Electric Energy Storage Arbitrage in Electric Power Markets with Wind Uncertainty

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Abstract

Electric energy storage (EES) can bring many benefits to power grids and to electric power markets. However, as private investors, they make profits mainly by price arbitrage. In this paper, the impacts of EES arbitrage in electric power energy markets are studied. The charging or discharging activities for large-scale EES may have huge impacts on real-time locational marginal prices (LMPs). An optimization model is proposed to maximize the arbitrager’s profits with the consideration of the charging and discharging impacts on the LMPs. Three charging policies are discussed to handle the uncertainty that the intermittent wind penetration brings to the power grid. Finally, an IEEE 118-bus system case study is carried out to evaluate the model and the different charging policies.

1. Introduction

The electric power industry is an $840-billion industry in the U.S., which represents approximately 3 percent of real gross domestic product [1]. Each year, electric utilities spend roughly 200 billion in their planning and operations [2]. Unlike other commodities in markets, the demand of electric power is considered to be very close to perfectly inelastic. Thus, the imbalance between supply and demand can cause severe issues in power grids. Consequently, one of the most challenging problems in electric power scheduling is the uncertainty. Generally, the uncertainties in electric power scheduling can be categorized as discrete or continuous uncertainties. Discrete uncertainties include the failure of a single generator or transmission line, also known as an N-1 contingency. Continuous uncertainties include the loads and renewable energy variations with respect to their predicted levels. Nowadays, more and more renewable energy resources (mainly wind and solar farms) are being built to provide electric power in order to lower generation costs and reduce environmental pollutions. According to the California Renewables Portfolio Standard (RPS) [3], eligible renewable energy resources in California will be increased to 33% of total production by 2020, compared to the current 12% nationwide renewable generation [4]. While it brings many benefits both economically and environmentally, on the other hand, the high penetration of intermittent renewables also complicates power scheduling due to the uncertain nature of the renewable energy.

With high renewable penetration in the power grid, electric energy storage (EES) is one way to assist handling resource uncertainties [5]. The technologies of EES are widely discussed in literature [5]-[10]. Pumped hydro storage (PHS), which pumps and discharges water from different reservoirs to store and release electric energy, is the most common and well-developed form of energy storage, representing approximately 3% of the world’s total installed power capacity, and 97% of the total storage capacity [6]. PHS is popular for its high round-trip charging efficiency (65-85%), large power capacity (100-1000MW), large energy storage capacity (12+ hours), long life (30-60 years), and low cycle cost; however, PHS is limited by its high capital cost ($100 million - $3 billion), long project lead time (typically ten years), environmental damages, and its unique geographical requirements [6]. Compressed air energy storage (CAES) is another currently suitable technique other than PHS for large-scale EES. CAES compresses the air into large storage reservoirs using cheaper off-peak electric power instead of expensive gas and releases the air for the conventional gas turbine cycles during peak hours [8]. CAES shares many of the same attractive characteristics of PHS, but has significantly lower capital cost and little environmental impact since the storage is underground [6]. Nevertheless, there are only two CAES plants in the world, one in Germany built in 1978 and another in Alabama, U.S. built in 1991 [6]. A variety of batteries have also been developed for EES including, but not limited to, lead-acid, nickel-cadmium, sodium-sulfur, lithium-ion, sodium-nickel-chloride, zinc-bromine, vanadium-redox.
and polysulphide-bromide batteries. Their characteristics are discussed in [5]-[10]. Other technologies such as flywheels, capacitors and supercapacitors, hydrogen storage, superconducting magnetic energy storage (SMES), and solar thermal energy storage have been developed or are under development for various EES purposes [5]-[10].

As for arbitrage in electric power market, EES owners buy the energy during the off-peak hours with low price and sell it in the peak hours with higher price in order to make profits. Therefore, EES devices should have higher power capacity (20+MW is desired) and higher energy storage capacity (4+ hours is desired). Currently, only PHS, CAES, sodium-sulfur battery and lead-acid battery are suitable for arbitrage in the markets.

In addition to responding to the intermittent renewable resources, introducing EES to the power grid can benefit the system in several ways. Large-scale EES was initially studied focusing on the peak load shaving effects. Social welfare can be improved by moving the peak load to the off-peak periods by utilizing EES and, as a result, the electricity price variability is typically reduced. Moreover, introducing EES can defer facility investments [11]. While such benefits exist, such welfare benefits may not be captured by the private investor of EES. As the private investors, they gain profits relying on pricing arbitrage. Since large-scale EES will smooth load and reduce price variance in different periods, the diminished value of arbitrage can be expected, which contradicts the investors’ motivation of investing in EES facilities. Therefore, despite the benefits that EES will bring to power systems, the current market settings give limited incentives for the private investors to build EES facilities. The ownerships of EES facilities, contracts for EES in the markets, and government subsidies have been discussed but not yet implemented [11]. An enhanced market design that is conducive to EES is left for future study.

