Firm Failure Timeline Prediction: Math Programming Approaches

Young U. Ryu
School of Management
The University of Texas at Dallas
ryoung@utdallas.edu

Abstract—Previous studies on firm failure prediction proposed models to predict only short- and fixed-term (normally one year) bankruptcies. Our study applies two math programming methods to predict firm failure timeline (i.e., longer- and variable-term bankruptcy): isotonic prediction and modified linear regression (a form of recurrence surface approximation). With these two methods, we also evaluate financial ratios that were previously used for fixed-term bankruptcy prediction. The experiments with actual firm data indicate that (1) both approaches generate good prediction outcomes but isotonic prediction performs better than modified linear regression and (2) the use of all twenty-three financial ratios for isotonic prediction result in the best prediction outcome after a feature selection preprocess is applied.

Keywords—bankruptcy prediction; isotonic prediction; modified linear regression; prediction method

I. INTRODUCTION

There have been numerous firm failure prediction models proposed since 1960. This trend of studies reflects the importance and benefits of accurate predictions of firm failure. All previous models, however, predict short- and fixed-term (e.g., one year) bankruptcies of firms. Essentially, the problem of bankruptcy prediction was treated as a classification problem, no matter what prediction methods were used (e.g., discriminant analyses [1]–[5], neural networks [6], [7], and decision trees [8], [9]). The principal goal is to classify firms distressed within a short time period based on a set of financial variables.

We view the problem of bankruptcy prediction as a regression problem in that predicted is when a firm will fail (if it ever does). Timeline prediction with regression (i.e., when will a firm fail) is more general and gives more valuable information than simple classification (i.e., will the firm fail in the next year). The difficulty, however, is in that data from both failed firms and healthy firms are to be used in the prediction model. That is, we need methods to deal with censored data. We use isotonic prediction [10] and modified linear regression based on recurrence surface approximation [11]. They are math programming models and thus can be efficiently and quickly solved by any linear programming solver. Further, customization and variation can be easily done for bankruptcy timeline prediction.

We collected real firm data from Standard & Poor’s Com-putstar North America database and performed experiments on them using two methods. As the results indicated, both methods resulted in good prediction outcomes, but isotonic prediction performed better.

II. BANKRUPTCY TIMELINE PREDICTION MODELS

A. The Problem

Suppose a finite set $A$ of firm data in a $d$-dimensional real space $\mathbb{R}^d$ is given, which is partitioned into two subsets $A_b$ with bankruptcy timeline function $p: A_b \to [l,u]$ and $A_s$ with non-bankruptcy survival timeline function $q: A_s \to [l,u]$, where $[l,u] \subseteq \mathbb{R}$. The goal of bankruptcy prediction models is to estimate a function of bankruptcy/survival time prediction $f: \mathbb{R}^d \to [l,u]$ that is as close to $p$ and $q$ as possible by minimizing the total estimation error or mis-prediction penalty

$$\min \sum_{i \in A_b} g(p(i) - f(i)) + \sum_{i \in A_s} h(q(i) - f(i)), \quad (1)$$

where $g$ and $h$ are mis-prediction penalty functions.

In bankruptcy timeline prediction on bankrupt firms, over-estimation is much worse than under-estimation. In firm survival time prediction on non-bankrupt firms, only under-estimation is explicitly considered since over-estimation is not observable. Thus, the functions $g$ and $h$ look like those in Figure 1:

$$g(x) = \begin{cases} -\alpha_b(x + \delta_{b,1}) & \text{if } x < -\delta_{b,1} \\ \beta_b(x - \delta_{b,2}) & \text{if } x > \delta_{b,2} \\ 0 & \text{otherwise} \end{cases} \quad (2a)$$

and

$$h(x) = \begin{cases} \beta_s(x - \delta_{s,2}) & \text{if } x > \delta_{s,2} \\ 0 & \text{otherwise} \end{cases} \quad (2b)$$

where $\alpha_b \gg \beta_b \geq 0$, $\delta_{b,1} \geq 0$, $\delta_{b,2} \geq 0$, $\beta_s > 0$, and $\delta_{s,2} \geq 0$.

