Constraint Satisfaction as a Basis for Designing Nonmasking Fault-Tolerance

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Abstract

We present a method for the design of nonmasking fault-tolerant programs. In our method, a set of constraints is associated with each program. Each of these constraints is continually satisfied under the execution of program actions, as long as faults do not occur. Whenever some of the constraints are violated, due to certain faults, all constraints are eventually reestablished by subsequent execution of the program actions. To design programs thus, two types of program actions are distinguished: "closure" actions and "convergence" actions. Closure actions are the actions that perform the intended computation of the program when all of the constraints are satisfied. Convergence actions are the actions that reestablish the constraints when they have been violated. Sufficient conditions for the validation of closure and convergence actions are formalized in terms of a "constraint graph". These conditions are illustrated by designing nonmasking fault-tolerant programs for diffusing computations, atomic actions, and token rings.

Keywords: design method, distributed constraints, closure, convergence, nonmasking fault-tolerance.

1 Introduction

One way to achieve program fault-tolerance is to ensure that when faults occur the program continues to satisfy its input-output relation. Systems designed thus are said to "mask" the effects of faults. Over the last few decades, masking fault-tolerant programs have been studied in great depth by the fault-tolerance community and, as a result, a variety of design methods have resulted. These include — to name but a few — methods based on redundancy, error correction codes, replication and voting, broadcast and agreement, and multiprocess synchronization.

In certain situations, however, the potential for program fault-tolerance either cannot or should not be realized via masking fault-tolerance. For instance, there are situations where achieving masking fault-tolerance is

- *Impossible* : e.g., there is no asynchronous distributed program whose processes reach consensus on a binary value and mask the effect of a process crash [1],
- *Impractical* : e.g., the amount of redundancy and synchronization required to implement fail-stop processors that mask the effect of byzantine faults can be prohibitively expensive [2], or
- *Unnecessary* : e.g., a call-back telephone service that eventually establishes a connection is useful even if it does not mask its initial failure to establish a connection.

In such situations, an alternative way to achieve program fault-tolerance is to ensure that when faults occur the input-output relation of the program is violated only temporarily. In other words, the program is guaranteed to eventually resume satisfying its input-output relation. Such "nonmasking" fault-tolerant programs work by restoring the program to some previously checkpointed state, by replaying some part of the computation or by repairing faulty program parts, whenever a violation of the input-output relation is detected. In comparison to masking fault-tolerant programs, the design of nonmasking ones has received lesser study.

In this paper, we present a method for the design of nonmasking fault-tolerant programs. The rest of the paper proceeds as follows. In Section 2, we define formally our programming notation. In Section 3, we describe our method for designing nonmasking fault-tolerant programs, based on closure and convergence actions. In Section 4, we define the notion of the constraint graph of a program. We employ this

notion, in Section 5, to present a sufficient condition for validating the closure and convergence actions of programs whose constraint graph is an out-tree. We illustrate the condition by designing a nonmasking fault-tolerant programs for diffusing computations. We then present more general sufficient conditions, in Section 6, for programs with self-looping constraint graphs and, in Section 7, for programs with cyclic constraint graphs. We illustrate these more general conditions by designing nonmasking fault-tolerant programs for atomic actions and token-rings. Finally, we make concluding remarks in Section 8.

2 Programs and Computations

A *program* is a finite set of variables and a finite set of actions. Each variable has a predefined nonempty domain, and each action has the form :

$$\langle \text{guard} \rangle \rightarrow \langle \text{statement} \rangle$$

A guard is a boolean expression over program variables. A statement updates zero or more of program variables and always terminates upon execution.

Let p be a program. A *state* of p is defined by a value for each variable of p (chosen from the domain of the variable). A *state predicate* of p is a boolean expression over the variables of p. An action of p is *enabled* at a state iff the guard of the action holds at that state.

An action of p *preserves* a state predicate R iff executing the action, starting from any state where the action is enabled and R holds, yields a state where R holds. A state predicate R of p is *closed* iff each action of p preserves R.

A *computation* of a set of actions of p is a fair, maximal sequence of steps; in every step, some action in the set that is enabled in the current state is executed. Fairness of the sequence means that each action in the set that is continuously enabled along the sequence is eventually executed. Maximality of the sequence means that if the sequence is finite then no action in the set is enabled in the final state.

