(i)}$, $h^{(i)}$, $\omega^{(i)}$, and $d^{(i)}$ are the *i*th-order initial moments of the random variables with distribution functions *B(t), H(t), W(t), and D(t)*.

It was shown above what replacements of symbols would have to be done in (17) when these formulas are used to evaluate the performance of the TAL of user U_s. The reliability of the DL in the interval between detected failures is characterized by indexes τ_{fDL}, τ_{rr}, and A_{rDL}. Then formulas (4) and (17) are used for performance modeling of the network DL with an unreliable and noisy line, with A_{rDL} substituted for A, τ_{rr}, - for $d^{(1)}$, and $1/\tau_{fDL}$ - for c.

4.7 Performance index calculation for network with adaptive routing

The probability of the timely packet delivery to its destination is found with formula (1). Index $q_s(\nu)$ is calculated with formulas (4) and (17) in which $\lambda = \gamma_u$, $A = A_{\Sigma TAL}$, $d^{(1)} = \tau_{r\Sigma TAL}$, $c = c_{\Sigma TAL}$. To calculate $A_{\Sigma TAL}$, $\tau_{r\Sigma TAL}$, $c_{\Sigma TAL}$, formulas (3), (5), (6), - (10), and (13) are used.

Index $q_N(\nu)$ in (1) is calculated as

$$q_N=\frac{1}{R}\sum_{i=1}^{M}P_i\,q_{Ni}=\frac{1}{R}\sum_{i=1}^{M}P_i\prod_{j\in\pi_i}q_j(\lambda_j)\ ,$$

where q_{Ni} is the probability of the timely packet delivery in VC_i between PSN_s and PSN_t and q_j (λ_j) - is the probability of the timely packet processing in PSN_j or transmission in DL_j for packet arrival rate λ_j.

Calculating q_j for the case when the jth network element is a DL was presented above. If the jth element is a PSN, the $M/D/\vec{\vec{1}}$ queue with an unreliable server is used for calculating q_j.

The mean time of the packet delivery from U_s to U_t is to be obtained by differentiating q:

$$T=q^{(1)}=q_s^{(1)}(\gamma_u)+\frac{1}{R}\sum_{i=1}^{M}P_i\sum_{j\in\pi_i}q_j^{(1)}(\lambda_j)+q_t^{(1)}(\gamma_u)$$

4.8 Performance index calculation for network with static routing

The final expressions for the case under consideration are as follows:

$$q_N=q_sq_t\sum_{i=1}^{M}\sigma_i\prod_{j\in\pi_i}q_j,\ T=q_s^{(1)}(\gamma_u)+\sum_{i=1}^{M}\sigma_i\sum_{j\in\pi_i}q_j^{(1)}(\lambda_j)+q_t^{(1)}(\gamma_u)$$

Here q_s, $q_s^{(1)}$ and q_t, $q_t^{(1)}$ are calculated by (4) and (17) with packet arrival rate $\lambda = \gamma_u$ for the steady-state reliability indexes of the TALs of users U_s and U_t, $A = A_{TAL}$, $d^{(1)} = \tau_{rTAL}$, and $c = 1/\tau_{fTAL}$.

Measures q_j, $q_j^{(1)}$ are found from the $M/D/\vec{\vec{1}}$ queue if the *j*th element is a PSN and from (4) and (17) if the jth element is a DL. In all these cases, the steady-state reliability indexes of network elements are used in the formulas.

5. Example of Numerical Analysis

A network with unreliable nodes and lines (TLs) is shown in Figure 2. Let us assume that there are no errors in the DLs and no failures in the TALs, and that all random variables have exponential distributions (including distribution $B(t)$). All nodes are identical, as are all TLs. The input data are as follows: frame (packet) length, 1024 bit; confirmation length, 128 bit; $\nu = 0.1$ 1/s; the maximum number of frame retransmissions $\xi = 10$; MTBF $\tau_f = 1000$ h and MTTR $\tau_r = 0.5$ h for each node; PSN switching throughput, 10 Mbit/s; MTBF $\tau_f = 100$ h, MTTR $\tau_r = 1$ h, and capacity, 2 Mbit/s for each TL; capacity of TAL, 64 Kbit/s, the message-exchanging node pairs are (1,4) and (2,3); there are 10 users attached to each PSN (1-4 in Figure 2); and the packet arrival rate from one user $\gamma_u = 10$ pack./s. The variable parameters whose values differ from the ones above are shown in the figures below.