The remainder of this paper is organized as follows: Section III gives the mathematical formulations. Section IV discusses three charging policies to hedge uncertainty for the arbitrager. Section V carries out a case study on the IEEE 118-bus system and shows the computational results. Finally, Section VI draws the conclusions.

2. Mathematical formulations

In the models in this paper, a power transfer distribution factor (PTDF) based direct current optimal power flow model (DCOPF) is adopted, which is a linear approximation of real power flow with a set of simplifying assumptions [12][13].

2.1. Location selection model

Since most energy markets in the U.S. now have a nodal market settlement structure, the location of EES will play an important role. The EES location selections depend on whether the owner of EES is a vertically-integrated utility or a for-profit private entity. For vertically-integrated utilities, the objective should be to minimize the total costs. Then the EESs are preferred at the locations where the whole system will benefit. On the other hand, in regards to for-profit private entities, the EES facilities are preferred at the locations where the owner can maximize the individual profits by arbitrage. The following model gives the optimal locations for minimizing total costs, the perspective of the vertically-integrated utility. Generally, these locations are not the optimal locations from a for-profit EES owner’s point of view. Arbitrage profits and market allocations will be compared when the EES facility is located at different buses.

\[
\begin{align*}
\text{min} & \quad \sum_{g} C_{p,t} + \sum_{n} B_{n} x_{n} \\
\text{s.t.} & \quad i_{nt} = \sum_{g \in G(n)} P_{gt} + \alpha_{n}^{-} r_{n} - \alpha_{n}^{+} r_{n} - D_{nt} \quad \forall n \in N, t \in T \quad (1) \\
& \quad \sum_{t} i_{nt} = 0 \quad \forall t \in T \quad (2) \\
& \quad -F_{l} \leq \sum_{t} \psi_{nt} i_{nt} \leq F_{l} \forall l \in L, t \in T \quad (3) \\
& \quad 0 \leq P_{gt} \leq P_{g}^{\text{max}} \quad \forall g \in G, t \in T \quad (4) \\
& \quad e_{nt} = e_{nt-1} + a_{n}^{+} r_{nt-1} - a_{n}^{-} r_{nt} \quad \forall n \in N, t \in T \quad (5) \\
& \quad E_{n}^{\text{min}} z_{n} \leq e_{nt} \leq E_{n}^{\text{max}} z_{n} \quad \forall n \in N, t \in T \quad (6) \\
& \quad 0 \leq a_{n}^{-} r_{nt} \leq U_{n} \quad \forall n \in N, t \in T \quad (7) \\
& \quad \sum_{n} z_{n} \leq K \quad (8) \\
& \quad z_{n} \in \{0,1\} \quad \forall n \in N \quad (9)
\end{align*}
\]

Equation (1) is the objective that minimizes the total dispatch costs and investment costs. Equations (2) and (3) restrict the relationship of power injections. When EES is charging, since the efficiency is not 100%, \( \alpha_{n}^{-} / \alpha_{n} \) unit energy is required in order to store \( \alpha_{n}^{+} \) unit energy in EES. In contrast, when EES is discharging \( \alpha_{n}^{+} \) unit energy, only \( \alpha_{n}^{-} \) unit energy can be utilized by the power grid. Equation (4) restricts the transmission line ratings. Equation (5) restricts the generation bounds. Equation (6) describes the EES energy balance relations. Equations (7) and (8) specify the energy and power capacity respectively. Equation (9) restricts the total number of EES facilities that can be built.
2.2. Security-constrained unit commitment and security-constrained economic dispatch

In the day-ahead market (DAM), the operators solve the security constrained unit commitment (SCUC) model to obtain unit commitment solutions. Then, they run the security constrained economic dispatch (SCED) model and calculate the locational marginal prices (LMPs). The basic procedure in DAM dispatch (SCED) model an (SCUC) model to obtain unit commitment solutions. Solve the security constrained unit commitment problems. Since the market model is deterministic based on predicted values.

The SCUC model is described as follows.

\[
\text{min } \sum_t \sum_g \left( C_g^u v_{gt} + C_g^w u_{gt} + C_g p_{gt} \right) \quad (11)
\]

\[
s.t. \quad p_{gt} + r_{gt} \leq p_{gt}^{\max} u_{gt} \quad \forall g \in G, t \in T \quad (12)
\]

\[
p_{gt} \geq p_{gt}^{\min} u_{gt} \quad \forall g \in G, t \in T \quad (13)
\]

\[
w_{nt} \leq W_{nt} \quad \forall n \in N, t \in T \quad (14)
\]

\[
\sum_g r_{gt} \geq 3\% \sum_g D_{nt} + 5\% \sum_g W_{nt} \quad \forall t \in T \quad (15)
\]

\[
p_{gt} - p_{gt,t-1} \leq R_g^u u_{gt} + v_{gt}^u \quad \forall g \in G, t \in T \quad (16)
\]

\[
v_{gt} \geq u_{gt} - u_{gt,t-1} \quad \forall g \in G, t \in T \quad (17)
\]