The formula (1) measures the overall prediction error penalty with adjustment based on relative seriousness of over-estimation and under-estimation in bankruptcy timeline prediction on $A_b$ and firm survival timeline prediction on $A_s$. Parameter $\delta_{b,1}$ denotes the amount of bankruptcy timeline over-estimation error tolerance. That is, for a bankrupt firm $i \in A_b$, if the bankruptcy timeline over-estimation error is less than $\delta_{b,1}$, then the over-estimation is not penalized in the model. Similarly, $\delta_{b,2}$ and $\delta_{s,2}$ denote bankruptcy timeline over-estimation error tolerance and non-bankruptcy survival timeline over-estimation tolerance, respectively. The parameters $\alpha_b$ and $\beta_b$ of the penalty function $g$ are penalties...
of over-estimation and under-estimation in bankruptcy time prediction on \( A_b \); the parameter \( \beta \) of the penalty function \( h \) is penalty of under-estimation of firm survival time prediction on \( A_s \). They are applied to prediction mistakes (i.e., \(|p(i) - f(i)| \) and \(|q(i) - f(i)| \)) minus prediction error tolerance.

B. Isotonic Prediction: A Non-Linear Prediction Model

The isotonic prediction model requires a binary relation \( S \) on \( \mathbb{R}^d \) (known a priori or discovered from the firm data set) such that the estimated function \( f \) must be monotone with respect to \( S \). That is,

\[
f(i) \geq f(j), \quad (i, j) \in S.
\]

For this reason, we will call the binary relation \( S \) an isotonic consistency condition [12], [13]. In many cases, the set \( S \) of the isotonic consistency condition consists of pairs \((i, j)\) with \( a_{i,h} \geq a_{j,h} \) for \( h = 1, 2, \ldots, d \) where \((a_{i,1}, a_{i,2}, \ldots, a_{i,d}) \) and \((a_{j,1}, a_{j,2}, \ldots, a_{j,d}) \) are the \( d \)-dimensional rectangular coordinate vectors of \( i \) and \( j \), respectively. Any binary relation would be sufficient to define \( f \) values on \( A \). However, to estimate \( f \) values on \( \mathbb{R}^d \), it must have a form of ordering relation. In general, an isotonic consistency condition is required to be a reflexive and transitive order relation (known as a quasi order) on \( \mathbb{R}^d \).

Let \( \pi_i = f(i) \) for \( i \in A \) (= \( A \)), \( p_i = p(i) \) for \( i \in A_b \), and \( q_i = q(i) \) for \( i \in A_s \). The minimization objective of (1) given penalty functions of (2a) and (2b) and the satisfaction of the isotonic consistency condition of (3) are achieved in the isotonic prediction model by the following linear program:

\[
\begin{align*}
\min \quad & a_b \sum_{i \in A_b} r_i + \beta \sum_{i \in A_s} s_i + \beta s_i \\
\text{s.t.} \quad & -r_i - r_i' \leq p_i - \pi_i \leq s_i + s_i' \quad \text{for } i \in A_b \\
& q_i - \pi_i \leq t_i + t_i' \quad \text{for } i \in A_s \\
& \pi_i - \pi_j \geq 0 \quad \text{for } i, j \in A, (i, j) \in S \\
& l \leq \pi_i \leq u \quad \text{for } i \in A \\
& r_i' \leq \delta_{b,1} \quad \text{for } i \in A_b \\
& s_i' \leq \delta_{b,2} \quad \text{for } i \in A_b \\
& t_i' \leq \delta_{s,2} \quad \text{for } i \in A_s \\
& r_i, r_i', s_i, s_i' \geq 0 \quad \text{for } i \in A_b \\
& t_i, t_i' \geq 0 \quad \text{for } i \in A_s
\end{align*}
\]

where

- \( r_i + r_i' \) is the bankruptcy timeline over-estimation error of bankrupt firm \( i \in A_b \), of which \( r_i \) is penalized;
- \( s_i + s_i' \) is the bankruptcy timeline under-estimation error of bankrupt firm \( i \in A_b \), of which \( s_i \) is penalized;
- \( t_i + t_i' \) is the survival timeline under-estimation error of non-bankrupt firm \( i \in A_s \), of which \( t_i \) is penalized.

