Seller Manipulation of Consumer Reviews under Competition

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Abstract
Consumer ratings in online marketplaces not only help consumers learn about the quality of sellers’ products and services, but also shape the competition among those sellers. Some sellers, taking the advantage of anonymity of contributing consumers, forge consumer reviews to boost their own ratings. This research uses a game theoretical model to explore the incentive mechanism of the manipulation of consumer reviews in a competitive environment. By examining the interaction between price competition and review manipulation, this paper shows that although forging consumer reviews can improve their perceived quality, high-quality sellers do not do so because they incur higher marginal costs. Only low-quality sellers fake consumer reviews. However, the manipulation of consumer ratings does not change the rankings of the perceived quality of sellers. This paper also shows how market characteristics, including consumer quality preference and manipulation cost influence the manipulation of consumer reviews.

1. Introduction

Consumer feedback systems are one of the many great innovations of the Internet. These feedback systems offer anonymity and convenience to contributors, and therefore, encourage a more honest and forthcoming sharing of purchase and consumption experience (Sun, et al. 2006). The systems also aggregate and quantify consumer reviews accessible to readers, and hence, turn different strangers’ viewpoints into an accurate numerical measure of business and product quality (Hu, Pavlou and Zhang 2006). It is not surprising that in a recent survey conducted by Nielsen, 68% of respondents indicated that they trusted online consumer reviews (Nielson 2013). For many consumers, it has become the norm to read online product reviews posted by their peers prior to purchase (BrightLocal 2014). Nevertheless, the anonymity of feedback contributors also permits some sellers to manipulate consumer reviews to their advantage. In the past, employees of a company were instructed to post favorable reviews of their products (CBC 2014), or unfavorable comments about their competitors (e.g., as done by the former CEO of Wholefoods [Reuters 2007]). Some companies also offer incentives to entice consumers to leave positive reviews (McGlaun 2009). Gradually, a market of fake consumer reviews has been formed, where sellers can pay hired guns to forge consumer opinions online (Shaffer 2013). A lawsuit recently settled between the Attorney General of New York and nineteen companies that paid for fake consumer reviews revealed just the tip of iceberg of such a practice (Consumer Reports 2013). Several empirical studies (e.g., Luca and Zervas 2013; Mayzlin, Dover and Chevalier 2013) also found that the use of fake consumer reviews is widespread in online marketplaces.

Given the prevalence of the manipulation of consumer reviews in competitive marketplaces, our understanding of the theoretical implications of such a practice on competition and its pertinent influencing factors is quite limited. In this paper, we model the sellers’ decisions of manipulation of consumer reviews on a competitive marketplace, and intend to answer the following research questions:

What type of sellers manipulates online consumer reviews?

How such manipulation influences the ranking of the perceived quality of sellers?

What factors contribute to the degree of consumer review manipulation?

How market price is influenced under consumer review manipulation?

We find that only low-quality sellers fake consumer reviews. High-quality sellers do not manipulate consumer reviews because they gain little from such a practice. However, the manipulation of consumer ratings does not change the rankings of the perceived quality of any pair of sellers. In addition, we also identify three market characteristics, namely, consumer quality preference, manipulation cost, and marketplace commission fee influence the manipulation of consumer reviews. Finally, when low-quality sellers manipulate their ratings, high-quality sellers have to lower their price, benefiting those consumers who purchase from them.

Below we review relevant literature on the manipulation of consumer opinions, and then we
present our analysis and discussion in the following sections. Proofs of findings are in the appendices.

2. Literature Review

Our paper is closely related to two seminar studies on the strategic manipulation of online consumer opinions. Dellarocas (2006) investigates the information transmission of product quality by a monopolistic seller through distorted consumer reviews. He finds that a higher degree of manipulation of consumer opinions benefits both firms and consumers. Mazylin (2006) studies firms’ manipulation strategies in a duopoly setting. In the presence of information asymmetry in product quality, firms are likely to promote inferior products more.

