An Economic Evaluation Tool of Inertia Services for Systems with Integrated Wind Power and Fast-acting Storage Resources

Ti Xu, Wonhyeok Jang and Thomas Overbye
Electrical and Computer Engineering
University of Illinois at Urbana-Champaign, Urbana, Illinois 61801
{txu20, wjang7, overbye}@illinois.edu

Abstract
As the increasing utilization of wind resources and lightweight gas turbine units, the power grid is shifting towards a situation with declining system inertia, which causes a larger frequency deviation after disturbances and threatens the reliability of the grid. Thus, we devote this work to develop an appropriate tool to determine the economic value of the inertia to the systems. To do so, we discretize and integrate the system dynamic equations into a modified version of the linear security-constrained unit commitment framework. Since the fast-acting storage devices are able to respond in a very short time, we also develop a storage control scheme to mimic the inertia and to provide primary frequency responses. We perform extensive studies on the IEEE 24-bus reliability test system and present some illustrative simulation results to demonstrate the effectiveness of the proposed method to assess the economic value of the power system inertia.

1. Nomenclature

\begin{align*}
i & \quad \text{Index of the conventional units from 1 to } I \\
n & \quad \text{Index of the simulation sub-periods from 1 to } N \\
k & \quad \text{Index of the dynamic simulation intervals from 0 to } K_s \text{ for each simulation sub-period} \\
\end{align*}

Variables - Dynamic Model

\begin{align*}
p_{i,n}^{p} & \quad \text{Output of each conventional unit [p.u.]} \\
r_{i,n}^{up} & \quad \text{Unit up spinning reserve [p.u.]} \\
r_{i,n}^{down} & \quad \text{Unit down spinning reserve [p.u.]} \\
r_{i,n}^{nons} & \quad \text{Unit non-spinning reserve [p.u.]} \\
p_{n}^{w} & \quad \text{Aggregated output from wind farms [p.u.]} \\
c_{i,n} & \quad \text{Production costs of each unit [$]} \\
c_{i,n}^{on} & \quad \text{Start-up costs of each conventional unit [$]} \\
u_{i,n} & \quad \text{State variable of each conventional unit} \\
u_{i,n}^{*,*} & \quad \text{Storage charging/discharging state variable} \\
p_{n}^{pp} & \quad \text{Storage charging rate [p.u.]} \\
p_{n}^{pr} & \quad \text{Storage discharging rate [p.u.]} \\
\varepsilon_{i} & \quad \text{Stored energy level in the storage [h]} \\
\end{align*}

Parameter

\begin{align*}
\Delta T & \quad \text{Step-size of each dynamic simulation interval [s]} \\
\alpha_{i,j}^{a}, \beta_{i,j}^{r} & \quad \text{Coefficients of the } j^{th} \text{ segment of the piecewise liner generation cost curve for each conventional unit [$/p.u.] } \\
o_{i}^{on} & \quad \text{Unit start-up cost [$]} \\
c_{i}^{up} & \quad \text{Unit up spinning reserve cost [$/p.u.] } \\
c_{i}^{down} & \quad \text{Unit down spinning reserve cost [$/p.u.] } \\
c_{i}^{nons} & \quad \text{Unit non-spinning reserve cost [$/p.u.] } \\
P_{i}^{cap}, P_{i}^{i} & \quad \text{Max/min generation limits of each unit [p.u.]} \\
\Delta P_{i}^{up} & \quad \text{Max ramping-up rate of each unit [p.u.]} \\
\Delta P_{i}^{down} & \quad \text{Max ramping-down rate of each unit [p.u.]} \\
d_{i} & \quad \text{Supplied load [p.u.]} \\
L_{i}^{up} & \quad \text{System min up spinning reserve [p.u.]} \\
L_{i}^{down} & \quad \text{System min down spinning reserve [p.u.]} \\
L_{i}^{nons} & \quad \text{System min non-spinning reserve [p.u.]} \\
H_{i} & \quad \text{Unit inertia [s]} \\
H_{i}^{pr} & \quad \text{Virtual inertia provided by storage [s]} \\
\end{align*}
2. Introduction

The increasing awareness of the global climate change stimulates the substitution of electricity generation from conventional plants by that from less- and zero-emitting generators, such as renewable energy resources [1]. Wind power is the most mature and cost-effective to harvest renewable energy with the global installed capacity of 369 GW by the end of 2014 [2]. Also, due to the higher heat rate and lower emission rate, the utilization of gas turbine units is continuously growing. As such, the power grid is shifting to a condition with less system inertia and thus being less capable to resist the system frequency deviation from its nominal value [3]. However, currently, there is no such an effective market framework providing economic incentives for market participants to offer inertia services [3].