The linear program of (4) gives \( f \) values on \( A \), but no functional form \( f \) on \( \mathbb{R}^d \). But, we can obtain tight bounds of \( f \) values on \( \mathbb{R}^d \). Further, based on these bounds, \( f \) values on \( \mathbb{R}^d \) can be estimated. For \( i \in \mathbb{R}^d \), the upper and lower bounds of \( f(i) \) are

\[
\begin{align*}
L_i & \leq f(i) \leq U_i \\
\text{with} \quad & L_i = \begin{cases} 
\max_j f(j) & \text{if } \exists j \in A. (i, j) \in S \\
1 & \text{otherwise},
\end{cases} \\
\text{and} \quad & U_i = \begin{cases} 
\min_k f(j) & \text{if } \exists j \in A. (j, i) \in S \\
u & \text{otherwise}.
\end{cases}
\end{align*}
\]

These upper and lower bounds of \( f \) may be sufficient for the actual use of the isotonic prediction model. However, if it is necessary to approximate \( f \) values on \( \mathbb{R}^d \), there are many possibilities. Depending on a specific prediction problem and the isotonic consistency condition \( S \), any reasonable approximation method can be used as long as the approximated function is a monotone function with respect to the isotonic consistency condition \( S \).
A way to approximate \( f \) on \( \mathbb{R}^d \) is
\[
f(i) = L_i + \rho(U_i - L_i), \quad \text{for some } 0 \leq \rho \leq 1.
\]
This satisfies the monotonicity requirement of \( f \) with respect to the isotonic consistency condition. Another possibility is to use \( \rho \) values varying upon specific points. Interpolation between the upper and lower bounds based on geometric distances belongs to this method. That is,
\[
f(i) = L_i + \rho_i(U_i - L_i), \quad \rho_i = \frac{LD_i}{UD_i + LD_i}
\]
where \( LD_i \) is the shortest distance from \( i \) to its lower bound points and \( UD_i \) the shortest distance from \( i \) to its upper bound points. Note, \( LD_i \) and \( UD_i \) can be obtained as follows: Let
\[
L_i = \{ j \in A : i \geq j \text{ and } f(j) = L_i \}
\]
\[
U_i = \{ k \in A : k \geq i \text{ and } f(k) = U_i \}.
\]
\( L_i \) is a subset of \( A \) such that \( f(j) \) is the lower bound of \( f(i) \) for all \( j \in L_i \); similarly, \( U_i \) is a subset of \( A \) such that \( f(k) \) is the upper bound of \( f(i) \) for all \( k \in U_i \). From these, we obtain:
\[
LD_i = \min_{j \in L_i} |i - j|,
\]
\[
UD_i = \min_{k \in U_i} |k - i|.
\]
Here, \( LD_i \) is the distance between \( i \) and the closest point in \( L_i \); similarly, \( UD_i \) is the distance between \( i \) and the closest point in \( U_i \).

C. Modified Linear Regression: A Linear Prediction Model

Recurrence surface approximation (RSA) [11] is a modified linear regression method original used to estimate survival time of breast cancer patients (i.e., cancer recurrence time after surgical removal of tumor). The RSA technique finds a \((d + 1)\)-dimensional linear surface (i.e., plane)
\[
f(i) = s(x) = w^T \cdot x + \gamma
\]
where \( x \) is the \( d \)-dimensional variable vector for the rectangular coordinates of \( i \), \( w^T \) is the transpose of a \( d \)-dimensional coefficient vector, and \( \gamma \) is a constant, such that it achieves the approximation error minimization of (1). Let \( \pi_i = f(i) \) for \( i \in A_b \cup A_s \), \( \rho_i = p(i) \) for \( i \in A_b \), and \( q_i = q(i) \) for \( i \in A_s \). The coefficients and the constant term of the plane (6) can be obtained by solving the following linear program:
\[
\begin{align*}
&\min \quad \alpha_0 \sum_{i \in A_b} r_i + \beta_0 \sum_{i \in A_s} s_i + \beta_1 \sum_{i \in A_s} t_i \\
\text{s.t.} \quad &-r_i - r'_i \leq \pi_i - \rho_i \leq s_i + s'_i \quad \text{for } i \in A_b \nonumber \\
&q_i - \pi_i \leq t_i + t'_i \quad \text{for } i \in A_s \nonumber \\
&\pi_i = w^T \cdot a_i + \gamma \quad \text{for } i \in A
\end{align*}
\]
\[(7)\]

In the original RSA model [11], \( \delta_{b,1} = \delta_{h,2} = \delta_{s,2} = 0 \); thus, \( r'_i = s'_i = 0 \) for all \( i \in A_b \) and \( t'_i = 0 \) for all \( i \in A_s \).