Our model comes at this issue from a different perspective. We consider manipulation of consumer reviews in an online marketplace—the most popular business infrastructure on the Internet, where multiple sellers compete with each other. This realistic setting allows us to examine the decisions behind manipulating consumer reviews by heterogeneous sellers. We find that some sellers never manipulate consumer opinions, regardless of the extent to which other sellers do.

Several studies provide strong empirical evidence that firms manipulate consumer reviews online. Mayzlin, et al. (2013) compare consumer reviews of the same hotels on Expedia.com where only genuine buyers can review, and onTripadvisor.com where anyone can comment. They find that for hotels that have a high incentive to manipulate consumer opinions, they receive more positive reviews on Tripadvisor than on Expedia, and their competitors receive more negative reviews on Tripadvisor than on Expedia. Hence, they indirectly find evidence of the manipulation of consumer opinions by some hotels in a competitive environment. Luca and Zervas (2013) study consumer reviews on Yelp.com, a major consumer review website for restaurants, and identify that roughly 16% of restaurant reviews on Yelp are fake. In addition, they also find that weak sellers are more likely to manipulate consumer opinions. This result is consistent with our theoretical findings.

3. Model Setup

We consider a marketplace where \( n \) sellers sell a product of experiential nature. The sellers compete with each other in an infinite repeated-game setting.

Sellers’ quality measure

Each seller sells one type of product. A product has two possible states during consumption: it either functions well, or not. When a product functions well, it provides a constant positive utility \( a \) to a consumer; zero utility when it does not. Denote \( \theta_i \) as the probability that the product of Seller \( i \) functions well. \( \theta_i \) essentially measures Seller \( i \)’s product quality. The concept of the product can also extend to services such as shipping and delivery. In this case, \( \theta_i \) becomes the probability that Seller \( i \) provides satisfactory product performance to a consumer. \( \theta_i \) is seller-specific and holds constant throughout the model.

The rating system of product quality and consumer-perceived quality

The marketplace maintains a consumer feedback system that freely provides potential consumers with an average rating score for each seller based on consumer reviews contributed in the previous period. Our setting highlights the critical importance of fresh consumer reviews on purchase decisions, as evidenced in a recent consumer survey (Anderson 2014). All major online marketplaces such as Amazon, eBay and Alibaba (Taobao) feature recent consumer ratings.

In each period, a cohort of consumers enters the marketplace without knowing sellers’ product quality. They rely on the average rating score from the feedback system to make their purchase decisions. Denotes \( \theta_{i,t}^p \) as the consumer-perceived quality of Seller \( i \) at Period \( t \). We let \( \theta_{i,t}^p \) equal to the average rating score of Seller \( i \).

Those consumers who eventually purchase a product are informed hereafter, and a fraction of them will report the state of their products to the feedback system. For tractability, we apply a unidimensional rating system where consumers rate the seller either “1” (i.e., contribute a positive review) if the product functions, or “0” if not (i.e., contribute a negative review). Based on those ratings, the feedback system computes and reports the average rating score, a value between zero and one, for each seller. When the fraction of contributing consumers who purchased functioning products equals to that of those who purchased malfunctioning products, the average rating score of a seller is equal to its true quality, i.e., the true probability that the seller’s product functions (we relax this assumption in the extension). In our repeated-game setting, those newly-computed average rating scores will then be used by the next cohort of consumers as their perceived quality in the next period.
Seller’s manipulation of consumer ratings

At the beginning of each period, a seller may boost its consumer-perceived quality in that period by purchasing fake positive consumer reviews (i.e., rating = “1”) to raise its own average rating score. Based on empirical observations, we assume that each fake consumer review is sold at a constant price $c_m$. Those fabricated reviews are deceptive enough that consumers will not suspect their authenticity.