As shown in Fig.1, the system primary frequency response consists of inertial responses and governor responses from the generation units. In this work, we aim to investigate the impacts of additional inertia on the post-contingency frequency responses of the systems, thus we only consider the approaches capable to provide inertial responses [4], [5]. Most of work on inertia focused on the dynamic simulations of the power systems [6], [7], [8]. But there are a limited number of papers working on the economic value assessment for the power system inertia as an ancillary service. In addition, some energy storage techniques, such as fast-acting batteries and flywheel storage systems [9], can quickly respond to the frequency variations via some specified control strategy. Thus, there is an acute need to construct an effective approach to determine the economic value of inertia services for systems with integrated wind resources. In this work, we develop a modified linear security-constrained unit commitment (SCUC) framework to compute the economic values of the inertia services. We carefully discretize the dynamic equations of a simplified system dynamic model and incorporate them into the SCUC framework as transient-stability constraints to represent the system dynamic performance requirements. The control scheme for fast-responding storage devices is addressed here as an alternative to provide inertia services. We perform extensive studies on the IEEE 24-bus reliability test system (RTS) [10], [11]. Several illustrative simulation results are presented to demonstrate the effectiveness of the proposed method to assess the economic value of the provided inertia services.

There are five more sections in this paper. In section 3, we describe the construction of the proposed evaluation tool. We discuss the equivalence transformation of nonlinear constraints into their linear forms in section 4. In section 5, we develop the control strategy for a fast-acting storage device to mimic the inertial response. Section 6 presents some selected illustrative simulation results and, in section 7, we conclude our work and provide directions for future work.

3. Economic Evaluation Framework for Inertia Services

In this section, we first briefly review the dynamic simulation and SCUC models. Then we devote to describe the modifications essential to integrate dynamic simulation and SCUC models to assess the economic value of the provided inertia services.

We show in Fig. 2 an overview of the construction of the proposed evaluation framework and summarize the construction process in three steps:

- discretizing the dynamic simulation model;
- integrating the discrete-time dynamic model into steady-state unit commitment model as transient-stability security constraints;
- transforming nonlinear constraints into equivalent, linear forms.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>$R_i$</td>
<td>Droop or regulation constant [s]</td>
</tr>
<tr>
<td>$T_{i,1}$</td>
<td>Governor lag time constant [s]</td>
</tr>
<tr>
<td>$T_{i,2}$</td>
<td>Turbine lead time constant [s]</td>
</tr>
<tr>
<td>$T_{i,3}$</td>
<td>Turbine lag time constant [s]</td>
</tr>
<tr>
<td>$D$</td>
<td>System load damping constant</td>
</tr>
<tr>
<td>$\Delta \omega_{\text{up}}$</td>
<td>Upper limit of frequency deviation [p.u.]</td>
</tr>
<tr>
<td>$\Delta \omega_{\text{down}}$</td>
<td>Lower limit of frequency deviation [p.u.]</td>
</tr>
<tr>
<td>$\bar{P}_{\text{max}}$</td>
<td>Max storage charging/discharging rate [p.u.]</td>
</tr>
<tr>
<td>$\eta^+, \eta^-$</td>
<td>Storage charging/discharging efficiency</td>
</tr>
</tbody>
</table>
where \( x \) and \( u \) represent the continuous (e.g., generator output) and discrete (e.g., generator state) variables, respectively, with the corresponding cost coefficients \( C_x \) and \( C_u \). Constants \( A, B \) and \( e \) are the parameters in the linear constraints. Values of the steady-state variables \( x \) determine the initial values \( z(0) \) for dynamic variables \( z \). In the transient stability model, \( z \) denotes as the generator dynamic variables and \( v \) refers to some constants, such as voltage reference points. \( E(u), F(u), G \) and \( e \) are the parameters used in the dynamic model. And \( E(u) \) and \( F(u) \) depend on the values of \( u \). \( \hat{z} \) is the discretized representation of variables \( z \); \( E' \) and \( E'' \) are the discrete (e.g., generator state) parameters used in the dynamic model. And discretized representation of \( F(u) \);