The RSA model (7) and the isotonic prediction model (4) are very similar, but there exist fundamental differences reflected in (7a) and (4a). The isotonic prediction model contains the constraint (4a) for the required use of isotonic consistency condition (3), but the resulting estimator is not restricted to be a linear function. On the other hand, the RSA model does not require any isotonic consistency condition, but generates a linear estimator (6).

III. EXPERIMENTS

A. Data

We selected 165 firms’ data of various sizes in various industries from Standard & Poor’s COMPSTAT North America database; 55 firms failed between 1996 and 2001, while the other 110 firms did not have any distress in those years. We collected 23 financial ratios of year 1993, 1994, and 1995; they are listed in the first column of Table I. The 23 financial ratios included in our experiments were used in previous studies [1]–[5], [8], [14], [15]. For instance, Altman’s seminal work [1] used the ratio of earnings before taxes and interest to total assets, the ratio of retained earnings to total assets, the ratio of sales to total assets, the ratio of working capital to total assets, and the ratio of market value of equity to total debt in the discriminant function to predict firms’ failure. Altman’s and three other authors’ financial ratios are listed in the third to sixth columns of Table I.

We had the firm failure time window of 5 years because previous studies [2], [14] found financial data of up to 5 years prior to firm failure were useful for prediction. It would be ideal to have data of similar size firms in similar industries within a narrow time line. Collecting all twenty three ratios of failed firms, however, was the major difficulty in the study. Thus, we had to select only 165 firms of various sizes in various industries with no missing data.

For the isotonic prediction, order restrictions of financial ratios (4a) are important. We used a previous study’s result [16], as shown in the second column of Table I. Here the plus symbol (+) indicates that a high financial ratio value signifies that the firm is healthy, while distressed firms tend to have high values of financial ratios marked with the minus symbol (−).

The prediction unit was month. Thus, isotonic prediction and modified linear regression predicted bankruptcy timeline in months (i.e., between 1 and 60 months). When the
Table I

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash Flow/Total Assets</td>
<td>+</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cash/Sales</td>
<td>−</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cash Flow/Total Debt</td>
<td>+</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Current Assets/Current Liabilities</td>
<td>+</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Current Assets/Total Assets</td>
<td>+</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Current Assets/Sales</td>
<td>+</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Earn. before Taxes and Int./Total Assets</td>
<td>+</td>
<td></td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Retained Earnings/Total Assets</td>
<td>+</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Net Income/Total Assets</td>
<td>+</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Debt/Total Assets</td>
<td>−</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sales/Total Assets</td>
<td>−</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Working Capital/Total Assets</td>
<td>+</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Working Capital/Sales</td>
<td>−</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quick Assets/Total Assets</td>
<td>+</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quick Assets/Current Liabilities</td>
<td>+</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quick Assets/Sales</td>
<td>−</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Market Value of Equity/Total Capitalization</td>
<td>+</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cash/Current Liabilities</td>
<td>+</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Current Liabilities/Equity</td>
<td>−</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inventory/Sales</td>
<td>−</td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Equity/Sales</td>
<td>−</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Market Value of Equity/Total Debt</td>
<td>+</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Net Income/Total Capitalization</td>
<td>+</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
</tr>
</tbody>
</table>

prediction outcome was over 60 months, we interpreted that the firm would not fail.

B. Results

We performed various experiments: with all 23 financial ratio values, Altman’s [1], Edminster’s [4], Beaver’s [14], and Zavgren’s [15] financial ratio values. In experiments, we used data of 1995, which is a year before the firm failure time window (between 1996 and 2001). Previous years’ data (i.e., 1993 and 1994 data) may be valuable in prediction; but the use of these data results in too many independent variables. Thus, instead of using raw data, we generated and used projection data with 1993, 1994, and 1995 data for each financial ratio.

Another important step was feature selection. It is a process of finding relevant ratios that would give the best prediction result. We used the backward sequential elimination method [17].

The set of 165 data points was randomly partitioned into two sets: one with 55 data points and the other with 110 data points. The smaller set was used for the feature selection. The larger set was used for training of prediction models and validation. Since only 110 data points were available, we performed the leave-one-out validation.