\[
\sum_{t=t-1}^{t+\Delta t} v_{gt} \leq u_{gt} \quad \forall g \in G, t \in T \quad (18)
\]

\[
\sum_{t=t-1}^{t+\Delta t} v_{gt} \leq 1 - u_{gt} \quad \forall g \in G, t \in T \quad (19)
\]

\[
i_{nt} = \sum_{g \in G(n)} p_{gt} + w_{nt} - D_{nt} \quad (20)
\]

\[
\sum_n i_{nt} = 0 \quad \forall t \in T \quad (21)
\]

\[
-F_l \leq \sum_n \psi_{nt} i_{nt} \leq F_l \quad \forall l \in L, t \in T \quad (22)
\]

\[
u_{gt} \in [0,1] \quad \forall g \in G, t \in T \quad (23)
\]

\[
0 \leq v_{gt} \leq 1 \quad \forall g \in G, t \in T \quad (24)
\]

\[
w_{nt}, r_{gt} \geq 0 \quad \forall N \in N, g \in G, t \in T \quad (25)
\]

The SCUC formulates the problem in 24 periods, where each period is one hour. Equation (11) is the objective to minimize the total costs including startup costs, no-load costs, and dispatch costs. While prior work approximates the generator cost curve as a quadratic function [12], the presented model assumes a piecewise linear formulation, which is consistent with existing market structures. Since the market model is non-convex, uplift payments may be required to ensure a non-confiscatory market. Equations (12) and (13) restrict the generation upper bound and lower bound. When the generator is not committed, i.e., \( u_{gt} = 0 \), the generation \( p_{gt} \) and \( r_{gt} \) is forced to be zero. Equation (14) restricts the wind power injection to be less than the maximum power that the wind farm can provide. Although the marginal cost of the wind power is zero, it is not always the case that dispatching all available wind power is beneficial. Due to transmission congestion and due to ramping limitations, it is at times cheaper to spill wind as opposed to displacing fossil fuel based production. Equation (5) specifies the system reserves in order to handle uncertainties. The ad-hoc 3+5 requirement rule, which requires the reserves to be more than 3% of the total predicted load and 5% of the total predicted wind power, is adopted [15]. Equations (16) and (17) restrict the hourly ramping up and ramping down constraints. Equation (18) specifies the relation between the unit commitment status and the startup status. Equations (19) and (20) impose the minimum up and down time constraints, which states that if a generator is turned on, it cannot be turned down in the next \( UT_g \) hours; similarly, if it is turned off, it cannot be turned on in the next \( DT_g \) hours. Equations (21) and (22) define the bus injection. Equation (23) imposes the transmission line rating. The generation shift factors are results from Kirchhoff’s laws. Equations (24)-(26) specify the variable restrictions. The startup variable is relaxed to be continuous but is guaranteed to be either 0 or 1, since (18)-(20) form the facet-defining constraint of \( u, v \) projection [16]. The SCED is described as follows.

\[
\text{min } \sum_t \sum_g C_g p_{gt} \quad (27)
\]

\[
s.t. \quad p_{gt} \leq p_{gt}^{\max} u_{gt} \quad \forall g \in G, t \in T \quad (28)
\]

\[
0 \leq w_{nt} \leq W_{nt} \quad \forall n \in N, t \in T \quad (29)
\]

\[
\sum_{g \in G(n)} p_{gt} + w_{nt} - i_{nt} = D_{nt} \quad \forall n \in N, t \in T \quad (30)
\]

\[
\sum_n i_{nt} = 0 \quad \forall t \in T \quad (31)
\]

\[
-F_l \leq \sum_n \psi_{nt} i_{nt} \leq F_l \quad \forall l \in L, t \in T \quad (32)
\]

\[
p_{gt} - p_{gt,t-1} \leq R_g^u u_{gt} + v_{gt}^u \quad \forall g \in G, t \in T \quad (33)
\]

\[
0 \leq v_{gt} \leq 1 \quad \forall g \in G, t \in T \quad (34)
\]

In the SCED with network constraints model, the objective (27) only minimizes the dispatching costs. Equations (28)-(34) are the same as (12)-(14), (21)-(25), (16)-(17), with the exceptions: one, for simplicity, the real-time market is modeled as an energy-only market; two, the unit commitment variables and the startup variables are fixed. The LMPs are the dual variables of equation (30).
2.3. Arbitrage model

In the following arbitrage models, it is assumed that the decision maker is a private investor who maximizes their arbitrage profits. EES is located at a single location \( k \).