3 A Method for Designing Nonmasking Fault-Tolerant Programs

To motivate our method, let us first observe that the input-output relation of (both sequential and concurrent) programs can be characterized by a state predicate that is true throughout the program execution [3, 4]. Such an *invariant* predicate serves two purposes. First, it identifies the set of "fault-free" states of the program; these are the states starting from which every computation of the program is guaranteed to meet the specification of the program, i.e., the safety and progress properties required of the program. Second, an invariant predicate constrains the design of program actions by requiring that the set of fault-free states be kept closed under the execution of program actions. Examples showing how to design program invariants appear in [5].

We next observe that the input-output relation of programs in the presence of faults can also be characterized by a state predicate that is true throughout program execution [6]. Such a state predicate identifies the *fault-span* of the program; i.e., the set of states that the program can reach in the presence of faults. (Note that the fault-span includes the fault-free states of the program.) Examples showing how to design fault-span predicates appear in [7]. These examples employ the view that all classes of faults can be represented as actions that change the program state [7, 8]. As a result of this view, a program fault-span identifies a set of states that is kept closed under the execution of program actions as well as fault actions.

We are now ready to give a formal definition of nonmasking fault-tolerance. Let S be a program invariant and T be a program fault-span, such that $(S \Rightarrow T) \wedge (S \not\equiv T)$. For a program to be T-tolerant for S, the following two requirements should be satisfied :

- Closure: Both S and T are closed under the execution of program actions.
- Convergence: Starting from any state where T holds, execution of program actions converges to a state where S holds.

The above formal definition suggests that nonmasking fault-tolerant programs can be designed ideally by separately designing two classes of program actions: "closure" actions and "convergence" actions. Closure actions are actions that perform the intended computation of the program when the program state satisfies the program invariant S; since their execution is meaningful only when S holds, closure actions are enabled only in states where S holds. Convergence actions are actions that restore the program from a state where the program fault-span T holds to a state where S holds; since their execution is meaningful only when S does not hold, convergence actions are enabled only in states where $\neg S$ holds.

When the program being designed is a distributed one, the design of closure and convergence actions needs further refinement. This refinement is due to the fact that actions of distributed programs can access and update only a limited part of the program state. Hence, closure actions cannot independently check whether S holds and convergence actions cannot independently establish states where S holds. To apply our method to distributed programs as well, we therefore propose to relax the restrictions on closure and convergence actions actions as follows: Closure actions may execute in states where S does not hold, provided their execution does not prevent the convergence actions from yielding states where S holds. Convergence actions may establish states where S holds, not in one step, but in some finite number of steps.

Our design method, then, is as follows. Assume that we are given a specification of a program and a set of fault actions. To design a program that both meets the specification and tolerates the fault actions, we first identify a "candidate" triple (p, S, T) where

- S is the program invariant, derived from the specification.
- T is the program fault-span, derived from S and the fault actions.
- p is the set of closure actions that perform the intended computation of the program when the program state satisfies S, and that each preserve S and T.

We then proceed to design convergence actions for the candidate triple (p, S, T), as follows. Recall that each convergence action can access and update only a part of the program state. Hence, we decompose S into a set of predicates, which we call the *constraints* in S, such that (i) each constraint in S can be independently checked and established, and (ii) the conjunction of all constraints in S together with T equivales S.

For each constraint c in S, we design one convergence action of the form:

$\neg c \quad \rightarrow$ " establish c while preserving T "

In other words, for each constraint c in S we design a convergence action that independently checks c and, if need be, establishes c while preserving T. Note that since the action is enabled only when $\neg S$ holds, the action trivially preserves S. To complete the design, it therefore only remains to perform "convergence validation"; i.e., to ensure that starting from any state where T holds, every computation of the closure and convergence actions converges to a state where S holds.

4 Constraint Graphs

The task of convergence validation, as presented above, is a nontrivial one. This is because closure actions may execute while convergence actions are executing; also, convergence actions for some constraints may violate some other constraints of the program invariant upon execution.

We therefore focus on the problem of convergence validation in the rest of this paper. More specifically, we formulate a set of sufficient conditions under which convergence validation for a candidate triple (p, S, T) and a set of convergence actions q is made feasible. Towards this end, we define the notion of a constraint graph, next.