Figures 4 and 5 display timely packet delivery probability q and network packet delay T, both of which depend on the maximum number of frame retransmissions ξ in data links for the node pair (1,4).

In Figure 4 the variable parameter is the packet arrival rate γ for node pair (1,4). Values Δq and ΔT show the differences between the performance indexes of the network with adaptive routing and static routing. The dependences for adaptive routing $q_{ar}(\xi)$ and $T_{ar}(\xi)$ have extrema in the vicinity of $\xi = 10$. Similar network performance indexes q_{sr} and T_{sr} for static routing are shown with broken horizontal lines for different values of γ. Indexes q_{sr} and T_{sr} do not depend on ξ.

If ξ is increased and the time for failure detection η is much longer than the MTTR of lines and nodes, then the network reacts to failures much too slowly, and therefore q_{ar} and T_{ar} deteriorate to the level of q_{sr} and T_{sr}, respectively. The property of adaptive routing, $q_{ar}(\xi) \Rightarrow q_{sr}$, $T_{ar}(\xi) \Rightarrow T_{sr}$ when $\xi \Rightarrow \infty$, is confirmed by formulas (6) through (15).

Figures 4 and 5 show that q_{ar} and T_{ar} deteriorate dramatically when $\xi \Rightarrow 0$. This happens because when $\xi \Rightarrow 0$ every frame transmission is detected as a failure by the DL layer protocol, thus causing multiple VC switching and the increase of the packet network delay. This is confirmed by formulas (10) and (12).

In Figure 4 the broken bold line shows mean packet network delay T_{ar} on the condition that transmission paths in the network are independent (have no common network elements). This result was obtained in [9]. It can be seen that the assumption of the path independence gives an error whose value depends on ξ and is substantial (Δ in Figure 4) in a large range of $\xi \in [10^2, 10^4]$. Numerical experiments on a computer show that the smaller the network element reliability, the larger the value of the relative error due to this independence assumption.

In Figure 5 the variable parameter is the MTTR of network elements τ_{fj}, which is the same for every network element. The behavior of the curves is similar to that in Figure 4.

6. Conclusion

The main contribution of this paper is an approach to network performability, reliability, and survivability evaluation based on a hierarchical system of interrelated methods and models using Boolean algebra, probability, and queueing theories. Both network reliability indexes and GOS measures, applied for evaluating the performability of VC network X.25/X.75 with adaptive and static routings, exposed to failure and transmission error action, are computed by models that take into account the network structure, the data link layer, and network layer protocols. It is shown with the help of a numerical example that there is an optimum value of the failure detection time found through the number of frame retransmissions on the DL layer in an unreliable network with adaptive routing. The numerical results of the network performance comparative analysis for adaptive and static routings are presented. The approach can be extended onto the more general models for integrated voice and data packet-switching networks.