Prior work frequently assumes that the arbitrageurs are price-takers, i.e., the participation of EES has little impact on the real-time LMPs and charging or discharging decisions are made based on the predicted LMP, \([11],[17]-[18]\). Due to the generally small size of EES, this assumption is valid. The EES owner runs the following model to maximize profits.

\[
\begin{align*}
\min & \quad \sum_t \lambda_{kt}(\alpha_t^c a_k - \alpha_t^p / \alpha_k) \\
\text{s.t.} & \\
& e_t = e_{t-1} + \alpha_t^c - \alpha_t^p \quad \forall t \in T \\
& E_{kt}^\text{min} \leq E_t \leq E_{kt}^\text{max} \quad \forall t \in T \\
& 0 \leq \alpha_t^c, \alpha_t^p \leq U_k \quad \forall t \in T
\end{align*}
\]

Equation (35) maximizes the profit with the predicted LMPs, \( \lambda_{kt} \), as a fixed constant. Equation (36) restricts the energy balance constraint and (37)-(38) specify the energy and power capacities respectively.

The price-taker assumption may not be true if the charging and discharging have a strong influence on its LMP. In this paper, the focus is on how the LMP changes induced by the charging or discharging may affect the optimal schedule for EES. Actually, the LMPs \( \lambda_{kt} \) become a function of charging or discharging decisions \( \alpha_t^c, \alpha_t^p \), i.e., the \( \lambda_{kt} \) will change as the \( \alpha_t^c, \alpha_t^p \) changes. Therefore, EES owner needs to consider this relation while making their charging decision to maximize profits. The following arbitrage model solves the optimization problem and returns the optimal charging and discharging decisions. The model is described in six parts: objective, primal constraints, dual constraints, strong duality constraint, linearizing constraints, and EES constraints. The model is described as follows.

**Objective:**

\[
\begin{align*}
\max \quad & \sum_t \sum_j (\eta_{jt} a_k - \eta_{jt}^+/\alpha_k) \\
\end{align*}
\]

Equation (39) is the objective to maximize the investor’s profits. In (35), when \( \lambda_{kt} \) is the function of \( \alpha_t^c, \alpha_t^p \), \( \lambda_{kt} \) also becomes the variable in the optimization problem. Then \( \lambda_{kt} \alpha_t^c \) and \( \lambda_{kt} \alpha_t^p \) become bi-linear terms, which causes the formulation to be non-linear and non-convex. Variable \( \eta_{jt} \) is introduced to linearize the bi-linear term. First, we assume that charging and discharging can be divided into several segments and each segment has to be fully charged or discharged. For instance, if an EES has power capacity of 100MW, then it can charge or discharge from 10MW, 20MW, 30MW, and up to 100MW. Each segment is 10MW in this case. Then the decision becomes whether to charge or discharge the first segment. The decision involves binary variables and when the bi-linear term consists of one continuous variable and one binary variable, the big-M method can be used to linearize the bi-linear term.

**Primal constraints:**

\[
\begin{align*}
p_{g_t}^\text{min} \tilde{u}_{gt} & \leq p_{gt} \leq p_{g_t}^\text{max} \tilde{u}_{gt} & \forall g, t \in T \quad (40) \\
i_{nt} = \sum_{g \in \mathcal{G}(n)} p_{gt} + w_{nt} - D_{nt} & + Q_k a_k (\sum_j \bar{x}_{jt}^c) - Q_k / \alpha_k (\sum_j \bar{x}_{jt}^p) \quad (41) \\
n = k, \forall t \in T & \quad (41) \\
i_{nt} = \sum_{g \in \mathcal{G}(n)} p_{gt} + w_{nt} - D_{nt} & \forall n \in N, n \neq k, t \in T \quad (42) \\
- F_t \leq \sum_n \psi_{nt} i_{nt} & \leq F_t \quad \forall t \in L, t \in T \quad (43) \\
\sum_n i_{nt} = 0 & \quad \forall t \in T \quad (44) \\
0 \leq w_{nt} & \leq W_{nt} \quad \forall n \in N, t \in T \quad (45) \\
p_{gt} - P_{g,t-1} & \leq R_t^{hr} \tilde{u}_{gt} + R_t^{sv} \bar{v}_{gt} \quad \forall g \in G, t \in T \quad (46) \\
p_{g,t-1} - p_{gt} & \leq R_t^{hr} \tilde{u}_{gt} + R_t^{sv} (\bar{v}_{gt} - \tilde{u}_{gt} + \tilde{u}_{g,t-1}) \quad \forall g \in G, t \in T \quad (47)
\end{align*}
\]

Equations (40)-(47) are the SCED primal constraints, the repeat of equation (28)-(34). The only change is (41), which includes the charging and discharging events at bus \( k \).

**Dual constraints:**

\[
\begin{align*}
- \phi_{gt}^+ + \phi_{gt}^+ + \lambda_{g(t)} & = 0 & \forall g \in G, t \in T \quad (48) \\
- \phi_{gt}^+ + \rho_{g,t+1}^+ - \rho_{g,t+1}^- & + \rho_{gt}^+ = C_g & \forall g \in G, t \in T \quad (49) \\
\lambda_{nt} - \omega_{nt} & \leq 0 & \forall n \in N, t \in T \quad (50) \\
- \lambda_{nt} - \sum_i \psi_{nt} (\mu_{it}^+ - \mu_{it}^-) + \tau_t = 0 & \forall n \in N, t \in T \quad (51)
\end{align*}
\]

Equations (48)-(51) are the SCED dual constraints. When taking the dual, the charging and discharging decisions are treated as known constants. The variables are generation level \( p_{gt}^+ \) dispatched wind power \( w_{nt} \), and bus injection \( i_{nt} \). Each constraint of (48)-(50) is one-to-one corresponding to the three variables and equation (51) specifies the sign restriction of each variable.

