When Cellular Capacity Meets WiFi Hotspots: A Smart Auction System for Mobile Data Offloading

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Abstract
The surge of social networking and video streaming on the go has led to the explosion of mobile data traffic. To minimize congestion costs for under-served demand (e.g., dissatisfied customers, or churn), the cellular service provider is willing to pay WiFi hotspots to serve the demand that exceeds capacity. In the present study, we propose an optimal procurement mechanism with contingent contracts for cellular service providers to leverage the advantages of both cellular and WiFi resources. We show the procedure of computing the optimal procurement mechanism with a tight integration of economics and computational technology. Simulation results show that the proposed procurement mechanism significantly outperforms the standard Vickrey-Clarke-Groves (VCG) auction in terms of the cellular service provider’s expected payoff.

1. Introduction
The surge of social networking and video streaming on the go has led to the explosion of mobile data traffic. Cellular providers today are struggling to meet the highly volatile mobile internet demand to reduce network congestion, because congested network often leads to undesirable outcomes such as user dissatisfaction and churn. Global mobile data traffic grew 70 percent in 2012 and will increase thirteen-fold between 2012 and 2017 [7]. The increasing popularity of smartphones has caused the surge in data usage. In 2012, a typical smartphone generated 50 times more mobile data traffic than a typical non-smartphone [7]. Cloud applications and services such as Netflix, YouTube, Pandora, and Spotify, contribute to the unprecedented growth of cellular traffic. Business demand is also one of the chief drivers behind this increase in data traffic as the workforce goes mobile, and data moves to the cloud.

The huge amount of data traffic poses a challenge to the network infrastructure: Cellular networks are overloaded and congested during peak hours because of the insufficient capacity. Network congestion can lead to a bad user experience and churn. The cellular networks, such as AT&T and Verizon, need to solve the challenge of effectively fulfilling the unmet demand from consumers for high network quality.

The previous literature proposed several solutions from both technical and economic aspects: (1) increasing the number of cellular base stations or deploying the cell-splitting technology; (2) upgrading the network to fourth-generation (4G) networks such as Long Term Evaluation (LTE), High Speed Packet Access (HSPA), and WiMax; (3) expanding capacity by acquiring of the spectrum of other networks, such as the attempted purchase of T-Mobile USA by AT&T; (4) adopting tiered pricing mechanisms (e.g. usage based price plans) to constrain the heaviest mobile data users, instead of using flat-rate pricing plans with unlimited data; and (5) offloading data traffic to WiFi networks [6,19].

Although all these solutions help solve the problem, each has its advantages and disadvantages. The first and second solutions are effective in the long term. However, they require significant investments, and getting government approval for building a new cell tower can take two years. Also, it is extremely expensive to increase the number of cellular base stations just for peak traffic demands. The health effect from cell tower radiation also raises a public concern. As a result, all cellular networks augment the first and second solutions with other approaches to expanding capacity. The third solution suffers from regulatory constraints. Cramton, Skrzypacez, and Wilson [11] show that an important market failure arises in spectrum auctions with dominant incumbents. They suggest that the Federal Communications Commission (FCC) should place limits on how much spectrum AT&T and Verizon are allowed to buy. This concern is also reflected in the action taken by the FCC to block the recent merger between AT&T and T-Mobile.

Because of these technical, economic and regulatory constraints, the fifth solution, using WiFi hotspots for mobile data traffic offloading, seems to be the most promising approach in augmenting the first
two solutions. WiFi hotspots refer to third-party hotspot owners, such as local restaurants, bookstores, and hotels, which offer WiFi service to their customers. WiFi offloading could potentially be a win-win solution: The cellular service provider achieves significant savings by not building more cellular base stations just for the peak traffic demands. The WiFi Hotspots gain additional revenue from their otherwise wasted spare capacity. WiFi offloading is particularly useful in supporting the mobile data demands during peak hours. Paul et al. [21] find that 28% of subscribers generate traffic only in a single hour during peak hours in a day. However, offloading data traffic to third-party WiFi hotspots is not purely a technology augmenting the existing cellular network. It is also a mechanism design problem, considering the economic incentives of third-party WiFi hotspots. Instead of focusing only on technical aspects, we need to combine both the technology of computing and auction theory to solve the challenge of effectively using WiFi hotspots. The tight integration of economics and computation technology in our system is seen as crucial to address issues surrounding the data traffic support for cloud-based services on mobile networks, such as business collaboration tools, which require sufficient download and upload speeds.