The feature selection results with the isotonic prediction method are shown in Table II. When all 23 financial ratios were used, the backward sequential elimination method with isotonic prediction selected 9 ratios, some of which were only 1995 ratios, some were only projected data, and others were both. When Altman’s ratios were used, all of them were selected but the ratio of earnings before taxes and interest to total asset was of 1995 and other four ratios were projected ones. The feature selection results for Beaver’s and Zavgren’s ratios are similar to that for Altman’s ratios. In the case of Edminster’s ratios, however, all of 1995 ratios and projected data were selected.

The prediction accuracy measured as validation errors is shown in Table III. Note that the prediction was for 60 months. The use of all ratios resulted in lower prediction errors; and when the feature selection method was used, prediction errors were lower. When all financial ratios were used and the feature selection preprocess was applied, the isotonic prediction method resulted in lowest prediction errors (6.55 months).

The feature selection results with the modified linear regression method are shown in Table IV. When all 23 financial ratios were used, 10 ratios were chosen for modified linear regression. Those the number of chosen ratios is similar to that for isotonic prediction, only 4 ratios overlap in selection. When the backward sequential elimination method with modified linear regression was used for Altman’s and other authors’ ratios, substantially many ratios were eliminated differently from feature selection for isotonic separation. In sum, we discovered that linear regression and isotonic separation used many different financial ratios for prediction.
The prediction accuracy measured as validation errors is shown in Table V. Feature selection reduced prediction errors significantly. For the modified linear regression method, the use of Zavgren’s [15] financial ratios resulted in lowest prediction errors (9.29 months). When prediction results are compared in our study, in general, the isotonic prediction method seems better than the modified regression method, though both of them generated low prediction errors. It is because isotonic prediction results in a non-linear predictor, which is more general than a linear predictor produced by the modified regression method.

IV. Concluding Remarks

Isotonic prediction and modified linear regression were evaluated for firm failure timeline prediction with firm data collected from Standard & Poor’s COMPUSTAT North America database. Our study indicated that both methods resulted in good prediction outcomes, but isotonic prediction performed a bit better.

Bankruptcy timeliness prediction requires further studies. We used one year data (i.e., data of one year before the prediction time window) and projection data obtained from data of three years before the prediction time window. But, we need more elaborated studies to establish how many years data should be used. We used a backward sequential elimination method for feature selection. The selected financial ratios need to be evaluated for one to clarify why they are useful attributes for firm failure prediction. Finally, bankruptcy timeline prediction methods based other techniques such as neural networks and decision tree induction techniques need to be developed. Since the bankruptcy timeline prediction problem is a very important problem that many firms and investors face with, they need various methods in order to avoid a potential bias of each method.

REFERENCES

Table IV
MODIFIED LINEAR REGRESSION: FEATURE SELECTION RESULTS

<table>
<thead>
<tr>
<th>Financial Ratios</th>
<th>All</th>
<th>Altman</th>
<th>Edminster</th>
<th>Beaver</th>
<th>Zavgren</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash Flow/Total Assets</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>Cash/Sales</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>Cash Flow/Total Debt</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>Current Assets/Current Liabilities</td>
<td>☒</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>Current Assets/Total Assets</td>
<td>☒</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>Current Assets/Sales</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>Earnings before Taxes and Int./Total Assets</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>Retained Earnings/Total Assets</td>
<td>☒</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>Net Income/Total Assets</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>Total Debt/Total Assets</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>Sales/Total Assets</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>Working Capital/Total Assets</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>Working Capital/Sales</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>Quick Assets/Total Assets</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>Quick Assets/Current Liabilities</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>Quick Assets/Sales</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>Market Value of Equity/Total Capitalization</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>Cash/Current Liabilities</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>Current Liabilities/Equity</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>Inventory/Sales</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>Equity/Sales</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>Market Value of Equity/Total Debt</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>Net Income/Total Capitalization</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
</tbody>
</table>

□☐: feature not selected  ☐☐: 1995 data used
☐☐: projection data used ☐☐: 1995 data and projection data used

Table V
MODIFIED LINEAR REGRESSION VALIDATION ERRORS

<table>
<thead>
<tr>
<th>Feature Selection Methods</th>
<th>Average Validation Errors (in Months)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All</td>
</tr>
<tr>
<td>No Feature Selection</td>
<td>13.61</td>
</tr>
<tr>
<td>Sequential Elimination</td>
<td>10.45</td>
</tr>
</tbody>
</table>


[13] T. Robertson, F. T. Wright, and R. L. Dykstra, Order Re-