**Strong duality constraint:**

\[
\begin{align*}
\sum_g \sum_t C_g p_{gt} = \sum_l \sum_n D_{nt} a_{nt} & - \sum_g \sum_t (p_{g,t}^\text{max} \phi_{gt}^- - p_{g,t}^\text{min} \phi_{gt}^+) - Q_k \sum_j (\eta_{jt} a_k - \eta_{jt}^+/\alpha_k) - \sum_i \sum_t F_i (\mu_{it}^+ + \mu_{it}^-) - \sum_n \sum_t W_{nt} \omega_{nt} - \sum_t \sum_n (R_t^{hr} \tilde{u}_{g,t-1} + R_t^{sv} \bar{v}_{gt}) \rho_{gt}^+ - \sum_g \sum_t (p_{g,t}^\text{hr} \tilde{u}_{gt} + R_t^{sv} (\bar{v}_{gt} - \tilde{u}_{gt} + \tilde{u}_{g,t-1}) \rho_{gt}^+ \\
\end{align*}
\]
Equation (51) equals the SCED primal objective to the dual objective. Based on the strong duality theorem, for a linear programming (LP), the primal feasible solution will have an objective that equals a dual feasible solution’s objective if and only if both the primal and dual are optimal. Therefore, by combining the primal constraints, the dual constraints, and the strong duality constraint together, this set of linear equations guarantees that the solution will be optimal if such a solution exists. Then the $\lambda_{nt}$ will be the true LMP at each bus in each period.

**Linearizing constraints:**

\begin{align}
\eta^+_{jt} &\leq M^+ x^+_jt & \forall j, t \in T \\
\eta^-_{jt} &\geq M^- x^-_jt & \forall j, t \in T \\
\eta_{jt} &\leq \lambda_{rt} - M^- (1 - x^-_jt) & \forall j, t \in T \\
\eta_{jt} &\geq \lambda_{rt} - M^+ (1 - x^+_jt) & \forall j, t \in T \\
\eta^+_{jt} &\leq M^+ x^+_jt & \forall j, t \in T \\
\eta^-_{jt} &\geq M^- x^-_jt & \forall j, t \in T \\
\eta_{jt} &\leq \lambda_{rt} - M^- (1 - x^-_jt) & \forall j, t \in T \\
\eta_{jt} &\geq \lambda_{rt} - M^+ (1 - x^+_jt) & \forall j, t \in T
\end{align}

Equation (53)-(60) linearize the bi-linear term using big-M method. First, assume $\lambda_{nt} \in (M^-, M^+)$. The goal is to equal $\eta_{jt}$ to $\lambda_{rt} x_{jt}$. Now only look at the charging decisions. When $x^+_jt = 0$, (53) and (54) force $\eta^+_jt = 0$; while the right hand side of (55) is positive, the right hand side of (56) is negative. When $x^-_jt = 1$, (55) and (56) force $\eta^-_jt = \lambda_{rt}$ while (53) and (54) are relaxed. By introducing the linearizing constraints, the original non-linear optimization problem is transformed to a mixed integer linear program (MILP) and commercial software, such as CPLEX, can solve this kind of problem very efficiently.

As described, the big-M value should be the lower and upper bound of the LMPs. The selection of the big-M value will affect the solution time of the MILP. In general, the tighter the big-M value is, the better the formulation will be. If the big-M value is chosen to be too big, then CPLEX, which uses branch and bound combined with cutting plane algorithms, will generate many cuts and, thus, will lead to a long solution time. The big-M value can be assigned based on historical LMPs; improvements to the big-M value selection are left for future work.

**EES constraints:**

\begin{align}
e_t = e_{t-1} + Q_k (\Sigma_j x^+_{jt-1}) - Q_k (\Sigma_j x^-_{jt-1}) & \forall t \in T \\
E^{min}_t \leq e_t \leq E^{max}_t & \forall t \in T \\
x^+_{jt+1,t} \leq x^+_jt & \forall j, t \in T \\
x^-_{jt+1,t} \leq x^-_jt & \forall j, t \in T \\
x^+_jt, x^-_jt \in \{0,1\} & \forall j, t \in T
\end{align}

Equation (61) restricts the energy storage balance constraint and (62) restricts the energy storage capacity. Equations (63)-(65) specify the relation and space of charging and discharging decisions. $Q_k$, the segment power capacity, can be assigned the value $U_k / |J|$, where $U_k$ is the total power capacity and $|J|$ is the number of segments.