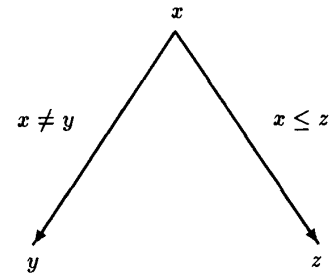
A *constraint graph* of q is a directed graph that has one edge for each action in q, such that

1. Each node of the graph is labeled with a set of variables that appear in q. Labels of nodes are mutually exclusive; thus, a variable appears in the label of only one node.
2. Each edge of the graph from node v to node w is labeled with an action of q that accesses variables in v and w and updates variables in w.

Since there is a bijection between constraints and convergence actions, for convenience, we sometimes ambiguously refer to the corresponding constraint as being the label of the edge.

Example : Consider a program that has three integer variable x, y, and z. Consider further that S is the conjunction of the constraints $x \neq y$ and $x \leq z$. Now, if a convergence action satisfies the first constraint by changing x if $x = y$ holds, it can violate the second constraint.

Alternatively, consider for the first constraint a convergence action that changes y if x equals y, and for the second constraint a convergence action that changes z to be at least x if x exceeds z. For these two convergence actions, the constraint graph is:



□

5 Designing Programs with Out-tree Constraint Graphs

In this section, we first identify in Theorem 1 a sufficient condition for the convergence validation of nonmasking fault-tolerant programs. The condition applies to programs whose convergence actions have an out-tree constraint graph. (An out-tree is a weakly connected directed graph one of whose nodes has indegree zero and the remaining of whose nodes have indegree one. See, for example, the constraint graph depicted above.) We then illustrate the condition by designing a nonmasking fault-tolerant program for diffusing computations.

Theorem 1 :
Let (p, S, T) be a candidate triple and q be a set of convergence actions.
If

- every closure action of p preserves each constraint in S, and
- the constraint graph of q is an out-tree,

then $p \cup q$ is T-tolerant for S.

[For want of space, we defer the proofs of all Theorems herein to the journal version of the paper.]

We illustrate Theorem 1 by designing a "stabilizing" program that maintains a diffusing computation. A stabilizing program [3] is one that exhibits an extreme form of nonmasking fault-tolerance: regardless of the state the program is started in, execution of the program converges to a state from where S holds. It follows that stabilizing programs can tolerate faults whose effect is to somehow corrupt the program state. Note that for stabilizing programs, the program fault-span T is the state predicate *true*; hence, each action of a stabilizing program trivially preserves T.

5.1 Stabilizing Diffusing Computations

Diffusing computations are used commonly in distributed systems to perform tasks that involve accessing or modifying the collective system state. Applications of diffusing computations include, for example, global state snapshot, termination detection, deadlock detection, and distributed reset. Typically, a distinguished process in the system periodically initiates a diffusing computation, which then propagates across the system to perform some subtask at each process. Having completely spanned the system, the computation then collapses back to the distinguished process.

Below, we specify and design an abstract version of a diffusing computation, that is independent of any specific application. (The resulting program is a simplified version of a program in [9].)

Specification :
Consider a finite, rooted tree. Desired is a program in which, starting from a state where all tree nodes are colored green, the root node initiates a diffusing computation. The diffusing computation then propagates from the root to the leaves, coloring the tree nodes red. Upon reaching the leaves, the diffusing computation is reflected back towards the root, coloring the tree nodes green. And the cycle repeats. The program should tolerate faults that arbitrarily corrupt the state of any number of nodes.

Design :
First, we characterize S. Let $c.j$ be the color of node j, and let $sn.j$ be a boolean session number that is used to distinguish "j has not started participating in the current diffusing computation" from "j has completed participating in the current diffusing computation". Also, let $P.j$ be the parent node of j in the tree (hence if j is the root then $P.j$ is j, else $P.j$ is the unique node from which there is an edge to j in the tree).

We postulate that when all j are colored green, all j have the same session number. Hence, to distinguish "j has not started participating in the current diffusing computation" from "j has completed participating in the current diffusing computation", it suffices that j toggles the value of $sn.j$ whenever j starts participating in a new diffusing computation.