Let $s_{i,t}$ be the total number of consumers who purchase from Seller $i$ at Period $t$, and let $h$ be the fraction of consumers who contribute their reviews to the marketplace. If Seller $i$ purchases $m_{i,t}$ fake positive reviews, its average rating score becomes $\theta_{i,t} = \frac{h s_{i,t} + m_{i,t}}{s_{i,t} + m_{i,t}}$. Clearly, when $m_{i,t} = 0$, $\theta_{i,t} = \theta_{i,t}^p$.

We acknowledge that under some conditions sellers may also forge negative reviews to smear their competitors (e.g., Mayzlin, et al. 2013). Nevertheless, our model excludes such a case for the following reasons. First, a seller gains little by doing so in a marketplace with many competing sellers. Unless the seller smears everyone, other sellers who are not smeared can free ride on this effort and become as competitive as the smearing seller. Second, many marketplaces allow buyers to rate only the sellers from whom they purchase. It is far more costly and difficult for a seller to have a real purchase from a competing seller and then leave a negative review (with the help of a third-party) than to fabricate a purchase of its own product with a positive review.

4. The Preliminary Analysis

We first derive the competitive equilibrium on the market. Without loss of generality, we normalize the mass of consumers to one. This one representative consumer’s utility function $u_i$ at Period $t$ is specified as in Daughety and Reinganum (2008).

$$u_i(s_{1,t}, s_{2,t}, \ldots, s_{n,t}) = \sum_{t=1}^{n} \alpha s_{i,t} - \frac{1}{2} \sum_{t=1}^{n} s_{i,t}^2 + \sum_{t=1}^{n} \sum_{j \neq i} \gamma s_{i,t} s_{j,t} - \sum_{t=1}^{n} \pi_{j,t} s_{j,t}$$

where, $\alpha$ is the utility the consumer receives when the product functions; $\gamma$ represents the substitute effect between the products from different sellers; $\pi_{j,t}$ is Seller $i$’s price at Period $t$.

Since the consumer cannot observe each seller’s true quality prior to purchase, the consumer makes the purchase decision based on those sellers’ average rating scores. Hence, the representative consumer optimizes her consumption choices based on consumer-perceived qualities $\{\theta_{i,t}^p\}$, instead of true quality $\{\theta_i\}$.

$$\max_{s_{1,t}, s_{2,t}, \ldots, s_{n,t}} u_i(s_{1,t}, s_{2,t}, \ldots, s_{n,t}) = \sum_{t=1}^{n} \alpha s_{i,t}^2 - \frac{1}{2} \sum_{t=1}^{n} s_{i,t}^2 + \sum_{t=1}^{n} \sum_{j \neq i} \gamma s_{i,t} s_{j,t} - \sum_{t=1}^{n} \pi_{j,t} s_{j,t}$$

The utility optimization leads to the following standard result of the demand function, as shown in Daughety and Reinganum (2008).

**Lemma 1.** The demand function for seller $i$ at Period $t$ is as follows:

$$s_{i,t} = \frac{1}{1-\gamma} \left[ \alpha \theta_{i,t} - \pi_{i,t} - \gamma \left( \alpha \sum_{t=1}^{n} \theta_{j,t} - \sum_{t=1}^{n} \pi_{j,t} \right) \right]$$

for $i = 1, \ldots, n$. (3)

**Seller competition in an infinitely repeated game**

The $n$ sellers in the marketplace compete with each other in an infinitely repeated game. At the beginning of each period, the sellers first simultaneously consider manipulating their ratings $\{\theta_{i,t}\}$ by determining the amount of deceptive reviews $\{m_{i,t}\}$ to purchase. $m_{i,t} \geq 0$, for $i = 1, 2, \ldots, n$. And then the sellers simultaneously set the prices $\{\pi_{i,t}\}$ for $i = 1, 2, \ldots, n$, to compete with each other. At each period, the marketplace charges each seller a variable amount fee that is $\beta$ percentage of the seller’s sales. For mathematical tractability, we use a constant time discount factor $\delta$ for all sellers.