**a. System Dynamic Model and Its Discretization**

Typically, the power system dynamic simulation involves solving a set of nonlinear differential and algebraic equations [12], which cast challenges to consider the dynamic performance criteria in SCUC framework. Discretization of continuous-time attributes, widely used in numerical solution methods for solving differential and algebraic equations, is an important step to represent the dynamic model in a steady-state problem. Thus, we adopt a discretized representation of the power system dynamic behaviors - specifically, the primary frequency response model - to be effectively integrated into the SCUC model as transient-stability constraints.

Consider a generation set of \( I \) conventional units, we utilize the primary frequency response analysis model (1)-(5), shown in Fig.3, from [13], [14], which 1) ignores the network effects; 2) assumes all the generators move coherently as a single lumped mass; 3) assumes that system load damping behaviors are lumped together and modeled as a single constant.

\[
\begin{align*}
\frac{d\Delta \omega}{dt} & = \sum_{i=1}^{I} p_i^{c,m} - d - D\Delta \omega \\
\frac{dp_i^{c,v}}{dt} & = \frac{1}{T_{i,3}} (p_i^{c,v} - p_i^{c,v}) \quad (2) \\
\frac{dp_i^{c,s}}{dt} & = \frac{1}{T_{i,3}} (p_i^{c,v} - p_i^{c,m}) \quad (3) \\
p_i^{c,v} & = p_i^{c,m} - \frac{1}{R_i} \Delta \omega \quad (4) \\
p_i^{c,m} & = \frac{T_{i,2}}{T_{i,3}} p_i^{c,v} + p_i^{c,s} \quad (5)
\end{align*}
\]

where non-windup limits are applied to (2): \( p_i^{c,v} \) will respond immediately to any change in \( p_i^{c,m} \), bounded in \([ p_i^{c,v}, \bar{p}_i^{c,v} ] \).

To focus on the nature of this work, we simply apply Forward Euler method [15] to discretize those dynamic equations (1)-(5) with a step-size of \( \Delta T_s \) indexed from 0 to \( K_s \):

\[
\Delta \omega[k+1] - \Delta \omega[k] = \sum_{i=1}^{I} p_i^{c,m}[k] - d - D\Delta \omega[k] \\
\frac{\Delta \omega[k+1] - \Delta \omega[k]}{\Delta T_s} = \sum_{i=1}^{I} 2H_i
\]

\[
\frac{p_i^{c,v}[k+1] - p_i^{c,v}[k]}{\Delta T_s} = \frac{1}{T_{i,3}} (p_i^{c,v}[k] - p_i^{c,v}[k]) \quad (7) \\
\frac{p_i^{c,s}[k+1] - p_i^{c,s}[k]}{\Delta T_s} = \frac{1}{T_{i,3}} (p_i^{c,v}[k] - p_i^{c,m}[k]) \quad (8) \\
p_i^{c,v}[k] = \min \{ p_i^{c,m}[k] - \frac{\Delta \omega[k]}{R_i}, \bar{p}_i^{c,v} \} \quad (9) \\
p_i^{c,m}[k] = \frac{T_{i,2}}{T_{i,3}} p_i^{c,v}[k] + p_i^{c,s}[k] \quad (10)
\]

After discretization, the algebraic equations (6)-(10) are capable to be integrated into an optimization formulation as transient-stability constraints.
b. Integration of Discretized Dynamic Equations into SCUC Framework

The SCUC model is used to determine the commitment plan that minimizes the electricity production cost while satisfying a set of physical and operational, security and reliability constraints [16]. In our work, we consider a contingency with a sudden change in load from \( d_n \) to \( d_n + \Delta d_n \) at some time for each sub-period\(^1\). We formulate the SCUC problem with three different categories of constraints:

- the steady-state constraints: (12)-(17);
- the transient-stability constraints: (18)-(21);
- the linkage between the steady-state and transient-stability constraints: (22)-(24);

\[
\min \sum_{s=1}^{S} \sum_{i=1}^{I} \left( c_{i,n}^{up} + c_{i,n}^{dn} + c_{i,n}^{cr} r_{i,n}^{up} + c_{i,n}^{cr} r_{i,n}^{dn} + c_{i,n}^{m} r_{i,n}^{m} \right) \quad (11)
\]

s.t.