We can now characterize S as follows : in the current diffusing computation, each j satisfies one of the following four conditions.

1. j and $P.j$ have both started participating: $c.j = c.(P.j) \ \wedge \ sn.j \equiv sn.(P.j)$,
2. j and $P.j$ have both completed participating: $c.j = c.(P.j) \ \wedge \ sn.j \equiv sn.(P.j)$,
3. j has not started participating and $P.j$ has: $c.j = green \ \wedge \ c.(P.j) = red \ \wedge \ sn.j \not\equiv sn.(P.j)$,
4. j has completed participating and $P.j$ has not: $c.j = green \ \wedge \ c.(P.j) = red \ \wedge \ sn.j \equiv sn.(P.j)$.

That is,

$$\begin{aligned} S &= (\forall j :: R.j) \text{ , where} \\ R.j &= (c.j = c.(P.j) \ \wedge \ sn.j \equiv sn.(P.j)) \vee \\ &\quad (c.j = green \wedge c.(P.j) = red) \ . \end{aligned}$$

Each state predicate $R.j$ can be independently checked and satisfied by the node j; hence, we consider each

$R.j$ to be a separate constraint of S. We now proceed to design the closure actions.

For initiating a diffusing computation at the root node, we consider

$$c.j{=}green \ \wedge \ P.j{=}j \ \rightarrow \ c.j\,,\, sn.j := red\,,\, \neg sn.j$$

For propagating a diffusing computation from $P.j$ to j, we consider

$$c.j{=}green \ \wedge \ c.(P.j){=}red \ \wedge \ sn.j{\not\equiv}sn.(P.j) \ \rightarrow \ c.j\,,\, sn.j := c.(P.j)\,,\, sn.(P.j)$$

For reflecting the diffusing computation from the children of j to j, we consider

$$c.j{=}red \wedge (\forall k : P.k{=}j \Rightarrow (c.k{=}green \ \wedge \ sn.j{\equiv}sn.k)) \ \rightarrow \ c.j := green$$

We observe that each of these closure actions preserves each constraint in S and, hence, also preserves S.

Finally, let us design for each constraint $R.j$ a convergence action of the form

$$\neg R.j \quad \rightarrow \quad \text{"establish } R.j\text{"}$$

We propose to establish $R.j$ by updating the variables associated with node j. As a result, the constraint graph edge associated with $R.j$ will be from node $P.j$ to j and, hence, the constraint graph will be an out-tree. From Theorem 1, it follows that the resulting program will be *true*-tolerant for S. In other words, the resulting program will be stabilizing fault-tolerant.

We note that there are several statements that establish $R.j$ as proposed above. For instance, "$c.j, sn.j := c.(P.j), sn.(P.j)$" could be used or "if $c.(P.j) = red$ then $c.j := green$ else $c.j, sn.j := green, sn.(P.j)$" could be used. We prefer the former statement, since it is identical to the statement of the propagation closure action. With this choice, these two actions can be combined to yield the action

$$\neg R.j \ \vee \ (c.j{=}green \ \wedge \ c.(P.j){=}red \ \wedge \ sn.j{\not\equiv}sn.(P.j)) \ \rightarrow \ c.j\,,\, sn.j := c.(P.j)\,,\, sn.(P.j)$$

which is equivalent to the action

$$sn.j{\not\equiv}sn.(P.j) \ \vee \ (c.j{=}red \ \wedge \ c.(P.j){=}green) \ \rightarrow \ c.j\,,\, sn.j := c.(P.j)\,,\, sn.(P.j)$$

Thus, our design yields the following stabilizing fault-tolerant program for diffusing computations:

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program *Diffusing-computation*
process $j : 1..N$;
var $c.j : \{green, red\}$;
$sn.j$: **boolean** ;
begin
$c.j{=}green \ \wedge \ P.j{=}j \ \rightarrow \ c.j\,,\, sn.j := red\,,\, \neg sn.j$
[] $sn.j{\not\equiv}sn.(P.j) \ \vee \ (c.j{=}red \ \wedge \ c.(P.j){=}green) \ \rightarrow \ c.j\,,\, sn.j := c.(P.j)\,,\, sn.(P.j)$
[] $c.j{=}red \ \wedge \ (\forall k :: P.k{=}j \ \Rightarrow \ (c.k{=}green \ \wedge \ sn.j{\equiv}sn.k)) \ \rightarrow \ c.j := green$
end

