Substation Local Voltage Controller using Synchrophasors

Sayed Mohammad Amelian, Vaithianathan “Mani” Venkatasubramanian, Javier Guerrero
School of EECS, Washington State University
Pullman, WA

Noah Badayos, Farrokh Habibi-Ashrafi, Backer Abu-Jaradeh, and Armando Salazar
Southern California Edison, Los Angeles, CA

Abstract
This paper continues development of a Substation Local Voltage Controller (SLVC) using synchrophasors. This SLVC is the substation counterpart of a hierarchical two-level voltage controller being developed at Southern California Edison (SCE) Inc. The controller is based on a linearized local reactive power flow approach for estimating the effects of any switching action within a substation on the corresponding voltage changes at the substation buses. Switching actions include tap changes at different sets of transformer banks, and switching in/out the shunt reactive devices (capacitors and reactors) at the substation. The formulation is tested on a power-flow model of a representative substation in the SCE network through simulations, and the local power-flow results are compared to the large-scale full power flow solutions. Different normal and contingency operation conditions as well as various combinations of switching actions are considered, and it is shown that the proposed formulation can estimate the voltage changes within the local network with acceptably small errors.

1. Introduction

One of the most important operational responsibilities in electrical power systems is the control of bus voltages and reactive power flows. This control action is called “Voltage/Reactive Power” or “Volt/Var” control. The ultimate goal of such control is to enhance system performance in terms of its security, service quality and economical operation [1].

In order to keep the system security high enough, we should manage our resources to have satisfactory voltage levels and sufficient reactive power reserves, to avoid voltage instability, while the system undergoes routine load (and generation) variations and when the system encounters various contingencies. The service quality and economical operation are related to maintaining the bus voltages within acceptable margins, as well as keeping reactive power flows (through the transmission network) as low as possible to minimize the transmission losses.

The main difficulties in Volt/Var control of the power systems are, first, to determine the desired (scheduled) voltage schedule for each bus, and second, to find a feasible combination of reactive resources that brings the voltages back to the tolerable thresholds which are reasonably close to the desired schedule values, when encountering voltage deviations. In North America, most of the operational reactive power management is carried out by switching of discrete var devices such as shunt capacitor or reactor banks, and Load Tap Changing (LTC) transformer banks. In fact, for utilities such as SCE that operate in deregulated market structures, the transmission operation cannot directly control generator voltage schedules. It is well known that voltage control and reactive power control are highly correlated [1]. The aim of these controls is to minimize transmission var losses because of reactive power flows and to manage reactive power flows so that adequate var reserves are available throughout the system.

Volt/Var control of the electrical networks has become more crucial in recent years due to significant changes in generation patterns from increased presence of renewables which necessitates a more efficient use of the grid infrastructure. Private ownership of most of the generation plants, and the increasing use of power electronic devices add to the uncertainty. The main difference of this volt/Var control, when compared to conventional frequency control of the power system, is that it is a localized problem, whereas the latter one is a system-wide phenomenon that entails control from a centralized point of view like conventional Automatic Generation Control (AGC). Considering this local nature and the discrete variety of var control devices, and given the possible interactions of such control actions, the volt/Var problem turns out to be difficult to solve for North American power system.
Different schemes for voltage control of the electrical power networks of the European countries were proposed during the '80s and early '90s [2]-[5]. From the outcomes of the corresponding studies, some common terms were defined, such as “primary”, “secondary” and “tertiary” voltage control, which constitute a similar hierarchy to that of the frequency control. Primary level is the fast control actions of synchronous generators AVRs within a small portion of the system around the affected buses. Secondary level is a slower one consisting of several AVRs of the machines in a regional scale. Tertiary level determines the set points of the AVRs in the other two levels [2],[3].

In the US, some preliminary voltage control algorithms were proposed in [6], [7], [8]. However, these controllers were never fully implemented. A hierarchical controller is proposed in [9] for the Chinese grid, which employs the basic concepts of the European approach, and is currently being tested on PJM interconnection [10].

In general, the Volt/var control at the secondary and tertiary levels can be accomplished by three basic approaches (from the practical standpoint): a) decentralized control, b) centralized control, and c) hierarchical control. Among these, the hierarchical control is the best scheme, since we want to make sure that the necessary coordination holds between the local controllers and the central coordinator. Considering the large number of synchrophasors installed in the North American power system and all around the world, developing control strategies based on synchrophasors and PMU data is becoming an interesting subject [11].

Considering the above-mentioned challenges, a hierarchical two-level voltage controller based on the PMU measurements has been proposed in [12], by dividing the control actions between local controllers in a substation level, and a centralized coordinator at the control center level. Details of the formulations of these substation level SLVC and the control center level Supervisory Central Voltage Coordinator (SCVC) can be found in [12]. These concepts will be summarized in Section 2 of this paper.