As more businesses employ mobile collaboration tools to increase productivity, mobile bandwidth availability becomes a key issue in marketing and operations for service providers. The objective of this study is to cope with the problem of peak data traffic by leveraging the advantages of both cellular and WiFi resources for service providers. There are several challenges in the design of this procurement auction system. First, a unique challenge in our problem setting is that the longer-range cellular resource introduces coupling between the shorter-range WiFi hotspots. We need to design an innovative procurement auction considering this different spatial coverage. Second, the data traffic is uncertain and changes quickly over time. It is critical to provide real-time support for computing the optimal contract. Dong et al. [13] propose a Vickrey-Clarke-Groves (VCG) type auction for mobile data offloading. A VCG auction is socially efficient, but it is not optimal for the cellular network (the buyer). A typical VCG mechanism leads to an overpayment to suppliers [10]. The simulation results in our study show that, compared to the standard VCG auction, a procurement auction with contingent contracts can significantly improve the cellular network’s expected payoff.

In this paper, we propose a procurement mechanism with contingent contracts to meet these challenges. The auction rule is contingent on demand uncertainty (i.e., consumers’ mobile data traffic). In the model, we partition the range of a cellular base station (a cell sector) into several regions. The cellular resource can serve data traffic in any region within the sector, whereas the WiFi resource can only serve local traffic. In the optimal design of such a procurement system, the economic model and the computing technology are complements. Recent advances in parallel computing makes it faster to find contingent contracts in large databases. With extremely fast computing speeds, our auction system has the potential to compute and implement a huge number of contingent contracts—a task was once considered computationally prohibitive, and significantly improve the cellular network’s expected gain.

2. Literature Review

Two streams of literature are related to this study. The first stream involves the study of three different auction schemes: quantity auctions, scoring auctions, and auctions with contingent contracts. Dasgupta and Spulber [12] extend the standard fixed quantity auction and study an auction that allows the quantity of the goods purchased to be endogenously based on the submitted bids. They consider a model with one buyer and a number of potential suppliers and ask the following question: How much of a product should be purchased and from which supplier? Our study differs from their approach in two critical ways: First, the unique feature of different spatial coverage makes a difference for the optimal auction design. Buying more resources from a local WiFi hotspot in one region frees up more cellular resources. Second, the auction rules are determined by the contingency terms. The terms of a contingent contract are not finalized until the uncertain demand is realized.

In many procurement situations, the buyer cares about other attributes in addition to price when evaluating the submitted bids. In a scoring auction, suppliers submit multidimensional bids, and the contract is awarded to the supplier who submitted the bid with the highest score according to the scoring rule. Che [8] develops a scoring procurement auction in which suppliers bid on two dimensions of the good. However, a scoring auction typically allows only sole sourcing, but in our procurement setting, offloading data traffic to multiple WiFi hotspots is naturally done. Our procurement mechanism is also different from multiple-object spectrum auction [14, 20, 24]: Instead of focusing on spectrum (cellular) resource, we look at an auction that considers the unique feature of WiFi hotspots. Because a hotspot can provide service only to customers who are physically nearby, service from one hotspot is not a direct substitute for service from a
different, far-away hotspot. Thus, standard procurement auctions will not yield a good outcome for the cellular service provider—isolated hotspots have too much monopoly power. The contribution of this paper is to show that under some conditions, the cellular service provider can in fact substitute bandwidth between hotspots. Hotspot A cannot provide service to a cellular customer near hotspot B, but by providing service to a customer near Hotspot A, it can free up some cellular capacity for the cellular service provider, which the cellular service provider can then use to serve the other customer.

Contingent contracts have been widely studied in economics literature [23] and applied in procurement auction [9]. The model in our study differs both in the application setting and auction formats.