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6 Designing Programs with Self-looping Constraint Graphs

In this section, we first identify in Theorem 2 a more general sufficient condition for the convergence validation of nonmasking fault-tolerant programs. The condition applies to programs whose convergence actions have constraint graphs with no cycles of length greater than 1. In other words, their constraint graph is either acyclic or every cycle in the graph is a self-loop. We refer to such constraint graphs as *self-looping* constraint graphs. Later in the section, we illustrate the condition by designing a nonmasking fault-tolerant program that implements an atomic action.

The basic problem with convergence validation when the constraint graph is self-looping is that if the edges in the constraint graph corresponding to two constraints have the same target node, then executing the convergence action of one of the constraints may violate the other constraint, and vice versa.

Example : For the constraint set $\{x \neq y\,,\ x \leq z\}$, consider a convergence action that changes x if x equals y, and a convergence action that changes x to be at most z if x exceeds z. For these convergence actions, it is possible that executing one can violate the constraint of the other, then executing the other can violate the constraint of the one, and so on.

In contrast to the above, consider for $x \neq y$ a convergence action that decreases x if x equals y, and for $x \leq z$ a convergence action that changes x to be at most z if x exceeds z. The first action preserves the constraint of the second action, and hence every computation of these two convergence actions is finite.

Theorem 2 :

Let (p, S, T) be a candidate triple and q be a set of convergence actions.

If
- every closure action of p preserves each constraint in S,
- the constraint graph of q is self-looping, and
- for each node j of the constraint graph of q, the convergence actions of edges with target j can be linearly ordered so that each action in the order preserves the constraints of the preceding actions in the order,

then $p \cup q$ is T-tolerant for S.

We illustrate Theorem 2 by designing a program that implements an atomic action. The program is motivated by a method due to Schlichting and Schneider [2], that shows how to design certain fault-tolerant programs using a sequence of "restartable" phases. A restartable phase is one that serves as its own recovery protocol: if execution of the phase is perturbed due to the occurrence of a fault, then recovery is achieved by executing the same phase all over again. Hence, if a computation of a sequence of restartable phases is perturbed due to the occurrence of a fault, then recovery of the computation is achieved by executing the sequence of phases starting from the phase whose execution was perturbed. Typically, to implement such a sequence of restartable actions, some form of stable storage is used that is unaffected by the faults.

6.1 Atomic Action

Specification :

Desired is a program *Atomic* that satisfies the Hoare-triple:

$\{x{=}M \wedge y{=}N\}$ *Atomic* $\{x{=}h.M \wedge y{=}h.N\}$,

where x and y are integer variables maintained in stable storage, M and N are integer constants, and h is a function from integers to integers. Program execution is to occur in a sequence of phases, and actions in each phase may update at most one variable in stable storage. Faults may corrupt the volatile storage of the program, but not the stable storage.

Design :

Let us design program *Atomic* to consist of two phases. In phase zero ($ph = 0$), $h.x$ and $h.y$ are computed and respectively assigned to fresh variables u and v, that are also maintained in stable storage. In phase one ($ph = 1$), u and v are respectively assigned to x and y.

Since actions in each phase update at most variable in shared storage, we propose that in phase zero, first, $h.x$ is assigned to u; and second, if $u = h.x$, then $h.y$ is assigned to v. Similarly, in phase one, first, u is assigned to x; and second, if $u = x$, then v is assigned to y. Let pc denote the program counter in each phase. Initially, $pc = 0$ in each phase.