One of the challenges in the SCE controller design is that each substation has only access to the local PMU measurements at that substation because of communication constraints. Therefore, carrying out a full nonlinear power-flow or state estimation even for the local neighboring network is not possible. Hence, the key point in the operation of either SLVC or SCVC is to properly predict or estimate the voltage changes after occurrence of a switching action without running a full power flow or state estimation algorithm. But, as it is assumed that all the internal bus voltages and line flows toward the adjacent buses of a particular candidate substation are measured by PMUs, voltage changes of the buses within the affected area can be approximated using a linearized reactive power-flow equations [1],[13], depending on local substation measurements only. This is called the Local Voltage Estimator (LVE) [12] and the corresponding formulation is described in Section 3.

This paper summarizes the concept of the LVE from [12] in Section 3, and proceeds to test its performance under different operational conditions and various switching actions, using detailed power-flow simulations in Section 4. The simulation results and analysis of testing this formulation on a typical 500/230/66 kV substation and surrounding network in Southern California are presented in Section 4. This includes voltage sensitivity analysis under different switching actions, calculation of the parameters for the LVE formulas, and finally, testing LVE as a whole by comparing the LVE results against the results from a large-scale full power flow solution. Section 5 discusses the results of simulations and the conclusions of this paper.

2. Overview of the Hierarchical Voltage Controller

The hierarchical design includes two levels and is shown in Figure 1 [12]:

a) Substation Local Voltage Controller (SLVC) which operates at the substation level using local measurements accompanied with supervisory guidance from the SCVC. The signals from central coordinator to local controllers include coordination (enable/disable) as well as voltage set-points for all the SLVCs, and are called Supervisory control signals in Figure 1. The local controller performs all the internal control calculations and actions purely based on local PMU measurements and the substation topology. Typical North American sub-transmission network (115 kV or 66 kV) has a radial configuration with no connections to neighboring bulk power substations, therefore, the sub-transmission level will be only managed by SLVC, whereas the bulk transmission buses (500 kV and 230 kV) will also be governed by SCVC through the enable/disable signals.

b) Supervisory Central Voltage Coordinator (SCVC) at the control center determines the overall voltage profile of the high voltage transmission network (230 kV and 500 kV) buses, and also coordinates SLVCs operation through enable/disable commands. To specify the optimal voltage schedules and controller operations for solving the voltage problems and other var related issues, SCVC carries out power-flow calculations of the whole network.
As of now, the SLVC has been tested in off-line and in real time simulation models for a substation in California. Owing to the lack of availability of large-scale real-time models of the California power grid, SCVC has been only tested using large-scale power-flow models of the SCE transmission network.

SCVC functions and SLVC controller actions are blocked during fault-on periods and other transient conditions including sudden load changes to prevent the proposed voltage controllers from interfering with fast dynamic reactive power resources such as generators and SVCs [12].

Figure 1. Hierarchical controller design [12]

3. Formulation of Local Voltage Estimator

This section provides the basic formulations for the LVE, defined previously from [12]. The main goal is to estimate the voltage changes of the corresponding substation buses (internal) as well as the immediate neighboring buses (external) that are directly connected to the internal buses through the transmission and sub-transmission lines, after occurring a capacitor/reactor switching or a transformer tap changing in the substation.

3.1. Capacitor/Reactor Switching

Let the transmission line between bus \( i \) and bus \( j \) be represented by \( \pi \)-equivalent model as shown in Figure 2. The reactive power line flow equation is

\[
Q_{ij} = f_{ij}(V_i, V_j) = -V_i^2(B_{ij} + B_{i0}) - V_j(G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij})
\]

where \( \delta_{ij} = \delta_i - \delta_j \) is the difference of two bus voltage angles.

Assuming that reactive line flow change is only related to terminal voltage changes, for typical operating conditions, the linearized reactive power line flow equation for a transmission line can be written as

\[
\Delta Q_{ij} = \frac{\partial f_{ij}}{\partial V_i} \Delta V_i + \frac{\partial f_{ij}}{\partial V_j} \Delta V_j
\]

where

\[
\frac{\partial f_{ij}}{\partial V_i} = V_j(B_{ij} + B_{i0}) + \frac{Q_{ij}}{V_i}
\]

\[
\frac{\partial f_{ij}}{\partial V_j} = V_i^2(B_{ij} + B_{i0}) + \frac{Q_{ij}}{V_j}
\]

Note that in (3), the partial derivatives are only dependent on line reactive power flow and voltage magnitude measurements. Thus, the line reactive power flow change could be estimated by (2), given the voltage changes at two terminal buses, without solving a full power flow.

From (2), the total reactive power change in bus \( i \) is given by,

\[
\Delta Q_i = \sum_{j \in J_i} \Delta Q_{ij} = \sum_{j \in J_i} \left( \frac{\partial f_{ij}}{\partial V_i} \Delta V_i + \frac{\partial f_{ij}}{\partial V_j} \Delta V_j \right)
\]

where \( J_i \) is the set of buses connected directly to bus \( i \).

For each internal bus, we will end up with an equation similar to (4), with the internal and external bus voltages as unknowns.