The entire arbitrage model combines (39)-(65); by solving this model, the optimal charging decision can be obtained with the consideration of the charging and discharging impacts on LMPs.

### 3. Electric energy storage charging policies

In the previous section, all models are deterministic models, i.e., the loads and the wind power are fixed at their predicted values. However, due to uncertainty, real-time LMPs will deviate away from their predicted values, which leads to sub-optimal arbitrage decisions, which can also cause negative profits. Three charging policies are discussed in this section. SCUC is first solved to obtain the unit commitment solution for the next 24 hours. Then the arbitrage model is solved under different polices to obtain the charging and discharging solutions. Finally, in the real-time market, the EES owner participates in the market based on the obtained charging schedule. The procedure is described in Figure 2.

![Figure 2. EES charging/discharging decision procedure](image)

#### 3.1. Forecasting-belief policy

In this policy, the EES owner makes the decision based on the day-ahead forecasting loads and wind power assuming that they are the true value in the real-time. The arbitrage model is run with the predicted loads and wind power. Once the EES owner obtains the charging solutions, they stick to the charging and discharging schedule in the real-time market for the next 24 hours.
3.2. Robust policy

In this robust policy, the EES owner first simulates several scenarios using Monte Carlo simulations. Then, they run the arbitrage model to estimate the best charging decision, the LMPs, and the total profits for each simulated scenario. Finally, they make the final charging decision based on these generated scenario solutions. Specifically, this policy calculates the lower and upper bound for the LMPs. Then the EES owner uses the maximum LMP when buying energy and the minimum LMP when selling the energy. In such manner, they hedge the risk of LMP variation that the wind and load uncertainty may bring to the power grid.

3.3. Dynamic policy

The wind power is highly related to the wind speed. In practice, the wind speed is usually predicted first and the wind power is calculated with some static non-linear function with respect to the wind speed [19]. A better wind speed prediction can be obtained when the predicted period is close to the current period. In this dynamic policy, the EES owner updates their charging decision periodically. Standing at the current hour, the EES owner can have a better wind speed prediction of the following few hours. At each period, they run the arbitrage model for the next 24 hours and they keep adjusting the charging decision dynamically. This policy takes the advantage of the better prediction of the wind power and more accurate real-time LMPs. Figure 3 describes the procedure of the dynamic policy.

4. Case study and computational results

In this case study, IEEE 118-bus test case is tested. There are 118 buses, 186 transmission lines, 54 generators and five wind farms are located at different buses as suggested in [20]. The total peak load is 4519MW. The average hourly wind power is 530.08MW, 11.73% of the peak load. A single EES facility is considered at a single certain bus, with power capacity 100MW and 12+ hour energy storage capacity.

First, the charging or discharging impacts on LMPs are examined. Table 1 and Table 2 show the LMP with and without consideration of charging or discharging inputs.

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</tr>
</tbody>
</table>

“DA_LMP” stands for the day-ahead LMP, which is the LMP without consideration of the charging or discharging. “RT_LMP” stands for the real-time LMP, which takes the charging or discharging into consideration. From Table 1, the LMP in period 4 changed dramatically when charging is taken into consideration. Originally, the day-ahead LMP was $0 without the consideration of charging impact. However, since the charging increases the load, the LMP increased to $11.36. Similarly, in period 21, the...
LMP dropped from $38.49 to $35.37. In period 22, the day-ahead LMP was $38.48; even still, charging does not occur in this period. It can be conjectured that if charging occurs, the real-time LMP would drop and make the arbitrage less profitable.

Table 2. **LMP comparison with and without charging or discharging under 150MW power capacity**

<table>
<thead>
<tr>
<th>Period</th>
<th>DA LMP</th>
<th>RT LMP</th>
<th>Charge</th>
<th>Discharge</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11.36</td>
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<td>3</td>
<td>11.36</td>
<td>11.03</td>
<td>1</td>
<td>0</td>
</tr>
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<td>24</td>
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<td>12.62</td>
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</tbody>
</table>

“TP” stands for the total profits obtained from arbitrage. “U” is the total units that EES charged during 24 hours. “P” is the arbitrage profit per unit, i.e., TP/U. “TC” is the total cost, which includes startup cost, no-load cost, and the variable operating cost. “TLP” is the total load payment. “TGR” is the total generation revenue. “TUL” is the total uplifts payment. “TGRent” is the total generation rent.