We can now characterize the program invariant of *Atomic* as

$$\begin{array}{lll} S = & ((ph{=}0 & \Rightarrow \; x{=}M \wedge y{=}N) \quad \wedge \\ & (ph{=}1 & \Rightarrow \; u{=}h.M \wedge v{=}h.N) \; \wedge \\ & (ph{=}2 & \Rightarrow \; x{=}h.M \wedge y{=}h.N) \; \wedge \\ & (ph{=}0 \wedge pc{=}1 & \Rightarrow \; u{=}h.x) \quad \wedge \\ & (ph{=}1 \wedge pc{=}1 & \Rightarrow \; u{=}x)) \end{array}$$

Since faults corrupt only the volatile storage and pc is the only program variable maintained in volatile storage, only the last two conjuncts of S can be violated when faults occur. Hence, we can characterize the program fault-span of *Atomic* as

$$\begin{array}{lll} T = & ((ph{=}0 \Rightarrow & x{=}M \wedge y{=}N) \quad \wedge \\ & (ph{=}1 \Rightarrow & u{=}h.M \wedge v{=}h.N) \; \wedge \\ & (ph{=}2 \Rightarrow & x{=}h.M \wedge y{=}h.N)) \end{array}$$

The last two conjuncts of S can both be independently checked and satisfied, and so we consider them to be separate constraints of S.

Following the two-phase description above, the closure actions are immediately suggested. For the first step of phase 0, we consider

$ph{=}0 \wedge pc{=}0 \quad \rightarrow \quad u, pc := h.x, 1$

For the second step of phase 0, we consider

$ph{=}0 \wedge pc{=}1 \wedge u{=}h.x \quad \rightarrow \quad v, ph, pc := h.y, 1, 0$

For the first step of phase 1, we consider

$ph{=}1 \wedge pc{=}0 \quad \rightarrow \quad x, pc := u, 1$

For the second step of phase 1, we consider

$ph{=}1 \wedge pc{=}1 \wedge x{=}u \quad \rightarrow \quad y, ph := v, 2$

We observe that each of the closure actions preserves both constraints as well as T.

Finally, we design the convergence actions for the two constraints, as follows (note that both constraints can be established by assigning the value 0 to pc, which corresponds to restarting the current phase of the program):

$ph{=}0 \wedge pc{=}1 \wedge u{\neq}h.x \quad \rightarrow \quad pc := 0$,

$ph{=}1 \wedge pc{=}1 \wedge x{\neq}u \quad \rightarrow \quad pc := 0$

We observe that each of the convergence actions preserves T. Also, the constraint graph for the convergence actions is a single node with a self-loop. Moreover, both convergence actions preserve each other's constraint. From Theorem 2, it follows that the resulting program is T-tolerant for S.

Thus, our design yields the following program for atomic actions:

```
program Atomic
var   ph : 0..2;
      pc : 0..1;
      x, y, u, v : integer ;
begin
   ph=0  ∧  pc=0                  →  u, pc := h.x, 1
[] ph=0  ∧  pc=1  ∧  u=h.x        →  v, ph, pc := h.y, 1, 0
[] ph=0  ∧  pc=1  ∧  u≠h.x        →  pc := 0
[] ph=1  ∧  pc=0                  →  x, pc := u, 1
[] ph=1  ∧  pc=1  ∧  x=u          →  y, ph := v, 2
[] ph=1  ∧  pc=1  ∧  x≠u          →  pc := 0
end
```

7 Designing Programs with Cyclic Constraint Graphs

In this section, we identify in Theorem 3 yet another sufficient condition for the convergence validation of nonmasking fault-tolerant programs. The condition applies to programs whose convergence actions have constraint graphs with cycles of length greater than 1.

Let us begin by observing that our definition of a constraint graph is sometimes "coarser" than need be: A cyclic constraint graph may become self-looping when its definition is refined to take into account (1) certain subsets of states or (2) certain subsets of constraints.

Consider a subset of states R of the program. If a constraint is true at each state in R, then in reasoning about states in R, the corresponding edge in the constraint graph can be ignored.

We describe three ways to use the notion of constraint graph, as refined above to take into account certain subsets of states. One possibility is that even if the constraint graph is cyclic, its restriction to those states where the fault-span predicate T holds may be self-looping, thereby enabling use of Theorem 2.

A second possibility is that T can be partitioned into a finite number of closed subsets of states. If the constraint graph for each of these partitions is self-looping, we may use Theorem 2 with respect to each of these partitions to validate the program actions.