To be able to solve the equations, there should be a direct relation between internal and external bus voltages. A constant \( a_{ij} \) can be defined as the sensitivity of the bus \( i \) voltage change, \( \Delta V_i \), to the change at bus \( j \), \( \Delta V_j \) for a reactive power injection at bus \( k \), \( \Delta Q_k \). Therefore, if a set of \( a \)-constants is known for each outbound transmission line, the internal buses could be related to the external buses by

\[
\Delta V_j = a_{ij} \Delta V_i
\]

Then, (4) can be re-written as

\[
\Delta Q_i = \sum_{j \in J_i} \left( \frac{\partial f_{ij}}{\partial V_i} \Delta V_i + \sum_{j \notin J_i} \left( \frac{\partial f_{ij}}{\partial V_j} a_{ij} \right) \Delta V_i \right) + \sum_{j \notin J_i} \left( \frac{\partial f_{ij}}{\partial V_j} \Delta V_j \right)
\]
where $J_d$ is the subset of internal buses connected to bus $i$, $J_a$ is the subset of external buses connected to bus $i$ and $J_e$ is the subset of buses with a fixed voltage (voltage-controlled buses, e.g. SVC or AVR controlled buses) connected to bus $i$. As a result, all voltages in the substation can be calculated when any reactive power injection $\Delta Q_k$ occurs at a bus $k$ inside the substation, using

$$
\Delta V = [B]^{-1} \Delta Q = [S] \Delta Q
$$

where $\Delta V = [\Delta V_1, \Delta V_2, ..., \Delta V_N]^T$ is the vector of voltage changes. $\Delta Q = [0, 0, \Delta Q_k, 0, ..., 0]^T$ is the vector of injection changes. $\Delta Q_k = Q_C$ is from the switching of capacitor/reactor at bus $k$.

$B$ is an $N$ by $N$ matrix whose elements are defined as

$$
B_{i,j} = \left[ \sum_{j \in J_d} \left( \frac{\partial f_j}{\partial V_i} \right) + \sum_{j \in J_a} \left( \frac{\partial f_j}{\partial V_i} \alpha_j \right) \right] (i = 1, 2, ..., N)
$$

$$
B_{i,j} = \left[ \frac{\partial f_j}{\partial V_i}; \ j \notin J_c; \ i, j = 1, 2, ..., N \right]
$$

$$
B_{i,j} = 0; \ j \in J_c
$$

where $N$ is the number of internal buses.

In our formulation, the parameters $\alpha_{ji}$ are assumed to be calculated constants from off-line full power flow. These parameters mostly depend on the location of the device (bus $k$) that hosts the discrete var control device. They do not vary much with load demand and network topology changes away from the substation. A detailed study of the sensitivity of these $\alpha$ parameters will be presented in Section 4. The modelling of transformer tap changes in the form of equivalent var injections on the secondary and primary side can be seen in [12] and is omitted here to save space. Note that the LVE does not need measurements from external buses since these can be calculated from the PMU measurements in the internal buses. The only measurements needed are the internal voltages and those of immediate neighbors, and the active and reactive power-flows involving local substation buses. These are all available from local PMU measurements.

4. Simulation Results and Analysis

In developing the LVE formulation, it was assumed that post-switching voltage change at each external bus connected to the substation under study is proportional to the local substation bus voltage by an a-priori known factor $a$ [12]. In this section, the goals are 1) to demonstrate that these linear relations between the voltage changes of internal and external buses hold in actual conditions, and 2) to show that the network topology and contingencies have a small effect on these calculated alpha values.

A typical substation in Southern California system, which has three 500 kV, 230 kV, and 66 kV voltage levels, is selected for carrying out the simulation and analysis. This substation X has connections in all the three levels, where the immediate neighboring buses are named based on the following: first letter (capitalized) of the bus name corresponding to its voltage level, where "A" indicates 500 kV, "B" indicates 230 kV, and "C" indicates 66 kV. The number after the letter, corresponds to the specific substations. AX, BX, and CX correspond to the external buses of different voltage levels, in descending order.

A large-scale 18,000 bus power system model from Western Electricity Coordinating Council (WECC) in peak load condition is used and the nonlinear power-flow solutions are computed using General Electric (GE) PSLF software.

The existing volt/var control devices at substation X (denoted sub. X in short) are as follows (it is specified if the device is switched in or out for the base case power flow model):

- 500/230 kV (AA) transformers: 2 transformers, both in service, base-case tap position=1.00 pu
- 230/66 kV (A) transformers: 5 transformers, three in service, base-case tap position=1.013 pu
- 500 kV regulating/switched capacitor: 2×150=300 Mvar, both switched out
- 230 kV regulating/switched capacitor: 1×79=79 Mvar, switched out
- 230 kV fixed capacitor: 79 Mvar, switched out
- 66 kV shunt capacitors: 4×28.8=115.2 fixed capacitors, all switched in
- 13.8 Reactors (on tertiary winding of 500/230 kV transformers): 8 reactors, all switched out

Also, to analyze the effects of network topology changes on the study, some different network topologies (contingencies) are considered, where aside from the base case, each of the Tier-1 (immediate connections of Sub. X) transmission (500 kV and 230 kV) lines are switched out, and in other cases, changes are made to the switched in/out condition of the Subs. X shunt reactive control devices. List of these cases are as follows: 1) Base Case, 2) BX-B1 230 kV line outage, 3) BX-B3 230 kV line outage, 4) BX-B4 230 kV line outage, 5) AX-A1 500 kV line outage, 6) AX-A2 500 kV line outage, 7) AX-A3 500 kV line outage, 8) 500 kV reg. cap. switched in, 9) 230 kV fixed cap.
4.1. Voltage Change Sensitivity Study

In order to show linear changes of system bus voltages as switching occurs within the internal buses of a substation, effect of Sub. X transformer tap changes of both 500/230 kV, and 230/66 kV banks are studied in this section. Changes in tap positions range from -5% to +5%, changed by the steps of 2%, while all the other controlled shunt devices and transformer taps are fixed to their corresponding values in normal (base case) operation condition. It should be noted that transformer sets regulate their LV side voltage.