Seller $i$’s total profit over an infinite time horizon is

$$\pi_i\{m_{i,t}, \theta_{i,t}\} = \sum_{t=1}^{\infty} \delta^t \left[ (1-\beta)\pi_{i,t}\theta_{i,t} - c_m m_{i,t} \right]$$. (4)

5. Steady-State Equilibrium

In the repeated game, a seller’s sales in one period determines the amount of the truthful consumer reviews it receives in the next period, which in turn, affects the seller’s consumer-perceived quality and

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1 In order to simplify the model, we assume the seller’s product costs are zero.
sales in every subsequent period. Therefore, the impact propagates across all future periods. A forward-looking seller will account for such impact when determining its consumer rating manipulation $m_{i,t}$ and pricing $p_{i,t}$. Specifically, considering Period $t = T$, Seller $i$’s optimal rating manipulation strategy $m_{i,T}$ and pricing $p_{i,T}$ are defined as follows.

Based on the first-order condition of $\pi_{i,w.r.t. m_{i,T}}$, we have

$$-c_m + (1 - \beta) \left\{ p_{i,T} \frac{\partial \delta_{i,T}}{\partial m_{i,T}} \phi_{i,T} + \sum_{t=T}^{\infty} \int_{m_{i,T}}^{\infty} p_{i,t+1} \frac{\partial \delta_{i,t+1}}{\partial m_{i,t+1}} \phi_{i,t+1} \right\} = 0$$

And $m_{i,T} = 0$ if $\frac{\partial m_{i,t}}{\partial m_{i,t}} \leq 0$.

Similarly, the first-order condition of $\pi_{i,w.r.t. p_{i,T}}$ yields

$$s_{i,T} + \frac{\partial \delta_{i,T}}{\partial p_{i,T}} \phi_{i,T} + \sum_{t=T}^{\infty} \int_{p_{i,T}}^{\infty} p_{i,t+1} \frac{\partial \delta_{i,t+1}}{\partial p_{i,t+1}} \phi_{i,t+1} = 0$$

In this paper, we confine our analysis to the steady state where each seller replicates its strategies as those in the previous period. In the following analysis, we drop the subscript $t$ for $m_{i,t}$, $p_{i,t}$, $s_{i,t}$, and $\theta_{i,t}$. We then simplify the two first-order conditions given above.

Lemma 2. In the steady state, for $i = 1, 2, \ldots, n$, Seller $i$’s rating manipulation strategy $m_i$ and pricing strategy $p_i$ are defined by (7) and (8) as follows.

$$-c_m + (1 - \beta) p_i \frac{\alpha}{1 - \gamma \varepsilon^m} \frac{h_i (1 - \theta)}{\sqrt{\alpha (1 - \beta)}} = 0$$

$$s_i = p_i \frac{1}{1 - \gamma \varepsilon^m} \frac{h_i (1 - \theta)}{\sqrt{\alpha (1 - \beta)}} = 0$$

Seller types and consumer review manipulation

We solve for the consumer perceived quality for each seller in the steady-state equilibrium. Based on this finding, we present the following proposition on which type of sellers manipulates consumer ratings.

Proposition 1. In the steady state, the sellers with true quality $\theta_i \geq \theta \equiv 1 - \frac{h_i}{\alpha (1 - \beta)}$ do not manipulate their ratings, i.e., $\theta_i^{\text{true}} = \theta_i$; other sellers purchase deceptive positive reviews to raise their consumer-perceived quality to $\theta_i^{\text{true}} = 1 - \frac{h_i}{\alpha (1 - \beta)}$.

Proposition 1 shows that only sellers of a relatively low quality buy deceptive reviews to raise their ratings; high-quality sellers do not manipulate their ratings. This finding is consistent with the empirical observations. The rationale is that, a low-quality seller sells fewer products than a high-quality seller, and hence, only have a few negative reviews (in absolute volume) to “bury” with fake positive reviews. Hence, the low-quality seller can significantly boost its perceived quality with a small number of fake positive reviews. In comparison, high-quality sellers attract a large amount of sales and receive a substantial number of truthful reviews. Consequently, in order to meaningfully boost their perceived quality, they need to purchase a greater number of fake reviews than low-quality sellers. When the true quality exceeds a certain threshold (i.e., $\theta$), the marginal cost of the raised average rating by forging consumer reviews exceeds the increased profit from the raised average rating score, and it is no longer worthwhile to do so.