Our research is also related to the computer science literature on mobile data offloading. Balasubramanian, Mahajan, and Venkataramani [5] design a WiFi offloading system to augment mobile 3G capacity. They find that for a realistic workload, WiFi offloading can reduce 3G usage by almost half for a delay tolerance of one minute. Dong et al. [13] propose a VCG procurement auction for mobile offloading to incentivize WiFi hotspot owners to be truthful in the bidding process. The integration of economics and computation technology allows us to improve the cellular network’s expected payoff in two important ways: (1) The appropriate mechanism design avoids overpayment in the VCG auction (economics); and (2) the parallel computing technology improves the performance of the procurement auction by computing and finding the optimal contract under each contingency on demand uncertainty (computation technology).

3. A Benchmark Model: Single WiFi Region

A cellular network provides service to its customers who demand for bandwidth to connect to the Internet. Congestion results when network capacity cannot satisfy instantaneous user demand. When the user demand for mobile data is below a certain threshold $X_B$, the cellular network face no additional cost except the sunk cost of buying the spectrum and keeping the system running. However, when the demand $\bar{X}$ exceeds the threshold, the cellular network incurs a cost of $C_0(\bar{X} - X_B)$. $X_B$ is the cellular capacity owned by the cellular network. The standard metrics used in the telecommunications industry to measure quality of service (QoS), such as Kleinrock delay formula, depend on the difference between user demand and capacity [22]. In our problem setting, $\bar{X} - X_B$ is the difference between user demand and cellular capacity. The cost function $C_0(\cdot)$ is strictly increasing and strictly convex, which captures the rapidly rising cost of congestion. A similar convex cost function has been widely used in modeling the congestion cost in the Internet [15]. Apparently, we have $C_0(x) = 0$ for any $x \leq 0$. Denote $c_0(x) = C_0(x)$ as the marginal cost of congestion.

We model the demand for bandwidth as a random variable $\bar{X}$ with a cumulative distribution function $G(\bar{X})$ in the support $[0,1]$. Given the unprecedented growth rate of mobile data demand and the high cost associated with congestion, the cellular network is interested in procuring spare resources from third-party WiFi hotspots.

If the cellular network purchases $Y_1$ units of bandwidth from the hotspots, then the expected reduction of congestion cost for the cellular network is

$$V(Y_1) = \int_0^{Y_1} c_0(\bar{X} - X_B) dG(\bar{X}) - \int_{X_B}^{X_B+Y_1} c_0(\bar{X} - X_B - Y_1) dG(\bar{X}),$$

which is the valuation that the cellular network attaches to the additional bandwidth $Y_1$. The first part $\int_0^{Y_1} c_0(\bar{X} - X_B) dG(\bar{X})$ is the expected congestion cost without procuring from WiFi hotspots, and the second part $\int_{X_B}^{X_B+Y_1} c_0(\bar{X} - X_B - Y_1) dG(\bar{X})$ is the expected congestion cost when the purchase quantity is $Y_1$. Because

$$V'(Y_1) = \int_{X_B}^{X_B+Y_1} c_0' (\bar{X} - X_B - Y_1) dG(\bar{X}) > 0$$

and

$$V''(Y_1) = - \int_{X_B}^{X_B+Y_1} c_0'' (\bar{X} - X_B - Y_1) dG(\bar{X}) - c_0(0) g(X_B + Y_1) < 0,$$

where $g(\cdot)$ is the density function of $\bar{X}$. $V(Y_1)$ is strictly increasing and strictly concave, which is not surprising given that the cost of congestion is convex.

We assume that the cost function for hotspot $i$ to provide capacity $Q$ to the cellular network is

$$C_i(Q, \theta_i) \equiv \int_0^Q c(q, \theta_i) dq, i = 1,2, \ldots, n,$$

where $c(q, \theta_i) \geq 0$ is the marginal cost function for hotspot $i$, and $\theta_i$ represents each hotspot’s private information about the cost of capacity provision. The cost of providing bandwidth for a hotspot is based on its instantaneous user demand and many other considerations that may not be revealed to the cellular network. We assume $c_0(q, \theta_i) \geq 0$ to capture the fact that the marginal cost of providing capacity for each hotspot increases as more capacity is provided to the cellular network. Marginal costs are increasing and convex in the cost parameter, $c_0 \geq 0$, $c_{q0} \geq 0$. Also, we assume $c_{q\theta} \geq 0$. Hotspots’ cost parameters are independently and identically distributed with a
It follows from Dasgupta and Spulber [12] that the optimal allocation can be implemented via a quantity auction (sealed bid) where

- The cellular network announces a payment-bandwidth schedule \( B = B(Q) \);
- Each hotspot chooses the bandwidth they want to sell given, \( B(Q) \); and
- The hotspot choosing to provide the highest capacity wins the auction and sells the chosen capacity to the cellular provider.