\[
p_{i,n}^{up} \leq P_{i,n} + r_{i,n}^{up} \leq u_{i,n} \tilde{P}_i, \quad L_{i,n}^{up} \leq \sum_{i=1}^{I} r_{i,n}^{up} \quad (12a)
\]

\[
p_{i,n}^{dn} \geq P_{i,n} - r_{i,n}^{down} \geq u_{i,n} \tilde{P}_i, \quad L_{i,n}^{down} \leq \sum_{i=1}^{I} r_{i,n}^{down} \quad (12b)
\]

\[
0 \leq r_{i,n}^{m} \leq \left( 1 - u_{i,n} \right) P_{i,n} + \tilde{L}_{i,n}^{m} \leq \sum_{i=1}^{I} r_{i,n}^{m} \quad (13)
\]

\[
\tilde{P}_i - u_{i,n-1} \left( P_{i,n} - \Delta P_{i,n}^{up} \right) - u_{i,n} \left( \tilde{P}_i - P_{i,n} \right) \geq p_{i,n}^{cr} - p_{i,n-1}^{cr} \geq 0 \quad (14)
\]

\[
\sum_{i=1}^{I} P_{i,n}^{up} + P_{n}^{dn} = d_n \quad (15)
\]

\[
c_{i,n}^{up} \geq \alpha_{i,n}^{up}, p_{i,n}^{up} + b_{i,n} u_{i,n} \quad (16)
\]

\[
c_{i,n}^{dn} \geq \alpha_{i,n}^{dn} \left( u_{i,n} - u_{i,n-1} \right), c_{i,n}^{m} \geq 0 \quad (17)
\]

The objective function (11) aims to minimize the total production costs, including the generation, unit start-up and reserve offering costs. The constraint (12) assures that each online unit generates electricity within its bounds. Both (12) and (13) connect the unit output, unit status and the scheduled reserves and guarantee the scheduled reserves satisfy the system-wise reserve requirements as well. Equation (14) represents the unit ramping constraints. The real power balance is represented in (15). Constraints (16) and (17) set the piecewise liner electricity generation costs and the unit start-up costs of each conventional unit, respectively.

\[
\sum_{i=1}^{I} 2u_{i,n} H_i \frac{\Delta \omega_n[k+1] - \Delta \omega_n[k]}{\Delta T_s} = \sum_{i=1}^{I} p_{i,n}^{cr}[k] + p_{n}^{dn} - d_n - D \Delta \omega_n[k] \quad (18)
\]

\[
p_{i,n}^{cr}[k] = \max \{ \min \{ p_{i,n}^{cr} \Delta \omega_n[k] / R_i, \tilde{P}_i^{cr} \}, \tilde{P}_i^{cr} \} u_{i,n} \quad (19)
\]

\[
\max \{ \min \{ p_{i,n}^{cr} \Delta \omega_n[k] / R_i, \tilde{P}_i^{cr} \}, \tilde{P}_i^{cr} \} u_{i,n} \quad (20)
\]

\[
\Delta \omega \leq \Delta \omega_n \leq \Delta \tilde{\omega} \quad (21)
\]

Constraints (18) - (21) are the modified version of dynamic equations (6) - (10). Nonlinear equation (18) quantifies the frequency deviation due to the imbalance between the unit mechanical input and electrical output, and (18) also guarantees that only those online units are able contribute to the system dynamic response. Constraint (19) assures that \( p_{i,n}^{cr} \) is zero when the unit is offline, and also represents that \( p_{i,n}^{cr} \) is bounded within \([\tilde{P}_i^{cr}, \tilde{P}_i^{cr}]\) and determined by \( p_{i,n}^{cr} \Delta \omega_n[k] / R_i \) for those online units. Constraint (21) sets the frequency deviation bounds. We note that the wind turbine inertia is very small compared to those of conventional units [17], [18]. The newer wind turbine models are type 3 and 4, of which the inertia is not seen by the grid. Thus, in this work, we neglect the inertia responses from wind turbines.