The analysis also suggests that, even if a marketplace does not take any disciplinary actions against sellers who manipulate consumer reviews, high-quality sellers still act “honestly” despite the fact that their competitive advantage in quality is shrinking, as suggested in the following corollary.

Corollary 1. In the steady state, for any two sellers with true quality $\theta_i$ and $\theta_j$, if $\theta_i > \theta_j$, then $\theta_i^{\text{true}} > \theta_j^{\text{true}}$. However, $\theta_i^{\text{true}} - \theta_j^{\text{true}} \leq \theta_i - \theta_j$.

Corollary 1 suggests that, rating manipulation does not change the relative rank of the seller’s perceived quality. Why will not a low-quality seller manipulate its perceived quality higher than that of a high-quality seller? Intuitively, if a low-quality seller boosts its perceived quality higher than that of a high-quality seller, it sells more products than the high-quality seller. Consequently, the manipulation backfires in the next period, as the low-quality seller will receive a great number of truthful negative reviews, much more than it can “bury” with fake reviews. The manipulation will be more costly if the seller intervenes, or its perceived quality will decrease if it purchases insufficient number of forged positive reviews to offset authentic negative reviews. Either way the low-quality seller’s future profit decreases. Thus, in the steady-state equilibrium, the low-quality seller would rather see its perceived quality lower than that of a high-quality seller, which it can profitably maintain with forged reviews. Therefore, rating manipulation reduces only the difference among the
sellers’ perceived quality, but does not alter the relative ranks of quality perception.

The comparative statics also reveal the impact of some factors on the distortion of perceived quality by fake reviews, as shown in Corollary 2.

**Corollary 2.**

\[
\frac{\partial(\theta_i^p - \theta_i)}{\partial \alpha} < 0, \quad \frac{\partial(\theta_i^p - \theta_i)}{\partial \beta} < 0, \quad \frac{\partial(\theta_i^p - \theta_i)}{\partial \alpha} > 0.
\]

Corollary 2 shows that when the fake consumer review is more expensive (i.e., a greater \(c_m\)), or when a higher fraction of consumers provide truthful reviews (i.e., a greater \(h\)), the distortion of the seller’s perceived quality will be smaller. The rationale is that an increase in \(c_m\) or \(h\) reduces the gain of rating manipulation, and hence dissuades such a practice. On the other hand, the greater the importance of the quality (i.e., a greater \(\alpha\)), the greater becomes the distortion of the sellers’ perceived quality. Intuitively, when consumers care more about product quality, the more effort some sellers put into raising their ratings, and hence results in more rating manipulation.

**Equilibrium price and consumer review manipulation**

Now we derive the equilibrium price and consumer review manipulation for as seller with a true quality \(\theta_i\).

**Proposition 2.** In the steady state, the amount of deceptive reviews purchased by each seller and its equilibrium price are determined by the market competition. \(p_i^e\) and \(m_i^e\) are defined as follows:

\[
p_i^e = \frac{1 - \gamma + n}{2 - 2\gamma + n} a \theta_i^p + \frac{\delta \gamma (1 - \gamma)}{2 - 2\gamma + n} (\theta_i^p - \theta_i) \frac{1 - \theta_i^p}{1 - \theta_i} + \frac{\gamma}{2 - 2\gamma + n} \sum_{j=1}^{n} \left( \frac{\theta_j^p - \theta_j}{1 - \theta_j} - \frac{\theta_j^p - \theta_j}{1 - \theta_j} \right);
\]

