Visualization of Bugs in Subtraction with Block-Box Model

Tsukasa HIRASHIMA, Mutsuko NOGUCHI, Akira TAKEUCHI
Kyushu Institute of Technology
680-4, Kawazu, Iizuka, Fukuoka, 820-8502 JAPAN
tukasa@ai.kyutech.ac.jp

Abstract
In this paper, an error-awareness learning environment targeting the bugs of multi-digit subtraction is described. We, first, introduce the block-box model where the minuend is embodied as blocks and the subtrahend is embodied as boxes. In the block-box model, the process of subtraction is embodied as concrete manipulation of putting the blocks in the boxes. The bugs in multi-digit subtraction can be visualized as irregular manipulation of the blocks and boxes. We have developed a learning environment where a learner subtracts a multi-digit number described with blocks and boxes by direct-manipulation. The learner then can reconstruct an erroneous subtraction including a bug with irregular manipulation of the blocks and boxes. Through this reconstruction, we expect that a learner will increase his/her understanding about the procedure of subtraction. In this paper, an experimental evaluation of the learning environment is also described.

1. Introduction

An error often provides a good learning opportunity for a learner to find a fault within his knowledge or to confirm its applicable range. By observing the error, the learner is aware of the necessity to correct his/her knowledge or to acquire new knowledge [1,2]. However, all errors don’t always trigger such learning. An error should satisfy the following conditions so that it is useful as a learning opportunity: (1) a learner is able to spot the error, (2) the learner is able to recognize the importance of the error, and (3) the spotted error suggests the method to correct the error. The first and second are mainly conditions of motivation and (3) is the method leading to adequate learning. Based on this consideration, we are investigating an environment that helps a learner to learn from errors [3]. We call this environment “Error-Awareness Learning Environment”.

In this paper, an error-awareness learning environment targeting the bugs of multi-digit subtraction is described. Although several researchers have investigated the bugs of subtraction, most of them pay attention to categorize the bugs and to develop the methods to diagnose the bugs from learners’ erroneous answers. Not enough attention is paid to the method of explanation how the bug is incorrect. From the standpoint of promoting learning from errors, we believe that the explanation of the bugs is important in using the errors as the learning opportunities.

We, first, introduce the block-box model where the minuend is embodied as blocks and the subtrahend is embodied as boxes. In the block-box model, the process of subtraction is embodied as concrete manipulation of putting the blocks in the boxes. The bugs in multi-digit subtraction can be visualized as irregular manipulation of the blocks and boxes. We have developed a learning environment where a learner subtracts a multi-digit number described with blocks and boxes by direct-manipulation. The learner then can reconstruct a bug procedure as irregular manipulation of the blocks and boxes. Through this reconstruction, we expect that a learner will increase his/her understanding about the procedure of subtraction. In this paper, an experimental evaluation of the learning environment is also described.

2. Block-Box Model

It is a promising approach to embody an abstract procedure to concrete manipulation in the learning of arithmetic [4,5]. In the learning of subtraction, the procedure of subtraction is often embodied as blocks and the deletion operations of the blocks [6]. In such cases, although the minuend is embodied as blocks, the subtrahend is not. However, the unembodied subtrahend is often difficult point of subtraction for learners. In this research, the subtrahend is also embodied as boxes. This embodiment is crucial in explaining the bugs in multi-digit subtraction. In this section, the block-box model is proposed to embody the subtraction procedure using the blocks and boxes. This will be followed by the explanation of the visualization of the bugs with the model.

2.1 Framework

In the block-box model, subtraction is mapped out by the manipulation of the blocks and boxes by the analogy of putting away the blocks into the boxes. Numbers in each digit are differentiated by the position and color of the blocks. Borrowing from the higher digit is described by changing one block in the higher digit to ten blocks in the lower digit.

Figure 1 shows the initial situation of 15-7. The minuend, that is 15, is described as one block in Ten’s place and five blocks in One’s place. The subtrahend is described by seven boxes in One’s place. Each box is put
in the same kind of block. In this example, the first step consists of five blocks filling up five boxes. This leaves two empty boxes remaining in One’s place. To fill up the remaining empty boxes, “borrowing of boxes” from the higher digit must be carried out. The single block that was in Ten’s place is moved to Exchange Area. In the Exchange Area, a single block of Ten’s place is exchanged to ten blocks of One’s place. Then, the single block becomes inactive which is shown by an X mark given to the block in the left column shown in Figure 2. This causes the ten blocks to become active which is shown by the X mark being removed from the blocks in the right column. The ten blocks are moved to One’s place of the minuend, and then two of them are put into the boxes in One’s place of the subtrahend. Finally, the remaining blocks are moved to One’s place of the answer, as shown in Figure 2.

2.2 Visualization of Bugs

We try to visualize the bugs in subtraction procedures by mapping them to irregular treatments of the blocks and boxes in the block-box model. For example, incorrect subtraction “15-7=12” includes a typical bug called “smaller from larger”. Visualization of this bug with the block-box model is shown in Figure 3. The two empty boxes were irregularly moved to One’s place of the answer column. Therefore, the bug is embodied as the irregular movement of the empty boxes.

3. Concluding Remarks

To confirm the effectiveness of the block-box model, we implemented a learning environment based on the model and used the environment in several arithmetic classes in three elementary schools. Based on the results of (1) comparison between the scores of pre-test and post-test and (2) questionnaire for teachers and learners, we concluded that (A) the learning environment was accepted as a learning tool by both the teachers and learners and (B) it is effective to improve learners’ understanding about subtraction. The details of the learning environment and the experiment will be reported in another paper.

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References