At last, we use equations (22) - (24) to connect steady-state and transient-stability constraints:

\[
\Delta \omega_n[0] = 0 \quad (22)
\]

\[
p_{i,n}^{cr}[0] = p_{i,n}^{cr}[0] = p_{i,n}^{up}[0] \quad (23)
\]

\[
p_{i,n}^{dn}[0] = (1 - T_{i,3}) p_{i,n}^{up}[0] \quad (24)
\]

c. Evaluation Process

To compute the economic value of the additional inertia into the grid, we perform simulations twice on the same system without any change except for modifying the total inertia value by a certain amount. The production cost decrement approximates the economic value of the system inertia increment.

4. Transformation of Nonlinear Constraints into Linear Constraints

Some nonlinear constraints, such as \( \Delta \omega_n[k] u_{i,n} \), and \( \max \{ \cdot, \cdot \} \), cannot be directly used in a linear optimization tool. Therefore, in this section, we

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\(^1\) Since positive frequency deviation is easy to be handled by curtail generation, we only consider the under-frequency contingency with a sudden load increase in this paper.
describe the equivalence transformation of nonlinear constraints into their linear forms.

a. Equivalence Transformation Concept

Consider the constraint \( z = x \times b \), with a continuous variable \( x \in [\bar{x}, \overline{\bar{x}}] \) and a binary variable \( b \). We propose an equivalent representation of 
\[ z = x \times b \]
by:
\[ x \bar{b} \leq z \leq \overline{x} b \]  
\[ x - (1 - b) \bar{x} \leq z \leq x - (1 - b) \overline{x} \]  
(25)  
(26)

When \( b = 0 \), \( x \) is 0 from (25) and satisfies constraint (26); when \( b = 1 \), \( z \) is \( x \) from (26) without violation of (25). We can see that (25) and (26) are equivalent to \( z = x \times b \).

Secondly, we consider the case \( z = \max \{x_1, x_2\} \), where \( x_1 \) and \( x_2 \) are continuous variables. We introduce binary variables \( b_1 \) and \( b_2 \), and propose an equivalent representation by:
\[ b_1 + b_2 = 1 \]  
\[ x_1 \leq z \leq x_1 + (1 - b_1) M \]  
\[ x_2 \leq z \leq x_2 + (1 - b_2) M \]  
(27)  
(28)  
(29)

where \( M \) is a sufficiently large positive number that makes the constraint set feasible. There are only two possible combinations of \( b_1 \) and \( b_2 \): \([1, 0]\) and \([0, 1]\). Under the condition \( x_1 > x_2 \), \( z \geq x_1 > x_2 \) from (28) and \( b_2 \) must be 0 with \( b_1 = 1 \), otherwise (29) is violated. We can carry a similar analysis on (28)-(29) for \( x_1 \leq x_2 \). Thus, (27)-(29) are proved to be equivalent to \( z = \max \{x_1, x_2\} \).

b. Equivalence Transformation Application

The first term that needs to be addressed is \( \Delta \omega_n[k] \mu_{i,n} \) from (18) and (19). We define a new variable \( z_{i,n}[k] = \Delta \omega_n[k] \mu_{i,n} \). Based on the analysis above, its equivalent representation is
\[ \Delta \omega \ u_{i,n} \leq z_{i,n}[k] \leq \Delta \bar{\omega} \ u_{i,n} \]  
(30)

\[ (1 - u_{i,n}) \Delta \bar{\omega} \leq \Delta \omega_n - z_{i,n}[k] \leq (1 - u_{i,n}) \Delta \bar{\omega} \]  
(31)

So, constraints (18) and (19) are modified into:
\[ \sum_{i=1}^{I} 2H_i \frac{z_{i,n}[k+1] - z_{i,n}[k]}{\Delta T_s} = \]  
\[ \sum_{i=1}^{I} p_{i,n}^c[k] + p_{n}^w - d_n - \Delta d_n - D \Delta \omega_n[k] \]  
(32)