\[
m_i^e = \frac{\alpha h}{2 - 2\gamma + n} \frac{\theta_i^p - \theta_i}{1 - \theta_i^p} - \frac{\delta \alpha h}{2 - 2\gamma + n} \frac{\theta_i^p - \theta_i}{1 - \theta_i} + \frac{\gamma}{(2 - 2\gamma + n)(1 - \gamma + n)} \sum_{j=1}^{n} \left( \theta_j^p - \theta_j \right) \frac{1 - \theta_j}{1 - \theta_j}
\]

where, \(\theta_i^p = \theta_i - \sqrt{\frac{h \alpha c_m (1 - \theta_i)}{\alpha (1 - \theta_i)}}\), and \(\theta_i^p = 1 - \sqrt{\frac{h \alpha c_m (1 - \theta_i)}{\alpha (1 - \theta_i)}}\).

Proposition 2 shows that the sellers’ prices and purchased fake reviews are affected by the market competition. Clearly, for those sellers who do not manipulate consumer reviews, \(\theta_i^{bps} = \theta_i\), \(m_i^{bps} = 0\). Next, we provide three findings based on Proposition 2.

**Corollary 3.** In equilibrium, the manipulation of consumer reviews by low-quality sellers decreases the price of high-quality sellers who do not manipulate their reviews.

Corollary 3 suggests that, since high-quality sellers do not manipulate their ratings to maintain the original advantage over low-quality sellers, consumer may benefit from the buying from high-quality sellers.

**6. Extension: Negativity Bias of Review Contribution**

The empirical literature on consumer satisfaction and WOM finds that consumers in different consumption states do not contribute their opinions in the same way. In many situations, dissatisfied consumers are more likely to voice their opinions than would satisfied ones (Anderson 1998, Bougie, Pieters and Zeelenberg 2003; Richins 1983). This phenomenon has been used to justify the manipulation of consumer reviews. When consumers disproportionately contribute negative ratings, sellers may desire to “correct” them by supplementing some positive ones, in the fear that negative ratings are more influential (Gershoff, Mukherjee, and Mukhopadhyay 2003). In the following analysis, we show the opposite may be true.

We modify the expression for average rating score by accounting for the negativity bias of review contribution.

\[
\theta_i^{bps} = \frac{h \theta_i + m_i}{h \theta_i + \mu h (1 - \theta_i) + m_i}.
\]

Where, \(h\) is the fraction of satisfied consumers who contribute positive ratings, and \(\mu h\) is the fraction of dissatisfied consumers who contribute negative ratings, \(0 < h < 1, \mu > 1\).

On substitution of \(\theta_i^{bps}\) into seller i’s profit function given in Eq. (4), and applying the same derivation through FOCs, we obtain the new perceived quality, denoted as \(\theta_i^{bps}\), given in the following proposition.

**Proposition 3.** In the steady-state equilibrium, seller i’s perceived quality is given by

\[
\theta_i^{bps} = 1 - \sqrt{\frac{\mu h c_m (1 - \theta_i)}{(1 - \theta_i) \alpha}}.
\]

Clearly, \(\theta_i^{bps} < \theta_i^{bps}\), suggesting that the existence of negativity bias of review contribution reduces the
magnitude of the perceived quality for sellers who consider manipulating consumer reviews. In other words, the nativity bias of review contribution reduces the gain of consumer review manipulation.

7. Conclusion

In this paper, we analyze sellers’ decisions for or against manipulation of consumer reviews in a competitive marketplace.

We find that only low-quality sellers manipulate consumer ratings, and high-quality sellers do not. Such manipulation will not change the rank of perceived quality among sellers (i.e., a low-quality seller will not be perceived to have higher quality than a high-quality seller). However, the difference of perceived quality of those sellers decreases as a result of the manipulation of consumer ratings. A direct result is a more intense competition among those sellers who are vertically differentiated.

We also identify several factors that influence sellers’ manipulation decisions. Those factors include consumer valuation, the cost of manipulation, and the fraction of contributing consumers. Managerially, by influencing the value of those parameters, a marketplace can influence sellers’ manipulative activities.