4.1.1. Base Case: Figure 3 shows bus voltage changes at Sub. X buses for the base case, where part (a) provides the results for tap changes in Sub. X 500/230 kV transformers and part (b) shows the results for tap changes in Sub. X 230/66 kV transformers. Regarding the 500 kV bus from part (a), it can be seen that its voltage increases almost linearly when tap position increases, from 1.057 for tap in -5% position to 1.067 for tap in +5% position, i.e. approximately 0.002 pu voltage change for every 2% tap change. Also, it can be seen from 230 kV and 66 kV bus voltages, that, changing the taps in order to increase the HV side voltage, decreases the LV side voltage (the typical transformer response) in a completely linear manner as well (for 230kV bus from 1.04 pu to 0.98 pu, i.e. 0.01 pu for every 2% tap change). From part (b), one can see that changing the taps of 230/66 kV transformers changes the voltage of 66 kV bus, again in a completely linear way (from 0.98 pu to 1.10 pu, i.e. 0.024 pu for every 2% tap change), while it has a very small effect on 500 kV and 230 kV bus voltages, since these two seem to be strong buses that are connected to the rest of the bulk transmission network.

![Figure 3. Sub. X bus voltages for the base case: (a) Tap changes in 500/230 kV transformers, (b) Tap changes in 230/66 kV transformers](image)

Figure 4 shows voltage changes of 500 kV buses in the base case. It can be seen from (a) that the voltages of neighboring buses outside of Sub. X also increase by some different yet lower slopes in an almost linear trend (A1 from 1.061 pu to 1.065 pu, A2 from 1.055 pu to 1.063 pu, A3 from 1.059 pu to 1.066 pu). Also, from (b), it can be seen that tap change in 230/66 kV transformers have almost no effect on voltages of external transmission buses, as the range of vertical axis is very limited (maximum change is for AX and is 0.001 pu for the whole tap change range).

![Figure 4. 500 kV bus voltages for the base case: (a) Tap changes in 500/230 kV transformers, (b) Tap changes in 230/66 kV transformers](image)

Figure 5 shows the voltage of 230 kV system buses in the base case. Part (a) shows that voltage of Sub. X 230 kV bus decreases very rapidly, but most of the other buses experience lower slopes of voltage changes, yet with a linear trend. However, there are some different trends in this bus set, where B5 230 kV bus voltage increases, and voltage B2 remains almost constant. Both of the buses are connected to Sub. X through links that are out of service, hence their voltage does not necessarily follow the changes of the Sub. X. However, in the case of B5, since its 500 kV side is also directly connected to Sub. X, as AX 500 kV bus voltage increases, B5 230 voltage also increases following its 500 kV side voltage. But, since the B2 is the maximum voltage level of its substation, its voltage does not change too much, and slight changes are due to the grid loop connection of 500/230 kV transmission network. Also, from part (b), for tap changes in 230/66 kV transformer, all 230 kV system bus voltages are almost fixed where BX 230 kV bus voltage slightly increases from 1.009 pu to 1.012 pu.

![Figure 5. 230 kV bus voltages in the base case: (a) Tap change in 500/230 kV transformers, (b) Tap changes in 230/66 kV transformers](image)

Figure 6 shows the voltage of some of the 66 kV system buses in the base case. Given that subtransmission lines are relatively short, the trend of voltage changes and corresponding range of the neighboring 66 kV buses closely follow that of CX 66
Since the last 5 cases changes of directly connected substations.

4.1.2. Contingency Cases: This subsection presents the results for contingency cases of 2 to 12. Whereas almost all of the voltage changes and trends are almost the same as that of the base case, the figures are presented in a comparative basis, where the corresponding changes of a single bus are plotted in a single figure for all of the twelve cases.

Figure 7 shows the results for the Sub. X 500 kV (AX) bus, for tap changes in (a) 500/230 kV, and (b) 230/66 kV transformers, respectively. It can be seen from part (a) that the voltage trend slopes are more or less the same, but each curve is moved up/down based on the corresponding change in system operation point. For instance when 500 kV cap. is switched in, voltage curve is moved upward, or when 66kV capacitors are switched out or reactors are switched in, the voltage is moved downward. Also, from part (b), it is obvious that changes in 230/66 kV transformers' tap have had almost no effect on Subs. X 500 bus voltage, considering the limited range of the voltage changes, seen from the range in vertical axis.

The results for other internal and external buses show almost the similar trend and have the similar analysis, hence omitted from this subsection, since do not provide any further outcomes. Specifically, the external buses follow the changes of the internal bus with the same voltage level, but to different extents, based on their electrical proximity to that internal bus. They also differ when their connecting lines to Sub. X becomes out-of-service, which is not important anymore, since we are only concerned with voltage changes of directly connected substations.