A third possibility is that all computations converge from T to S in two stages. Let R be a closed state predicate such that $S \Rightarrow R$ and $R \Rightarrow T$. In the first stage, starting from any state where T holds each computation reaches a state where R holds. In the second stage, starting from any state where R holds each computation reaches a state where S holds. (Gouda and Multari call this a convergence stair of height two [11].) In this case, the constraint graph for R may self-looping, whereas the constraint graph for T is cyclic. As a result, convergence validation may be carried out in two corresponding stages, and Theorem 2 may be used for the second stage.

An alternative approach to convergence validation when the constraint graphs are cyclic is to partition the constraints in a hierarchical manner. If the constraint graph when refined taking into account the subset of constraints in each layer of the hierarchy is self-looping, then convergence validation is made feasible as follows.

Let q' be a subset of the convergence actions of a program p. A *constraint graph* of q' is a directed graph that has one edge for each action in q', such that

1. Each node of the graph is labeled with a set of variables that appear in q'. Labels of nodes are mutually exclusive; thus, a variable appears in the label of only one node.
2. Each edge of the graph from node v to node w is labeled with an action of q' that accesses variables in v and w and updates variables in w .

Theorem 3 :
Let (p, S, T) be a candidate triple and q be a set of convergence actions that is partitioned into subsets numbered 0, 1, ... , $M-1$.

If
- for each partition, each closure action of p preserves each constraint in that partition whenever all constraints in a lower numbered partition hold,
- for each partition, the convergence actions of the constraints in that partition preserve each constraint in a lower numbered partition l whenever all constraints in partitions numbered lower than l hold,
- for each partition, the constraint graph is self-looping, and
- for each partition, the convergence actions of edges whose target is node j in the constraint graph of that partition can be linearly ordered so that each action in the order preserves the constraints of the preceding actions in the order,

then $p \cup q$ is T-tolerant for S.

[In the journal version of this paper, we illustrate Theorem 3 by designing a stabilizing token ring program, that is due to Dijkstra [3]. More recently, we have used Theorem 3 in designing a novel program for tree reconfiguration [10].]

8 Concluding Remarks

In this paper, we have developed a method for the design of nonmasking fault-tolerant programs. Our method distinguishes two types of program actions and exploits the dependencies between the constraints that together comprise the program invariant, for designing program actions that satisfy the closure and convergence properties necessary for nonmasking fault-tolerance.

Closure properties are a special class of the safety properties of programs. Safety properties are used in design methods to constrain the design of program actions. For example, an invariant predicate constrains the design of program actions so that the program specification can be satisfied. In this paper, we have shown how a fault-span predicate constrains the design of program actions so that nonmasking fault-tolerant can be achieved.

Convergence properties are a special class of the progress properties of programs. The standard approach for proving that computations of a program progress towards satisfying some state predicate is to exhibit a "variant" function. A variant function is a mapping from the program state space to a set that is wellfounded under a relation $<$, such that in each step of the computation the variant function value does not increase (with respect to $<$) and eventually decreases, until the desired state predicate is satisfied. Exhibiting variant functions is a nontrivial task, especially when the program in question is distributed. In this paper, we have shown how to simplify the problem of exhibiting variant functions, in terms of the sufficient conditions on the closure and convergence actions and the constraint graph.

To further develop the methodology, one issue that we are studying is the role of fairness. The fairness requirement on program computations is often unnecessary. (In fact, each of the programs derived in this paper is correct even when the fairness requirement is ignored; to see this, observe that each computation of the closure actions is either finite or has a state where S holds [the argument for the token-ring program requires an additional similar requirement for the convergence actions of each layer].) Hence, it will be useful to characterize sufficient conditions under which program actions guarantee convergence to S without requiring fairness.

It will also be useful to develop systematic methods of refining programs that preserve the property of convergence to fault-free states. For instance, recall that one of the closure actions in the stabilizing diffusing computation involves accessing the state of a node and all its children nodes in the out-tree. This action has high atomicity and may therefore be unsuitable for a distributed implementation. In [9], we present a refinement of this system that yields actions with low atomicity and preserves the property of convergence. We study refinement issues in a companion paper.

Finally, we note that our approach here has been to design the fault-tolerance of a program hand-in-hand with the rest of the program. Alternative design approaches are also worthy of study. For example, we could design first a fault-intolerant version of the program, then transform the fault-intolerant program into one that is nonmasking fault-tolerant. We have studied this alternative approach elsewhere [12, 13].

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