A Measure of Dissimilarity between the Structure of Instructional Material and the Structure of the Learner’s Understanding

Nobuyoshi YONEZAWA 1, Kazuhito HIRAI1, Shizuaki TAKAHASHI1, Youzou MIYADERA2 and Keizo NAGAOKA3

1)Department of Computer Science and Communication Engineering, Kogakuin University, 2665-1 Nakano-machi, Hachioji-shi, Tokyo 192-0015, Japan. E-mail: ct72058@ns.kogakuin.ac.jp
2)Department of Mathematics and Information Science, Tokyo Gakugei University
3)National Institute of Multimedia Education, Ministry of Education and Science

Abstract
The present paper investigates the construction of a navigation system that assists the learner in linking learning tasks so that the structure of the learner’s understanding will conform to the structure of instructional material. As an initial step, a measure of the dissimilarity between two digraphs expressing the structure of instructional material and the structure of learner’s understanding, respectively, is defined. Details concerning the navigation will be reported at a later date.

1. Introduction
The structure of instructional material is expressed via a digraph having no directed cycles conceptually. The vertex $v_i$ and the directed edge $(v_i, v_j)$ of the digraph correspond to the learning task and the order relation of the learning task, respectively. A topological order for the structure of instructional material is a sequential listing of learning tasks.

The system doesn’t show the structure of instructional material to the learner and presents only the learning tasks in order of the sequential listing. The learner then solves the tasks in the order in which they were presented. After learning, the learner constructs a digraph, referred to as the structure of the learner’s understanding, by linking the learning tasks.

The learner’s degree of understanding is measured based on the difference between the structure of the learner’s understanding and the structure of the instructional material.

2. Digraph
Let $G_T$ be the structure of the instructional material designed by a teacher. Figure 1 shows an example of $G_T$. The vertex without the output edge is called a root, whereas the vertex without the input edge is called a leaf. The root is a learning target, whereas the leaf is the most basic learning task. The structure of the learner’s understanding as constructed by the learner after learning is designated as $G_S$. Figure 2 shows an example of $G_S$. The two digraphs $G_T$ and $G_S$ have the same vertices, and are respectively given as $G_T = [V(G_T), E(G_T)]$, $G_S = [V(G_S), E(G_S)]$.

Members of a set $E(G_T)$ are the edges of $G_T$. If $E(G_T) = E(G_S)$, then $G_T$ is equal to $G_S$. If the number of members in a set $A$ is expressed as $N(A)$, then $N(V(G_T)) = n$.

The dissimilarity between $G_T$ and $G_S$ is defined in Section 3. As the dissimilarity is made smaller, the similarity between $G_T$ and $G_S$ increases. If the dissimilarity is 0, then $E(G_T)$ is equal to $E(G_S)$, that is, $G_T = G_S$.

3. Dissimilarity between two digraphs
A sequence of directed edges

\[(v_{j_{c-1}}, v_{j_c}) \quad c = 1, 2, 3, \ldots, x\]

from $v_{j_0}$ to $v_{j_x}$ in $G_T$ is called a path. Let $P< - v_j - >^T$ be the set of all paths from a number of leaves to a root, which passes through a vertex $v_j$. $P< - v_j - >^T$ is divided into two sets $P< - v_j - >^T$ and $P< - v_j - >^T$, where $P< - v_j - >^T$ and $P< - v_j - >^T$ are the set of all paths from a number of leaves to $v_j$ and the set of all paths from $v_j$ to a root, respectively. Let $E< - v_j - >^T$ be a set in which the members are the edges that form $P< - v_j - >^T$, and
let \( E^v_j \) be a set in which the members are the edges that form \( P^v_j \). In the same way, \( P^v_j, P^v_j, P^v_j, P^v_j, E^v_j \) for \( G_S \) is defined.

We define the distance between \( v_i \in V(G_T) \) and \( v_j \in V(G_T) \) as follows:

\[
d_{ij}^{TT} = N(E < v_i > T \nabla E < v_j > T) + N(E < v_i > T \nabla E < v_j > T)
\]

where \( \nabla \) is the symmetric difference \(( A \setminus B ) - ( A \cap B )\).

In the same way, the distance between \( v_i \in (G_T) \) and \( v_j \in (G_T) \) is given by

\[
d_{ij}^{TS} = N(E < v_i > T \nabla E < v_j > T) + N(E < v_i > T \nabla E < v_j > T)
\]

For example,

\[
E^v_2 = \emptyset, E^v_2 = \{(v_2,v_3), (v_3,v_5)\}
\]

\[
E^v_3 = \{(v_1,v_3), (v_2,v_3)\}, E^v_3 = \{(v_3,v_5)\}
\]

\[
E^v_3 = \{(v_1,v_3), (v_2,v_3)\}, E^v_3 = \{(v_3,v_5)\}
\]

\[
d_{22}^{TT} = 0, d_{22}^{TS} = N((v_2,v_3), (v_1,v_3))=2
\]

\[
d_{32}^{TS} = N((v_3,v_1), (v_2,v_3)) + N((v_2,v_3), (v_4,v_5))=3
\]

\[
d_{32}^{TS} = N((v_3,v_1), (v_2,v_3)) + N((v_2,v_3), (v_4,v_5))=5
\]

As the difference between \( G_T \) and \( G_S \) increases, the value of \( d_{ij}^{TS} \) increases.

All \( d_{ij}^{TT} \) of \( G_T \) are expressed in a \( n \times n \) square matrix \( d^{TT} \) having the components \( d_{ij}^{TT} \). \( d^{TT} \) is a symmetric matrix having diagonal components equal to 0. All \( d_{ij}^{TS} \) between \( G_T \) and \( G_S \) are expressed in a \( n \times n \) square matrix \( d^{TS} \) having the components \( d_{ij}^{TS} \). \( d^{TS} \) is an asymmetric matrix, and the diagonal components are not necessarily 0.

We define the dissimilarity between \( G_T \) and \( G_S \) as follows:

\[
D^{TS} = \frac{1}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} |d_{ij}^{TS} - d_{ij}^{TT}|
\]

where \( n = N(V(G)) \).

4. Conclusions

\( d^{TT} \) and \( d^{TS} \) are shown in Tables 1 and 2. The dissimilarity \( D^{TS} \) between \( G_T \) and \( G_S \) is 0.8. The shape of the digraph and the dissimilarity visually coincide approximately. The navigation system, which assists the learner in linking learning tasks based on the dissimilarity, will be reported in a future study.

Reference