4.2. Calculation of Alpha Constants

In this section, calculation of the previously-defined alpha values (based on eq. 5) are carried out for each of the directly connected buses to subs. X. From the figures in section 3.1., it was obvious that initial tap position doesn't affect the voltage changes in a single case, i.e. the slope of voltage changes in a particular bus is the same from either -5%→+3% or +1%→+3%. However, for having more accurate results, α values are calculated for different initial tap positions, for example from -5% to -3%, -1% to +1%, and so on. Moreover, it is calculated for different shunt switching actions at the Subs. X, where we have five different var injecting shunt devices (switching of 13.8 kV reactors in, 66 kV capacitors out, a 500 kV cap. step in, 230 kV cap. step in, 230 kV fixed cap. in).

The obtained results are the trend of α values for each bus in all of the twelve cases introduced before, to enable us to find out whether we can assign a single constant α value for each bus. Since the last 5 cases correspond to different combination of the shunt devices being switched in or out, in these cases the α results would have some missed points, which are omitted here to prevent intricacy, whereas the computed α values for these five cases are approximately the same as the other seven cases.

Figure 8 presents the results for B1 230 kV bus, where part (a) corresponds to calculated values based tap changing in transformer taps, and part (b) corresponds to those calculated based on shunt device switching. There are total of 120 step tap changes (12 no. of cases × 2 no. of transformer sets × 5 step changes of each transformer set), hence total 120 calculated α values. In these figures, a green line indicating the average value of α values for all the step changes is plotted, as well as two red lines that indicate boundary of an assumed permissible error of ±5% in α value calculation. Case numbers along the x-axis are according to the numbers given in the very beginning of this section, where different cases are separated by vertical solid lines. Also, in each case, corresponding results to 500/230 kV tap changes are on the left side of a dashed line and those of 230/66 kV tap changes are on its right side. As can be seen from part (a), there are some differences between the calculated values for tap changes in 500/230 kV and 230/66 kV transformers.

However, the calculated values remain almost constant in different cases, except for case 2 which is the corresponding BX-B1 230 kV line outage. Since there is no direct connection between BX and B1 buses anymore, the corresponding α will be automatically set to zero in eq. (5), and there is no need to even calculate it. Also, it can be seen that in cases 11 (all 66 kV fixed capacitors switched out) and 12 (all reactors switched
in) there is a slight deviation in calculated alpha based on 230/66 kV transformer tap changes. In addition, in the main study, again tier-2 line contingencies were also considered where once more, the changes were less effective compared to tier-1 contingencies. The main outcome from part (a) is that, the alpha values remain almost constant, separately for each set of transformer banks, for different initial tap positions in almost all the cases.

Figure 8 shows the calculated alpha values for bus B1 based on shunt device switching within the subs. X. Here, the values for all the cases are more or less the same, except case 2, where BX-B1 connection is lost. There is no different value depending on the switching of different device types on different voltage levels (cap. on 66 kV, reactor on 13.8 kV, etc.). It should be pointed out that the alpha values are similar when calculated based on shunt switching as well as the calculation based on 500/230 kV transformer tap.

Figure 9 presents the results for B3 230 kV bus, where it can be seen that, again, there is difference between the calculated values for tap changes in 500/230 kV and 230/66 kV transformers. However, the calculated values remain almost constant in different cases, except than case 3 which is the outage of BX-B3 connections. Also, note that the alpha value becomes negative when changing 500/230 kV transformers in this case 3, because of the existing 500 kV connection, between the two substations, where although BX 230 kV voltage is decreased, B3 230 kV voltage is increased, because of the increase in its 500 kV side voltage as a result of increase in AX 500 kV bus voltage. Nevertheless, as discussed before, the $\alpha$ will be set to zero, in such cases when the direct connection is lost. A big difference between Figure 9.(b) and the previous B1 bus as in Figure 8.(b), is the fact that switching of 500 kV capacitor results in a different $\alpha$ value, when compared to switching of other shunt devices. Again, this happens because of indirect/loop connection of this B1 230 kV bus with the AX 500 kV.

Figure 10 provides the results for B5 230 kV bus. The main point from part (a) is that the value for all the 500/230 kV tap changes is negative, which is happened because BX-B5 230 kV line is out of service in the base case power flow, while it is connected to Subs. X indirectly, from its 500 kV side, hence showing opposite behavior to Subs. X 230 kV bus voltage, when changing tap of AA banks. Like the previous analysis, there are small differences in the cases 5 and 6, where indirect and short connections of B5 to the Sub. X through 500 kV lines are affected. Also, as can be seen from Figure 11.b, again due to the indirect/loop connection of B5 230 kV to AX 500 kV, capacitor switching on AX 500 kV, results in a different alpha value, compared to other shunt devices switching.