This quantity auction is optimal for the cellular network if we assume that a single winner emerges. Given the payment-bandwidth schedule \( B(Q) \), the hotspots’ bidding strategy is denoted by \( Q(\theta) \): A hotspot with private cost parameter, \( \theta \in [\theta^l, \theta^u] \), bids \( Q(\theta) \). Let \( \theta^* \) be a threshold cost parameter: Hotspots for which the cost parameter exceeds \( \theta^* \) do not bid, while those with \( \theta < \theta^* \) bid according to \( Q(\theta) \). This represents the individual rationality constraint.

**Proposition 1 (Single Region).** In the optimal quantity auction, the payment-bandwidth schedule \( B^*(Q) \) and the optimal bidding strategy \( Q^*(\theta) \) are given by the following equations:

\[
B^*(Q(\theta)) = C(Q(\theta), \theta) + \frac{\int_{\theta}^{\theta^*} (1-F(x))^{n-1} C_0(Q(x), \theta) dx}{(1-F(\theta))^{n-1}},
\]

\[
V^*(Q(\theta)) = C_0(Q(\theta), \theta) + C_0(\theta) H(\theta).
\]

The cellular network’s expected profit is

\[
n \int_{\theta}^{\theta^*} (1-F(\theta))^{n-1} F'(\theta) [V(Q) - C(Q, \theta) - C_0(Q, \theta) H(\theta)] d\theta.
\]

Under asymmetric information, this is the highest expected profit for the cellular network when it must procure from a single winning hotspot.

Note that the hotspot with the lowest \( \theta \) always wins the auction. In equation 5, \( n(1-F(\theta))^{n-1} F'(\theta) \) is the density of the lowest \( \theta \). The cellular network’s benefit is the expected reduction of the congestion cost, which is given by equation 1. \( C(Q, \theta) + C_0(Q, \theta) H(\theta) \) is the "virtual cost" the cellular network pays to the winning hotspot. Under complete information, the payment to the winning hotspot is the cost \( C(Q, \theta) \). The information asymmetry is reflected in the term \( C_0(Q, \theta) H(\theta) \), which is the information rent of the winning hotspot.

4. **Multiple WiFi Regions**

In the benchmark model, we assume that only a single hotspot wins the auction. However, the WiFi capacity for one hotspot is limited, and relying on multiple hotspots is optimal because of the convexity of the congestion cost functions. The benchmark model also assumes that the range of a cellular base station is the same as the range of a hotspot. However, cellular resources and WiFi resources actually have different spatial coverages. In suburban areas, a typical cellular base station covers 1-2 miles (2-3 km) and in dense urban areas, it may cover 1/4 - 1/2 mile (400-800 m). A typical WiFi network has a range of 120 feet (32 m) indoors and 300 feet (95 m) outdoors. Therefore, we need to partition a cell sector into several regions. In Figure 1, a red circle is a WiFi region. Usually, a WiFi region has several WiFi hotspots that are close together. Note that each WiFi hotspot is an independent decision maker, and in the current model, we do not consider the collusion among WiFi hotspots.

**Figure 1. Multiple WiFi regions**

Now suppose there are \( M \) WiFi regions in a cell sector, \( 1, 2, \cdots, M \), and the demand for region \( m \) is \( \bar{X}_m \). The demand vector \((\bar{X}_1, \bar{X}_2, \cdots, \bar{X}_M)\) has a joint distribution function \( G(\bar{X}_1, \bar{X}_2, \cdots, \bar{X}_M) \). We assume the same congestion cost function of the cellular network for all regions. Cellular resources can serve traffic in any region \( m \), whereas WiFi hotspots in region \( m \) can only serve local traffic. In Figure 1, a hotspot located in a WiFi region cannot serve the demand in another region. A unique challenge in the procurement auction is that the longer range cellular resource introduces coupling between the shorter range WiFi hotspots. In this section, we derive the optimal auction rule under different spatial coverages.