Table 3. **Arbitrage comparison under different locations**

<table>
<thead>
<tr>
<th>BUS</th>
<th>TP</th>
<th>U</th>
<th>P</th>
<th>TC (Sk)</th>
</tr>
</thead>
<tbody>
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<td>15,103</td>
<td>900</td>
<td>16.78</td>
<td>906</td>
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<td>70</td>
<td>13,767</td>
<td>900</td>
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<td>911</td>
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<td>2,291</td>
<td>900</td>
<td>4.58</td>
<td>923</td>
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<td>716</td>
<td>500</td>
<td>1.43</td>
<td>924</td>
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<td>89</td>
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<td>14,170</td>
<td>900</td>
<td>15.74</td>
<td>907</td>
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<td>117</td>
<td>15,428</td>
<td>900</td>
<td>17.14</td>
<td>908</td>
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</table>

From Table 2, it can be seen that the LMP change is more frequent among 24 hours; especially in periods 4, 5, and 14, the LMPs change dramatically. Therefore, large amount of EES injection will affect the real-time LMP, which is expected. When the EES owner makes the arbitrage decision, this effect should be taken into consideration.

Next, different locations of EES are examined for arbitrage. Moreover, the market welfare allocations are listed for comparison. Table 3 shows the results.

As discussed in Section III.A, the locational model is run assuming the EES owner is a vertically-integrated utility. Specifically, all 118 buses are examined for their potential load-shifting savings. A couple of potential EES locations are compared. Bus 67 gives the optimal location for load-shifting savings. Bus 24 gives the minimum savings per MW. Bus 8 gives the minimum total savings. Bus 70 gives the maximum total charged MW. Bus 60 is the wind farm location with maximum average wind power. Bus 89 is the bus with maximum load. Buses 28, 79, and 117 are randomly selected buses.

Thirty real-time wind scenarios are generated and all results are shown as the average of all 30 scenarios. In Table 3, all results are based on the forecasting-belief policy.

From the results, bus 67 maximizes the social welfare whereas bus 117 gives the maximum arbitrage profits. This confirms that the best EES location for a vertically-integrated utility is different from the best location for a for-profit private investor. Although bus 67 does not give the highest arbitrage profits, the profits at this location are still very high. Bus 60 gives the minimum arbitrage profits since the wind farm is also located at this bus. The marginal cost of the wind power is considered to be zero; thus, it can be expected that the LMPs at this bus are low and flat among all 24 periods. While PHS is suggested to build along with the wind farm to flatten the variance of the wind power, the arbitrage value at a bus with wind farm is not prominent. This result also communicates that a joint investment effort between the two resources is likely preferred over independent investments.

Different load profiles and different efficiencies are
also tested to evaluate the arbitrage effects. The results are listed in Table 4 and Table 5. “H”, “M” and “L” stand for high, medium and low load profiles respectively. From Table 4, the arbitrage profits will drop as the load drops. When load is low, the expensive generators may not be turned on and, thus, the variation in LMPs is lower, which reduces arbitrage opportunities. From Table 5, the arbitrage profits will drop as the round-trip efficiency goes down.

### Table 4. Arbitrage comparison under different load profiles

<table>
<thead>
<tr>
<th>Load</th>
<th>TP</th>
<th>U</th>
<th>P</th>
<th>TC (Sk)</th>
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<tbody>
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<tr>
<td>M</td>
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<td>L</td>
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<td>1,100</td>
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<table>
<thead>
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<th>TGR (Sk)</th>
<th>TUL (Sk)</th>
<th>TGRent (Sk)</th>
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<tr>
<td>M</td>
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<td>426</td>
<td>67</td>
<td>114</td>
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<tr>
<td>L</td>
<td>272</td>
<td>213</td>
<td>62</td>
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### Table 5. Arbitrage comparison under different EES efficiency

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<th>P</th>
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<table>
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<th>TGR (Sk)</th>
<th>TUL (Sk)</th>
<th>TGRent (Sk)</th>
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<td>574</td>
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<tr>
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<td>1,759</td>
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<td>1,428</td>
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<td>594</td>
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</table>

Finally, the arbitrage behaviors under different policies are compared. The results are shown in Table 6. “P1”, “P2” and “P3” stand for the policy 1, 2 and 3 respectively. The high load profile and medium load profile are tested.

### Table 6. Arbitrage comparison under different policies

<table>
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<tr>
<th></th>
<th>P1H</th>
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<th>P3H</th>
<th>P1M</th>
<th>P2M</th>
<th>P3M</th>
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<td>914</td>
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<td>378</td>
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<tr>
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<td>534</td>
</tr>
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<td>422</td>
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<tr>
<td>TGRent (Sk)</td>
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From Table 6, the robust policy makes more profits than the forecasting-belief policy since it takes different real-time wind power uncertainty into consideration. The robust policy is a conservative policy, i.e., charges less frequently than the other two policies. The dynamic policy minimizes the total cost of the system but not necessarily returns the maximum arbitrage profits. This contradicts what was intuitively expected, that a dynamic policy that frequently updates its scheduling would be preferred. The reasons might be the inaccurate forecasts and the frequent charging and discharging. More tests are desired to confirm this result.