Figure 11 presents the results for C1 66 kV bus, where from part (a) for tap changing, alpha values remain constant in all the cases, regardless of being AA or A transformer banks. The point is that the direct CX-C1 66kV link is out-of-service in the base case, but it is connected to Sub. X through other 66 kV lines, as well as some indirect 230 kV lines, hence causing it to be affected "hugely" by the tap changes in Sub. X, as can be inferred from the average alpha value of 1.1, even bigger than 1.0. Also, the average value is the same for calculation based on shunt switching.
In this section, tabular representation of calculated alpha values in the last section is presented. As a result of analysis of the studies in the last section, it can be concluded that calculated alpha values can be clustered into four main groups, based on the corresponding switching actions, where alpha values in each group are more or less the same, for any of the considered contingency cases. These groups are as follows:

- **Group 1**: Tap changes in Sub. X 500/230 kV transformers
- **Group 2**: Tap changes in Sub. X 230/66 kV transformers
- **Group 3**: Sub. X 500 kV shunt device switching
- **Group 4**: Switching of other shunt reactive devices at Sub. X

Table 1 summarizes the mean and standard deviation values of these four clusters of alpha values for each bus as calculated over the corresponding set of values in all the considered cases. Also, in the last two columns, mean and standard deviation of alpha values as calculated over all the considered cases (not based on the clusters) are presented for comparisons. As can be seen, standard deviation of the values in each cluster is either zero or so small that, when compared to the mean value, the variance can be neglected as a very good approximation. Also, from the last two columns, it can be seen that the standard deviation of the alpha values for some buses are high, which were identified in previous sub-section (highlighted here). We can interpret from this Table 1 which of the neighboring buses need to be considered in the form of different alpha values for different switching actions.

### 4.4. LVE Testing

To check the performance of LVE, a fixed set of $\alpha_{ij}^k$ parameters should be used for predicting/estimating voltage changes considering different switching actions within a substation and compare the result with that of the full power flow model. Thus, in this section, the local voltage estimator (LVE) is tested by forming the $\mathbf{B}$ matrix based on different sets of alpha values, as
discussed in the last section. Each set of alpha values was generated by computing for the change in voltage magnitude at each substation bus after switching shunt elements (capacitor or reactor) at internal buses, or by changing the tap of AA or A bank transformers. The intention of this study is to determine how the choice of alpha constants impacts the voltage estimates for specific combinations of switch actions. All the estimates are directly compared against a full power flow solution, following the same set of switch combinations. To facilitate this, a custom framework is developed using C#, which can run the full power flow in the PowerWorld software and retrieve the results, while simultaneously running the matrix computations of LVE equations (equations 7 to 8).

Table 1. Mean and Standard Deviation (S.D.) of α values for different groups for all the cases