Then, to further linearize equation (33) that contains both \( \max \{z, 0\} \) and \( \min \{z, 0\} \), we introduce three binary variables \( b_{1,i,n}[k], b_{2,i,n}[k], b_{3,i,n}[k] \) such that:
\[ b_{1,i,n}[k] + b_{2,i,n}[k] + b_{3,i,n}[k] = 1 \]  
(34)

\[ \overline{P}_{i,n}^c u_{i,n} \leq \overline{P}_{i,n}^c u_{i,n} \]  
(35)

\[ \underline{P}_{i,n}^c u_{i,n} \leq \overline{P}_{i,n}^c u_{i,n} \]  
(36)

\[ \overline{P}_{i,n}^c u_{i,n} \leq \overline{P}_{i,n}^c u_{i,n} \]  
(37)

\[ \underline{P}_{i,n}^c u_{i,n} \leq \overline{P}_{i,n}^c u_{i,n} \]  
(38)

\[ \overline{P}_{i,n}^c u_{i,n} \leq \overline{P}_{i,n}^c u_{i,n} \]  
(39)

As a result, the complete linear SCUC model is described using (11)-(17), (20)-(24), (30)-(32) and (34)-(39). The number of variables in the constructed model depends on the adopted SCUC and dynamic models. In general, the scale of the problem is is in the order of \( O(I N K) \).

5. Storage Control Strategy to Mimic Inertia

Current storage technologies enable storage of some types to respond up to its maximum charging/discharging rate within 1 ms [19], [20]. In this paper, we also address the utilization of storage devices as a potential method to provide some additional inertia into the grid during the system transient processes.

a. Mathematic Representation of the Storage Control Strategy

In consideration of an energy storage device connected to the grid, we modify equation (1) into
\[ \frac{d \Delta \omega}{dt} = \frac{\sum_{i=1}^{I} p_{i,n}^m - d - D \Delta \omega + p^s}{\sum_{i=1}^{I} 2H_i} \]  
(40)

Without any specified control strategy, the storage output \( p^s \) is always fixed at its scheduled constant value \( p^{s_0} \). We make use of these fast-responding advantages to design such a control algorithm to
provide some virtual inertia into the power system. The core idea of this design is to control the storage output such that the system frequency response behaves as if there is an additional inertia of value \( H^* \) with the storage output remaining at the initial value \( P^0 \). Thus, (40) can be re-expressed by:

\[
\frac{d\Delta \omega}{dt} = \sum_{i=1}^{j} p_{i}^{*m} - d - D\Delta \omega + p^{x0}
- 2H^* \sum_{i=1}^{j} 2H_i + H^*
\tag{41}
\]

From (40) and (41), we can have:

\[
P^* = p^{x0} - 2H^* \frac{d\omega}{dt} = p^{x0} - 2H^* \frac{d\Delta \omega}{dt}
\tag{42}
\]

Similarly, we can derive \( p_{n}^*[k] \) by

\[
p_{n}^*[k] = p_{n}^*[0] - 2H^* \frac{\Delta \omega_{n}[k+1] - \Delta \omega_{n}[k]}{\Delta T_s}
\tag{43}
\]

Equation (43) is sufficient for us to generate the control signal for storage devices. However, the storage output cannot violate its bounds \([-P^*, \overline{P}^*]\), bringing a nonlinear constraint into the optimization problem. Similarly, we can introduce binary variables \( q_{1,n}[k], q_{2,n}[k], q_{3,n}[k] \) to linearize the constraints by:

\[
q_{1,n}[k] + q_{2,n}[k] + q_{3,n}[k] = 1
\tag{44}
\]

\[
p_{n}^*[k] \geq p_{n}^*[0] - 2H^* \frac{\Delta \omega_{n}[k+1] - \Delta \omega_{n}[k]}{\Delta T_s} - (1 - q_{1,n}[k])M
\tag{45}
\]

\[
p_{n}^*[k] \leq p_{n}^*[0] - 2H^* \frac{\Delta \omega_{n}[k+1] - \Delta \omega_{n}[k]}{\Delta T_s} + (1 - q_{1,n}[k])M
\tag{46}
\]

\[
p_{n}^*[k] \geq \overline{P}^* - (1 - q_{2,n}[k])M
\tag{47}
\]

\[
p_{n}^*[k] \leq \overline{P}^* + (1 - q_{3,n}[k])M
\tag{48}
\]

As for steady-state constraints for storage devices, constraint (50) limits that the storage must maintain a level within its physical bounds. Equation (51) guarantees that storage can only be in charging or discharging state and bounds its output. Equation (52) re-expresses the real power balance and (53) is the energy balance equation for storage with consideration of charging/discharging efficiency. Constraint (54) serves as the linkage between steady-state and dynamic constraints.