### 5. Conclusions

In this paper, EES arbitrage in electric energy markets is studied. When EES power capacity is large, the charging or discharging events may change the real-time LMP from the day-ahead predicted value. Since the arbitrager bids in the real-time markets, the potential LMP changes should be taken into consideration. The test case shows that the LMP in real time changes significantly in some periods with charging or discharging. If the EES owner makes arbitrage decision only based on the day-ahead LMP, it might end up losing money. Locations of the EES facility play a very important role in the arbitrage. From test case results, the best location of EES for a private investor is likely to be different from the best location if the EES owner is a vertically-integrated utility. Nevertheless, in many cases the two have similar beneficial impacts on both arbitrage profits and social welfare improvement, i.e., the location that maximizes arbitrage profits usually has a high social welfare and the location that maximizes social welfare usually has relatively high arbitrage profits. When the location of EES is selected at some inappropriate bus, the benefits of arbitrage may be little. Load levels and EES round-trip efficiency would also affect the arbitrage profits. The arbitrage decision will also be affected by the wind power penetration. Better prediction of wind power can help improve the arbitrage profits.

Although EES brings many benefits to the power grid planning and operation, making profits through price arbitrage in real-time market has low return on investment and may be very vulnerable considering the high penetration of renewable energy. It is also vulnerable to the placement of the storage; over the long time horizon to recover the investment cost, the ideal arbitrage location is likely to change. Current market design does not incentivize private investors to build EES. Furthermore, existing market bidding structures are designed for fossil fuel based generators, not other forms of resources. Storage resources naturally offer a different product, a different service to the industry but yet they are forced to conform to the bidding and characteristics of a fossil fuel generator.
based on how supply resources are modeled with electricity market auctions. It is time for market design reform, a market structure that acknowledges the variation in products offered by different supply resources instead of conforming to one such structure. As long as the market structures conform to only one dominant resource type, the incentives are not what they should be to encourage investment in alternative resources that are critical to the operations of future power systems. While this is a critical topic and one of high interest, at this time this challenge is left to future work.

6. Nomenclature

Sets:
- $G$: Set of generators
- $G(n)$: Set of generators at bus $n$
- $J$: Set of energy storage unit charging segments
- $L$: Set of transmission lines
- $N$: Set of buses
- $S$: Set of scenarios
- $T$: Set of time periods

Generator Parameters:
- $C_g$: Generation variable cost of generator $g$
- $C_{NL}^g$: No-load cost of generator $g$
- $C_{st}^g$: Startup cost of generator $g$
- $DT_g^b$: Minimum down time of generator $g$
- $g(n)$: Bus location of generator $g$
- $P_{max}^g$: EcoMax of generator $g$
- $P_{min}^g$: EcoMin of generator $g$
- $R_h^g$: Hourly ramping rate of generator $g$
- $R_{sh^g}$: Shutdown ramping rate of generator $g$
- $R_{st}^g$: Startup ramping rate of generator $g$
- $UT_{g}^b$: Minimum up time of generator $g$

Energy Storage Parameters:
- $B_n$: Cost of building energy storage unit at bus $n$
- $E_{max}^n$: Maximum capacity of energy storage unit at bus $n$
- $E_{min}^n$: Minimum capacity of energy storage unit at bus $n$
- $K$: Maximum number of energy storage units can be built
- $Q_n$: Power capacity of energy storage unit for one segment at bus $n$
- $U_n$: Power capacity of energy storage unit at bus $n$
- $α_n$: Efficiency of energy storage unit at bus $n$

Transmission Network Parameters:
- $F_l$: Line rating of transmission line $l$
- $D_{nt}$: Predicted load at bus $n$ in period $t$
- $W_{nt}$: Predicted wind generation at bus $n$ in period $t$
- $ψ_{nt}$: Generation shift factor on transmission line $l$ from bus $n$ to the reference bus

Variables:
- $ε_t$: Energy storage level in period $t$
- $i_{nt}$: Power injection at bus $n$ in period $t$
- $α^+_g/α^-_g$: Charging/discharging power in period $t$
- $p_{gt}$: Generation of generator $g$ in period $t$
- $r_{gt}$: Reserves of generator $g$ in period $t$
- $u_{gt}$: Binary unit commitment status of generator $g$ in period $t$
- $v_{gt}$: Startup status of generator $g$ in period $t$
- $w_{nt}$: Utilized wind generation at bus $n$ in period $t$
- $x^+_gt/x^-_gt$: Binary charging/discharging power decision of segment $m$ in period $t$
- $z_n$: Binary indicator if build energy storage unit at bus $n$
- $η^+_gt/η^-_gt$: Linearized variable of segment $m$ in period $t$
- $λ_{nt}$: Locational marginal price at bus $n$ in period $t$
- $μ^+_lt/μ^-_lt$: Flowgate marginal price on transmission line $l$ in period $t$
- $ρ^+_gt/ρ^-_gt$: Dual variable for ramping up/down constraint of generator $g$ in period $t$
- $τ_t$: Dual variable for total injection constraint in period $t$
- $φ^+_gt/φ^-_gt$: Dual variable for generation capacity constraint of generator $g$ in period $t$
- $ω_{nt}$: Dual variable for wind generation constraint at bus $n$ in period $t$

References