In addition, we find that sellers gain less in the presence of negativity bias of the review contribution. This is because the negativity bias decreases the gap of perceived qualities between a low-quality seller and a high-quality seller.

Our findings pave the way for future research on regulatory practices. In particular, studies on how platforms can reduce the degree of consumer review manipulation bear both theoretical and managerial importance. In addition, behavioral research on whether consumers are aware of fake reviews and how the knowledge affects their decision making is also important to quantify the possible impact of review manipulation on consumer welfare, and is worth pursuing.

8. Appendix

Proof to Lemma 1:

Take the first order condition of (2) with respect to $s_{i,t}$, we have reversed demand functions $p_t = \alpha \theta^p_t - s_{i,t} - \gamma \sum_{(j \neq i)} s_{j,t}$ for $i = 1, \ldots n$.

Sum up the $n$ reversed demand functions, we have $\sum_{i=1}^n p_t = \alpha \sum_{i=1}^n \theta^p_t - \gamma \sum_{i=1}^n s_{i,t}$, or

$\sum_{i=1}^n \theta^p_t = \frac{1}{1 - (1 - \gamma)n} \left( \alpha \sum_{i=1}^n \theta^p_t - \sum_{i=1}^n p_t \right)$.

Substitute the above equation into the reversed demand functions, we have

$s_{i,t} = \frac{1}{1 - \gamma} \left[ \alpha \theta^p_t - p_t - \gamma \left( \frac{1}{1 - \gamma} \right) \frac{\sum_{i=1}^n \theta^p_t - \sum_{i=1}^n p_t}{\gamma (1 - \gamma)n} \right]$ for $i = 1, \ldots n$.

Q.E.D.

Proof to Lemma 2:

As $s_{i,t} = \frac{1}{1 - \gamma} \left[ \alpha \theta^p_t - p_t - \gamma \left( \frac{1}{1 - \gamma} \right) \frac{\sum_{i=1}^n \theta^p_t - \sum_{i=1}^n p_t}{\gamma (1 - \gamma)n} \right]$ for $i = 1, \ldots n$, in the steady state equilibrium,

$\frac{\partial s_{i,t}}{\partial p_t} = \frac{\alpha \theta^p_t - p_t - \gamma \left( \frac{1}{1 - \gamma} \right) \frac{\sum_{i=1}^n \theta^p_t - \sum_{i=1}^n p_t}{\gamma (1 - \gamma)n}}{(1 - \gamma)n^{\frac{1}{2}}} = \alpha \left( \frac{1}{1 - \gamma} \right) \frac{\gamma (1 - \gamma)n}{(1 - \gamma)n^{\frac{1}{2}}} = \alpha \frac{\gamma (1 - \gamma)n}{(1 - \gamma)n^{\frac{1}{2}}}$.

Similarly, we have

$\frac{\partial s_{i,t}}{\partial \theta^p_t} = \frac{\gamma (1 - \gamma)n}{(1 - \gamma)n^{\frac{1}{2}}} \frac{\gamma (1 - \gamma)n}{(1 - \gamma)n^{\frac{1}{2}}} = \alpha \frac{1}{\gamma (1 - \gamma)n^{\frac{1}{2}}}$.

Substitute the above derivatives back, we have

$-c_m + (1 - \beta) \left[ p_t \frac{\alpha h_s (1 - \theta^p) h_s (1 - \theta^p)}{1 - (1 - \gamma)n^{\frac{1}{2}}} \right] + \delta \beta \left[ p_t \frac{h_s (1 - \theta^p) h_s (1 - \theta^p)}{1 - (1 - \gamma)n^{\frac{1}{2}}} \right] + \delta^2 p_t \left( \frac{\alpha h_s (1 - \theta^p)}{1 - (1 - \gamma)n^{\frac{1}{2}}} \right) + \delta^3 p_t \left( \frac{\alpha h_s (1 - \theta^p)}{1 - (1 - \gamma)n^{\frac{1}{2}}} \right) + \cdots = 0$