\[
e_n \leq \varepsilon_n \leq \overline{E}
\tag{50}
\]

\[
u_{n}^+ + u_{n}^- = 1
\tag{51a}
\]

\[
0 \leq p_{n}^{*+} - u_{n}^+ \overline{P}^* \leq 0 \leq p_{n}^{*-} - u_{n}^- \overline{P}^*
\tag{51b}
\]

\[
\sum_{i=1}^{j} p_{i}^{*+} + p_{n}^{*+} - p_{n}^{*-} - p_{n}^{*-} = d_{n}
\tag{52}
\]

\[
\varepsilon_n = \varepsilon_{n-1} + p_{n}^{*+} - \frac{p_{n}^{*-}}{\eta}
\tag{53}
\]

\[
p_{n}[0] = -p_{n}^{*+} + p_{n}^{*-}
\tag{54}
\]

b. Numerical Example

<table>
<thead>
<tr>
<th>Table 1 Parameters in Numerical Example</th>
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<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>value(s)</td>
</tr>
</tbody>
</table>

Here we provide a numerical example to illustrate the proposed control strategy of storage to mimic inertia. Consider a system with ten 120-MW (=1.2 p.u.) conventional units with inertia varying from 3.5s to 2.5s, in the decrement of 0.1s and time constants shown in Table 1. For the contingency with a sudden load increase of 1 p.u. from the base load 10 p.u. and the initial storage output of 0, we perform dynamic simulations using equations (40) and (43) for storage with the virtual inertia value of 0s, 3s, 6s and 9s. We present the storage output and frequency deviation in Fig.4. We note that the higher the virtual inertia is, the better frequency response the system has and the higher charging/discharging capacity the storage needs to have.

\[
\text{Figure 4: Dynamic simulation at time resolution of 0.05s}
\]

We also compare the influences of using the storage control scheme with equations (40) and (43), and using the (41) with a fixed initial storage output (=0 p.u.) on the dynamic simulation results with the virtual inertia value of 6s. As shown in Fig.3, the nearly indistinguishable curves verify the correctness of the proposed control scheme.
For the purposes of comparisons among time-resolutions, we perform dynamic simulations using a 0.01-s and 0.05-s (Fig.4) time step and display the results with a finer time resolution in Fig.6. The results clarify that a step size of 0.05s is roughly good enough for utilization by dynamic constraints in the SCUC. For more realistic simulations, we perform simulations setting $H = 9s$ with considering the limited storage charging/discharging capacity. We note in Fig.7 the storage with a tighter bound on the charging/discharging rates has less capability to mimic the inertia.

In addition, we perform studies to investigate the impacts of different initial outputs $p_{s0}$ on the performances of the fact-acting storage to mimic then inertial responses, in consideration of the charging and discharging capacities to be 10 MW. The results shown in Fig. 8 indicate that the storage devices in charging status help the system arrest the frequency while those in discharging status further worsen the minimum frequency compared to the case without storage. The observation is reasonable since there are more conventional units online with higher outputs to contribute to frequency regulation when the storage device is withdrawing electricity from the grid. Furthermore, we note that the storage with -10 MW initial output can contribute to the inertial responses right after the contingency occurs while the storage with 10 MW initial output starts to help arrest the frequency deviation only after the frequency has been dropping for a second.

A comprehensive analysis of the impacts of storage parameters on the frequency controls in the system are performed and the results presented in this section demonstrate that the storage devices can be controlled in such a way to provide a certain amount of inertia services to the power systems with some initial working status. In this paper, we only consider the utilization of storage to mimic inertial responses with their initial scheduled output fixed at zero because this work aims to determine the value of additional inertia into grid instead of valuing the contribution of the fact-acting storage devices. The capability of fact-acting storage devices to respond to frequency deviation is simply one of the methods used to mimic inertia response.