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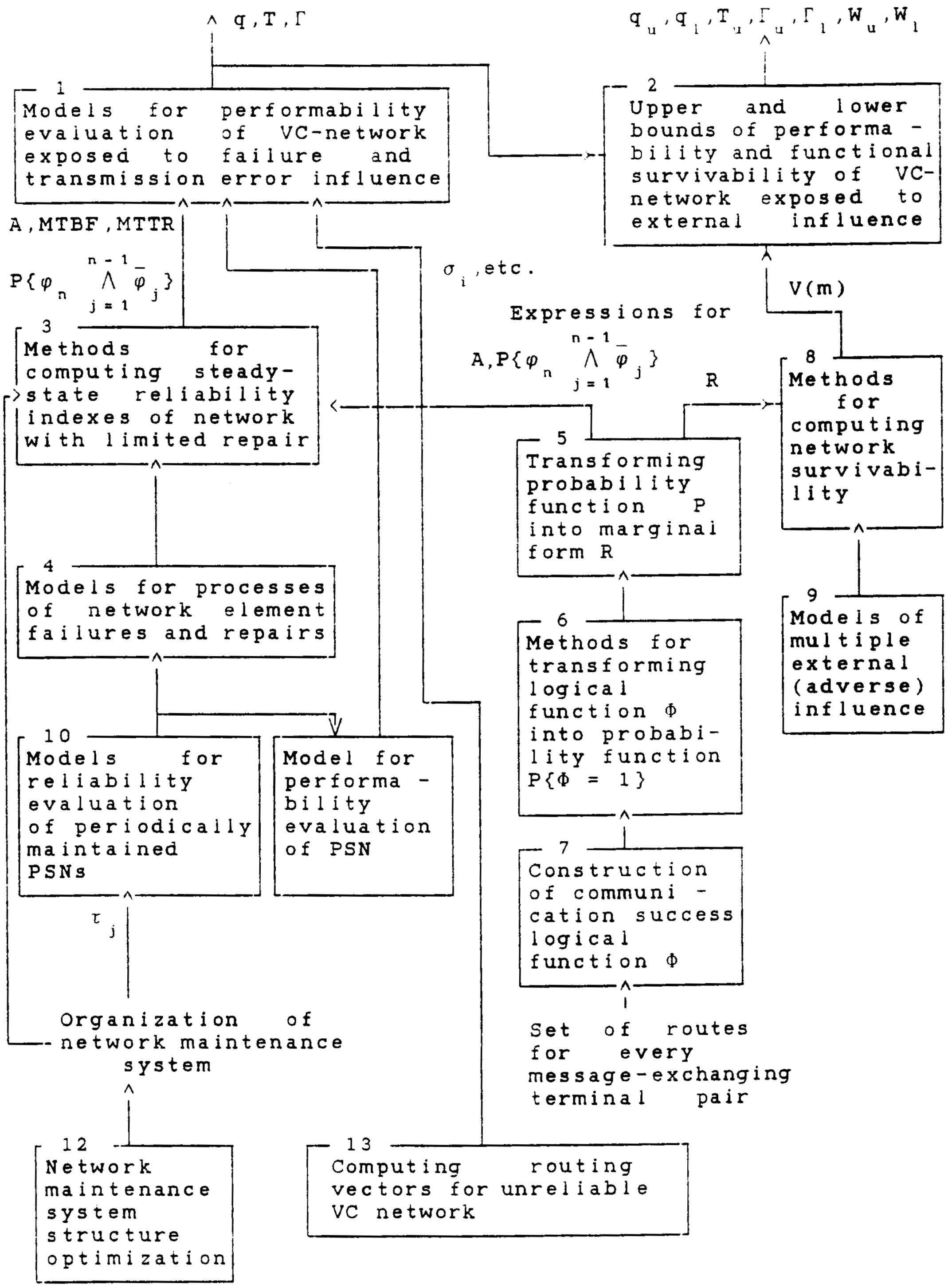

q, T, Γ
$q_u, q_l, T_u, \Gamma_u, \Gamma_l, W_u, W_l$
1 Models for performability evaluation of VC-network exposed to failure and transmission error influence
2 Upper and lower bounds of performability and functional survivability of VC-network exposed to external influence
A, MTBF, MTTR
$P\{\varphi_n \bigwedge_{j=1}^{n-1} \bar{\varphi}_j\}$
σ_i, etc.
V(m)
Expressions for
$A, P\{\varphi_n \bigwedge_{j=1}^{n-1} \bar{\varphi}_j\}$
R
3 Methods for computing steady-state reliability indexes of network with limited repair
8 Methods for computing network survivability
5 Transforming probability function P into marginal form R
4 Models for processes of network element failures and repairs
9 Models of multiple external (adverse) influence
6 Methods for transforming logical function Φ into probability function P{Φ = 1}
10 Models for reliability evaluation of periodically maintained PSNs
Model for performability evaluation of PSN
7 Construction of communication success logical function Φ
τ_j
Organization of network maintenance system
Set of routes for every message-exchanging terminal pair
12 Network maintenance system structure optimization
13 Computing routing vectors for unreliable VC network

Figure 1.