or

$s_t = p_t \frac{\alpha h_s (1 - \theta^p) h_s (1 - \theta^p)}{1 - (1 - \gamma)n^{\frac{1}{2}}} + \delta \beta \left[ p_t \frac{h_s (1 - \theta^p) h_s (1 - \theta^p)}{1 - (1 - \gamma)n^{\frac{1}{2}}} \right] + \delta^2 p_t \left( \frac{\alpha h_s (1 - \theta^p)}{1 - (1 - \gamma)n^{\frac{1}{2}}} \right) + \delta^3 p_t \left( \frac{\alpha h_s (1 - \theta^p)}{1 - (1 - \gamma)n^{\frac{1}{2}}} \right) + \cdots = 0$.

Q.E.D.

Proof to Proposition 1:

From (9), $p_t = \frac{\delta}{1 - \gamma}$, substitute into (8), we have

$c_m = \frac{\delta}{(1 - \gamma)}$.
In the steady state, \( \theta_i^p = \frac{h_s^* \delta_i + \alpha \eta_i}{h_s^* \gamma + \alpha \eta_i} = 1 - \frac{h_s^*(1-\theta_j)}{h_s^*(1-\theta_j)} = 1 - \sqrt{\frac{c_{p_i}^{\alpha h_i(1-\theta_j)}}{\alpha(1-\theta_j)}} \)

As \( \theta_i^p > \theta_j \) when \( m_1 > 0 \), we have \( 1 - \sqrt{\frac{c_{p_i}^{\alpha h_i(1-\theta_j)}}{\alpha(1-\theta_j)}} > \theta_i \), or \( \theta_i < 1 - \sqrt{\frac{c_{p_i}^{\alpha h_i(1-\theta_j)}}{\alpha(1-\theta_j)}} \).

According to Proposition 1, Seller dy is \( S_2 \). Thus,

Q.E.D.

\[ A \]

Proof to Corollary 2:

Case 1. \( \theta > \theta_1 > \theta_j \). Both sellers manipulate consumer ratings according to Proposition 1. Note that \( \frac{\delta \theta_i^{p}}{\alpha_1} = \frac{1}{2} \sqrt{\frac{c_{p_i}^{\alpha h_i(1-\theta_j)}}{\alpha(1-\theta_j)}} > 0 \), and hence, \( \theta_i^{p} > \theta_j^{p} \).

Case 2. \( \theta_1 > \theta > \theta_j \). According to Proposition 1, Seller i does not manipulate its consumer rating: \( \theta_i^{p} = \theta_i \), and Seller j does: \( \theta_j^{p} = 1 - \sqrt{\frac{c_{p_j}^{\alpha h_j(1-\theta_j)}}{\alpha(1-\theta_j)}} \). Thus, \( \theta_i^{p} > \theta_j^{p} \).

Case 3. \( \theta > \theta_1 > \theta_j \). Neither seller manipulates consumer ratings. Their perceived quality is just their true quality. Q.E.D.

**Proof to Corollary 2** is straightforward by taking the respective partial derivatives.
\[ p_i = \frac{1-\gamma+\gamma}{2-2\gamma+\gamma} \alpha \theta_i + \frac{\delta \alpha(1-\gamma)\theta_i - \theta_i}{1-\theta_i} + \frac{\gamma}{2-2\gamma+2} \left[ \delta \sum_{i=1}^{n} (\theta_i^p - \theta_i) \right] \]

\[ \theta_i \leq h_i \] 

\[ \beta \] 

\[ \psi = 1-\gamma+\gamma \]

\[ \alpha \]

\[ \mu \]

\[ \bar{\delta} \]

\[ \bar{\delta} \]
Since $e_i^{*}$, we have $S$ since $e_i^{*}$.

The comparison of $\theta_i^{np*}$ and $\theta_i^{*}$ is straightforward for $\mu > 1$.

9. References