6. Illustrative Simulation Study

We performed extensive simulation studies and devote this section to present some illustrative results to verify the capability of the proposed method to quantify the economic value of inertia services for power systems with wind resources and fast-responding storage installation. The studies were performed on a modified version of the 24-bus IEEE RTS, of which the generator parameters, such as droop or regulation constants, are obtained from [21].
All generators are assumed to provide primary frequency responses with their outputs no violation of the corresponding max/min limits. We scale 2004 WECC load data [22] to the same load level with an annual peak load of 2,850 MW. The load damping rate is assumed to be zero. The up/down/non-spinning reserve requirements are set at 5%, 2% and 5%, respectively. We consider 2-MW MM 92 [23] as the wind turbine model and use historical wind speed measurement from [24] to build the wind power portfolio. We assume that the storage device has sufficiently large charging/discharging capacity to provide the proposed inertia service. An under-frequency contingency is considered with sudden load change of a 100-MW (=1 p.u.) increment and the minimum frequency requirement is set at 59.7 Hz.

a. Case Study I – Varying Load Level

For illustration purposes, we start with a simulation case study for a single hour on the test system with load levels varying from 1,000 MW to 2,500 MW, in 100-MW increment, and no wind farms. We perform simulations with and without an additional inertia of 0.3 s.

We summarize in Fig. 9 the computed production costs under different load conditions. The broken line indicates the difference between production costs with and without the additional inertia, which represents the economic benefit brought into system by introducing the additional inertia. For a fixed load level lower than 2,200 MW, the introduction of additional inertia reduces the total production costs because the system can make use of the additional inertia without the need to put more units online for maintaining the post-contingency system frequency above the required level. We also note that, as the load increases, the economic benefit of the additional inertia declines and becomes zero after the load reaches 2,200 MW. The results are reasonable since a higher load level requires more units online, resulting in the higher total system inertia value and less need of the additional inertia to resist frequency deviation such as to satisfy frequency requirement.

b. Case Study II – The Impacts of Increasing Virtual Inertia

In the second case study, we aim to investigate the impacts of the increasing virtual inertia provided by the storage with sufficiently large capability and capacity. We perform simulations on the test system with a fixed load at 2,000 MW and no wind power, and present in Table 2 the economic value of the additional inertia for cases with virtual inertia ranging from 0 to 5 s, in the increment of 0.5 s, and from 5 to 25 s, in the increment of 5 s. As the additional inertia increases, its economic value becomes larger because the higher virtual inertia enables the system to schedule the units with lower costs and lower inertia to supply load. The corresponding post-contingency frequencies are shown in Fig. 10 (without frequency constraint) and Fig. 11 (with frequency constraint). The Fig. 12 presents the simulated storage outputs for each additional virtual inertia value.
c. Case Study III – Deepening Penetration of Wind Power Resources

In the third study, we investigate the impacts of wind resource integration on the economic value of power system inertia. First, we perform simulations on the system with a fixed load at 2,200 MW with wind power penetration varying from 0% to 30%. We present in Fig. 13 the average energy costs w.r.t. deepening wind power penetration w/o/w the frequency constraint. The energy production cost declines as the wind-based generation increases, but such a contribution becomes less significant for the case with the frequency constraint. This is because the wind farm output replaces the generation of conventional units so that both of the total production costs and the total system inertia decreases. The declining inertia value results in re-scheduling among conventional units to assure a sufficient system inertia value.

Then, we run simulations on the system with 30% electricity supplied by wind energy and display the hourly load, wind farm outputs and the corresponding economic values of the 0.3-s additional inertia. As we see, the economic value is very low except for the hours with low load levels and high wind farm outputs. Specifically, it is more of need for the system to request additional inertia services during low net-load hours, such as early morning when load has not climbed up while wind energy reaching at a high value.

7. Conclusion and Future Work

In this work, we modified the security-constrained unit commitment framework for the effective integration of dynamic performance constraints. We also developed the control strategy for the fast-acting storage devices to provide inertia service into power grid. We applied the proposed simulation framework on a modified version of IEEE 24-bus reliability test system. The presented simulation results verified that the proposed tool is capable to compute the economic value of inertia services under different wind power and load scenarios.

The proposed tool provides the basis to construct a comprehensive market simulation tool with integration of dynamic performance security constraints. The comparison of the accuracy of the adopted dynamics model with that of a full simulation is of huge interest as well. The impacts of wind turbines capable to provide frequency regulation services will be considered. We will address these needs in our future work on valuing the inertial services. In addition, in our future research on evaluation of the fact-acting storage contribution to the power systems, we will consider to use fast-acting storage devices to provide energy, ancillary and primary frequency regulation services and study its contribution to the power systems.

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9. References