<table>
<thead>
<tr>
<th>Bus Name</th>
<th>Bus kV</th>
<th>Group 1 Mean</th>
<th>Group 1 S.D.</th>
<th>Group 2 Mean</th>
<th>Group 2 S.D.</th>
<th>Group 3 Mean</th>
<th>Group 3 S.D.</th>
<th>Group 4 Mean</th>
<th>Group 4 S.D.</th>
<th>Average α Mean</th>
<th>Average α S.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>500</td>
<td>0.20</td>
<td>0.02</td>
<td>0.34</td>
<td>0.02</td>
<td>0.22</td>
<td>0.02</td>
<td>0.21</td>
<td>0.02</td>
<td>0.26</td>
<td>0.07</td>
</tr>
<tr>
<td>B3</td>
<td>500</td>
<td>0.09</td>
<td>0.02</td>
<td>0.27</td>
<td>0.02</td>
<td>0.60</td>
<td>0.03</td>
<td>0.25</td>
<td>0.02</td>
<td>0.22</td>
<td>0.13</td>
</tr>
<tr>
<td>B5</td>
<td>500</td>
<td>-0.13</td>
<td>0.02</td>
<td>0.26</td>
<td>0.01</td>
<td>1.09</td>
<td>0.07</td>
<td>0.26</td>
<td>0.02</td>
<td>0.18</td>
<td>0.30</td>
</tr>
<tr>
<td>C1</td>
<td>66</td>
<td>1.09</td>
<td>0.00</td>
<td>1.10</td>
<td>0.00</td>
<td>1.09</td>
<td>0.00</td>
<td>1.09</td>
<td>0.00</td>
<td>1.09</td>
<td>0.00</td>
</tr>
<tr>
<td>C2</td>
<td>66</td>
<td>1.01</td>
<td>0.00</td>
<td>1.01</td>
<td>0.00</td>
<td>1.01</td>
<td>0.00</td>
<td>1.01</td>
<td>0.00</td>
<td>1.01</td>
<td>0.00</td>
</tr>
<tr>
<td>C3</td>
<td>66</td>
<td>0.59</td>
<td>0.01</td>
<td>0.53</td>
<td>0.00</td>
<td>0.61</td>
<td>0.01</td>
<td>0.58</td>
<td>0.01</td>
<td>0.57</td>
<td>0.03</td>
</tr>
<tr>
<td>C4</td>
<td>66</td>
<td>1.03</td>
<td>0.00</td>
<td>1.03</td>
<td>0.00</td>
<td>1.03</td>
<td>0.00</td>
<td>1.03</td>
<td>0.00</td>
<td>1.03</td>
<td>0.00</td>
</tr>
<tr>
<td>C5</td>
<td>66</td>
<td>1.07</td>
<td>0.00</td>
<td>1.08</td>
<td>0.00</td>
<td>1.08</td>
<td>0.00</td>
<td>1.08</td>
<td>0.00</td>
<td>1.08</td>
<td>0.00</td>
</tr>
<tr>
<td>C6</td>
<td>66</td>
<td>-0.12</td>
<td>0.01</td>
<td>0.01</td>
<td>0.00</td>
<td>1.03</td>
<td>0.06</td>
<td>0.20</td>
<td>0.01</td>
<td>0.07</td>
<td>0.28</td>
</tr>
<tr>
<td>C7</td>
<td>66</td>
<td>1.05</td>
<td>0.00</td>
<td>1.05</td>
<td>0.00</td>
<td>1.05</td>
<td>0.00</td>
<td>1.05</td>
<td>0.00</td>
<td>1.05</td>
<td>0.00</td>
</tr>
<tr>
<td>C9</td>
<td>66</td>
<td>1.04</td>
<td>0.00</td>
<td>1.04</td>
<td>0.00</td>
<td>1.04</td>
<td>0.00</td>
<td>1.04</td>
<td>0.00</td>
<td>1.04</td>
<td>0.00</td>
</tr>
<tr>
<td>C11</td>
<td>66</td>
<td>1.06</td>
<td>0.00</td>
<td>1.07</td>
<td>0.00</td>
<td>1.07</td>
<td>0.00</td>
<td>1.07</td>
<td>0.00</td>
<td>1.07</td>
<td>0.00</td>
</tr>
<tr>
<td>C12</td>
<td>66</td>
<td>1.09</td>
<td>0.00</td>
<td>1.10</td>
<td>0.00</td>
<td>1.09</td>
<td>0.00</td>
<td>1.09</td>
<td>0.00</td>
<td>1.09</td>
<td>0.00</td>
</tr>
<tr>
<td>C13</td>
<td>66</td>
<td>1.01</td>
<td>0.00</td>
<td>1.01</td>
<td>0.00</td>
<td>1.01</td>
<td>0.00</td>
<td>1.01</td>
<td>0.00</td>
<td>1.01</td>
<td>0.00</td>
</tr>
<tr>
<td>C15</td>
<td>66</td>
<td>1.06</td>
<td>0.00</td>
<td>1.06</td>
<td>0.00</td>
<td>1.06</td>
<td>0.00</td>
<td>1.06</td>
<td>0.00</td>
<td>1.06</td>
<td>0.00</td>
</tr>
<tr>
<td>C16</td>
<td>66</td>
<td>1.03</td>
<td>0.00</td>
<td>1.03</td>
<td>0.00</td>
<td>1.03</td>
<td>0.00</td>
<td>1.03</td>
<td>0.00</td>
<td>1.03</td>
<td>0.00</td>
</tr>
<tr>
<td>C17</td>
<td>66</td>
<td>1.03</td>
<td>0.00</td>
<td>1.03</td>
<td>0.00</td>
<td>1.03</td>
<td>0.00</td>
<td>1.03</td>
<td>0.00</td>
<td>1.03</td>
<td>0.00</td>
</tr>
<tr>
<td>C18</td>
<td>66</td>
<td>1.00</td>
<td>0.00</td>
<td>1.00</td>
<td>0.00</td>
<td>1.00</td>
<td>0.00</td>
<td>1.00</td>
<td>0.00</td>
<td>1.00</td>
<td>0.00</td>
</tr>
<tr>
<td>A1</td>
<td>500</td>
<td>0.37</td>
<td>0.03</td>
<td>0.61</td>
<td>0.03</td>
<td>0.54</td>
<td>0.03</td>
<td>0.60</td>
<td>0.03</td>
<td>0.52</td>
<td>0.12</td>
</tr>
<tr>
<td>A2</td>
<td>500</td>
<td>0.80</td>
<td>0.05</td>
<td>0.83</td>
<td>0.04</td>
<td>0.82</td>
<td>0.05</td>
<td>0.83</td>
<td>0.04</td>
<td>0.82</td>
<td>0.05</td>
</tr>
<tr>
<td>A3</td>
<td>500</td>
<td>0.67</td>
<td>0.04</td>
<td>0.72</td>
<td>0.04</td>
<td>0.70</td>
<td>0.04</td>
<td>0.72</td>
<td>0.04</td>
<td>0.70</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Figure 15. Voltage estimate mean error from comparison of LVE and full power flow solutions

For each alpha setting, a B matrix is computed and several switching combinations of shunt capacitors, reactors, and transformer tap changes are applied. The voltage estimates from each combination was then compared to a full power flow computation by simply computing the percent difference (error) in the estimated voltage magnitudes. The alpha values used were computed by inducing a voltage change in the full power flow simulation, using a specific switching action described in the Table 2. Note that a more detailed clustering of alpha values, based on different switching actions, is considered here. While it is hypothesized that selection of the best alpha will greatly depend on the anticipated switching action, it is ideal to have a single set of alpha for most if not all types of switching combinations to simplify the estimation process. The switching combinations tested on each alpha set are as shown in Table 3 below. The base case used in the test has four 28.8 Mvar capacitors of 66 kV bus switched in by default, and all other shunt elements are switched out.