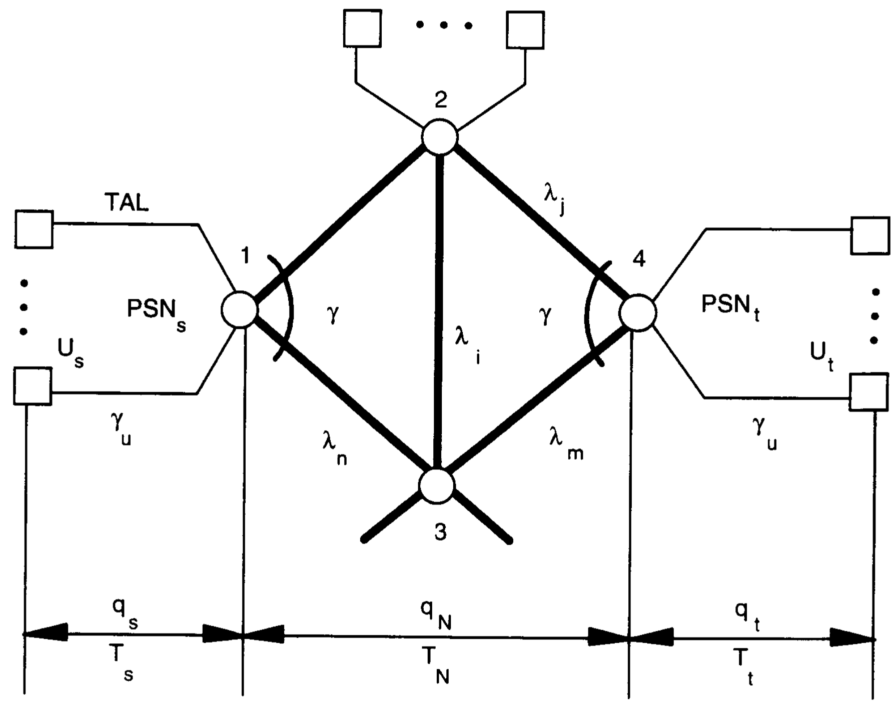

Figure 2.

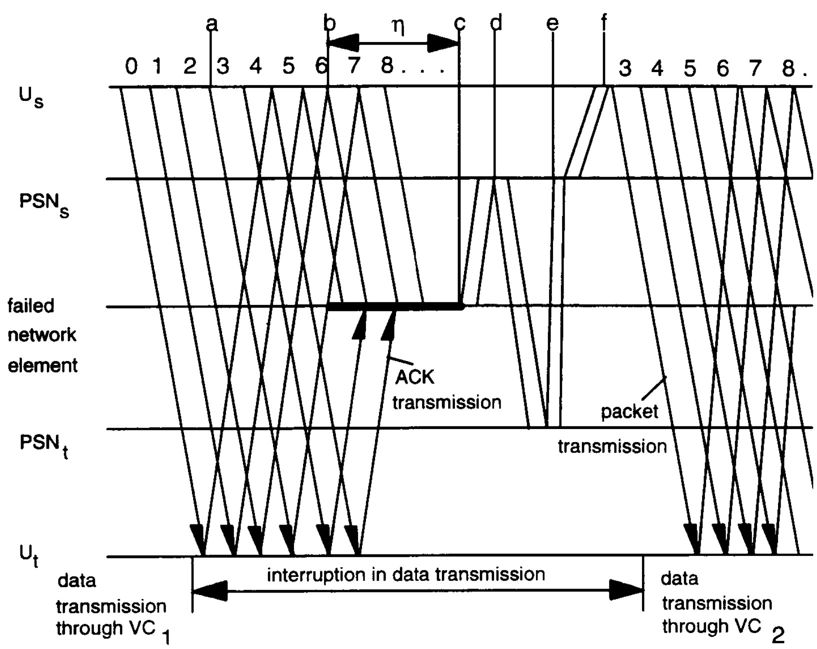

Figure 3.