Figure 15 presents the voltage estimate mean error as compared between LVE and full power flow solution. The most notable outcome from all the estimation error tests is that the errors are all well below 1%. Comparing the mean voltage errors among
alpha settings from groups I to VIII within each combination test type from A to S, it can be seen that most of the values have practically the same error rates, except for alpha groups V and VI, whose results stand out in most combination types.

Table 2. Alpha Groups

<table>
<thead>
<tr>
<th>No.</th>
<th>Alpha Groups</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>500 kV 150 Mvar Cap On</td>
</tr>
<tr>
<td>II</td>
<td>230 kV 79.2 Mvar Cap On</td>
</tr>
<tr>
<td>III</td>
<td>66 kV 28.8 x 2 Mvar Cap Off</td>
</tr>
<tr>
<td>IV</td>
<td>13.8 kV -45 x 2 MvarReac On</td>
</tr>
<tr>
<td>V</td>
<td>AA transformer bank 1 tap up</td>
</tr>
<tr>
<td>VI</td>
<td>AA transformer bank 1 tap down</td>
</tr>
<tr>
<td>VII</td>
<td>A transformer bank 1 tap up</td>
</tr>
<tr>
<td>VIII</td>
<td>A transformer bank 1 tap down</td>
</tr>
</tbody>
</table>

Table 3. Switching Combinations

<table>
<thead>
<tr>
<th>No.</th>
<th>Switching Combinations</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>500 kV 79.2 Mvar Cap</td>
</tr>
<tr>
<td>B</td>
<td>500 kV 79.2 Mvar Cap + 230 kV 150 Mvar Cap</td>
</tr>
<tr>
<td>C</td>
<td>230 kV 150 Mvar Cap</td>
</tr>
<tr>
<td>D</td>
<td>230 kV 150 Mvar Cap + 66 kV 28.8 x 2 Mvar Cap Off</td>
</tr>
<tr>
<td>E</td>
<td>66 kV 28.8 x 2 Mvar Cap Off</td>
</tr>
<tr>
<td>F</td>
<td>500 kV 79.2 Mvar Cap + 66 kV 28.8 x 2 Mvar Cap Off</td>
</tr>
<tr>
<td>G</td>
<td>500 kV 79.2 Mvar Cap + 230 kV 150 Mvar Cap + 66 kV 28.8 x 2 Mvar Cap Off</td>
</tr>
<tr>
<td>H</td>
<td>13.8 kV -45 x 2 MvarReac On</td>
</tr>
<tr>
<td>I</td>
<td>500 kV 79.2 Mvar Cap + 13.8 kV -45 x 2 MvarReac On</td>
</tr>
<tr>
<td>J</td>
<td>230 kV 150 Mvar Cap + 13.8 kV -45 x 2 MvarReac On</td>
</tr>
<tr>
<td>K</td>
<td>AA bank 2 taps up</td>
</tr>
<tr>
<td>L</td>
<td>AA bank 2 taps down</td>
</tr>
<tr>
<td>M</td>
<td>A bank 2 taps up</td>
</tr>
<tr>
<td>N</td>
<td>A bank 2 taps down</td>
</tr>
<tr>
<td>O</td>
<td>AA bank 1 tap up + 500 kV 79.2 Mvar Cap</td>
</tr>
<tr>
<td>P</td>
<td>AA bank 1 tap up + 230 kV 150 Mvar Cap</td>
</tr>
<tr>
<td>Q</td>
<td>AA bank 1 tap up + 13.8 kV -45 x 2 MvarReac On</td>
</tr>
<tr>
<td>R</td>
<td>A bank 1 tap up + 230 kV 150 Mvar Cap</td>
</tr>
<tr>
<td>S</td>
<td>A bank 1 tap up + 66 kV 28.8 x 2 Mvar Cap Off</td>
</tr>
</tbody>
</table>

5. Conclusions

This paper tested the concept of local reactive power flow model, through comprehensive study of a typical 500/230/66 kV substation and surrounding network in Southern California.

First, linear behavior of voltage changes in a local network under the switching effect of discrete control devices was established through voltage sensitivity analysis for transformer tap changing actions. It was found out that voltage changes on each internal bus cause external bus voltages to follow the behavior of the internal bus in the same voltage level, but to different extents, based on their electrical proximity and status of connection to the internal bus. Next, it is observed that although a bus may be directly disconnected from the internal bus, but still may get affected by voltage changes of other internal buses with higher voltage levels, due to the loop connections mainly in 500 kV and 230 kV network. Another significant effect of these indirect connections is different calculated alpha values for different sets of switching control actions. Also, alpha values of 66 kV level, do not significantly change, with line outages on 500 and 230 kV levels.

Finally, it was concluded that defining different sets of alpha parameters in the LVE formulation based on the type of switching action results in accurate estimation of post-switching voltages with less than 0.5% prediction error compared to full nonlinear power-flow models of the large power system. Therefore, the studies in this paper effectively validate the suitability of LVE formulation for carrying out local power-flow computations using local synchrophasor measurements for any SLVC.

6. References