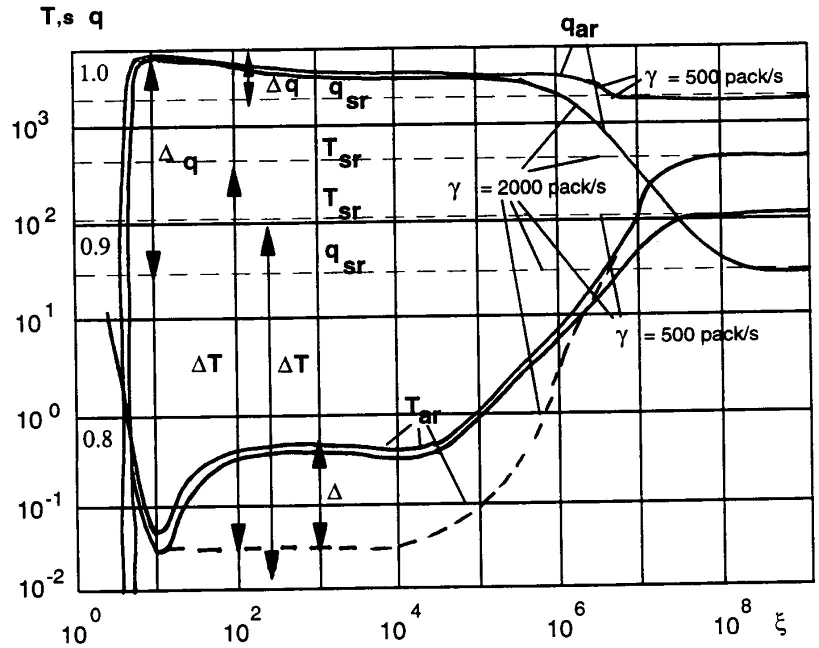

Figure 4.

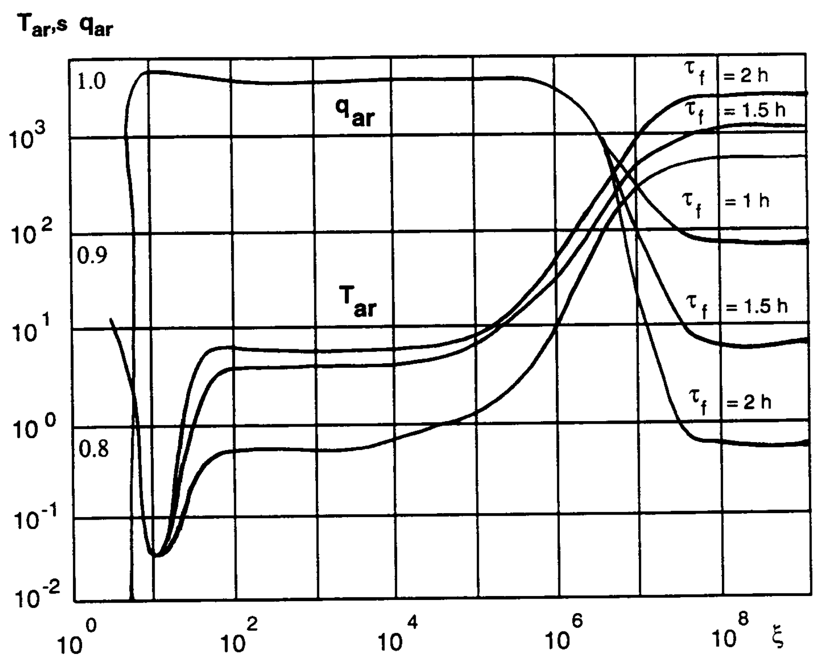

Figure 5.