The cellular network follows a two-stage decision procedure. In the first stage, it purchases WiFi capacity from hotspots in different regions. In the second stage, the cellular network adjusts the allocation of cellular resources across regions.
We first focus on the optimization problem in the second stage. If the cellular network purchases \( Y_m \) units of bandwidth from hotspots in region \( m \), then the expected congestion cost is
\[
\min_{y_1, y_2, \ldots, y_M} \int_0^1 \int_0^1 \int_0^1 \sum_{m=1}^M C_0(\bar{X}_m - y_m)dG(\bar{X}_1, \bar{X}_2, \ldots, \bar{X}_M) \\
\text{s.t. } \sum_{m=1}^M y_m = X_B, \text{ } y_m \geq 0, \text{ for } m = 1, 2, \ldots, M. \tag{6}
\]

where \( y_m \) is the amount of cellular capacity allocated to region \( m \). The cellular network can adjust the allocation of cellular resources across regions through varying \( y_m \). Purchasing more capacity from a local WiFi hotspot frees up more cellular resources, which can be allocated to other regions.

Similarly, without hotspots, the expected congestion cost is
\[
\min_{y_1, y_2, \ldots, y_M} \int_0^1 \int_0^1 \int_0^1 \sum_{m=1}^M C_0(\bar{X}_m - y_m)dG(\bar{X}_1, \bar{X}_2, \ldots, \bar{X}_M) \\
\text{s.t. } \sum_{m=1}^M y_m = X_B, \text{ } y_m \geq 0, \text{ for } m = 1, 2, \ldots, M. \tag{7}
\]

Because \( C_0(\cdot) \) is convex, using Jensen’s inequality, we have
\[
\sum_{m=1}^M \int_{\bar{X}_m}^{\bar{X}_m - y_m} C_0(x)dx \\
\geq \int_{\bar{X}_m}^{\bar{X}_m - y_m} C_0(x)dx \\
= \sum_{m=1}^M C_0(\bar{X}_m - y_m) \geq M \cdot C_0(\bar{X} - y). \tag{8}
\]

If we define \( \bar{X} = \bar{X}_1 + \bar{X}_2 + \cdots + \bar{X}_M \) as the total average excess demand of the sector, \( \bar{X} = X/B \) can be interpreted as the average excess demand across regions. The optimal allocation of cellular resources should be \( y_m^* = (\bar{X}_m - \bar{X}) - (Y_m - \bar{Y}) \) with using WiFi hotspots and \( y_m^* = \bar{X}_m - \bar{X} \) without using hotspots.

For such allocations of cellular resources across regions to be feasible, we need \( y_m^* \geq 0 \), or equivalently,
\[
\frac{X_B}{M} \geq \left( \frac{Y_m}{M} - \frac{1}{M} \sum_{i=1}^M Y_i \right) - \left( \frac{\bar{X}_m}{M} - \frac{1}{M} \sum_{i=1}^M \bar{X}_i \right). \tag{9}
\]

for \( m = 1, 2, \ldots, M \). The condition is more likely to be satisfied if bandwidth demand and hotspots supply are relatively homogeneous across regions or if \( X_B \) is relatively large. We assume inequality 9 is always satisfied.

The expected reduction of congestion cost for the cellular network after the procurement of hotspot bandwidth is
\[
V(Y_1, Y_2, \ldots, Y_M) = M \int_0^1 \int_0^1 \int_0^1 C_0(\bar{X})dG(\bar{X}_1, \bar{X}_2, \ldots, \bar{X}_M) - M \int_0^1 \int_0^1 \int_0^1 C_0(\bar{X} - \bar{Y})dG(\bar{X}_1, \bar{X}_2, \ldots, \bar{X}_M).
\]

Because the valuation function is only a function of \( X_1, \ldots, X_M \) through \( \bar{X} \), we denote the distribution of \( \bar{X} \) as \( \bar{G} \) and rewrite the valuation as
\[
V(Y_1, Y_2, \ldots, Y_M) = V(\bar{Y}) = M \int_0^1 C_0(\bar{X})d\bar{G}(\bar{X}) - M \int_0^1 C_0(\bar{X} - \bar{Y})d\bar{G}(\bar{X}) \tag{10}
\]

Note the similarity between the valuation function for the case of a single region (equation 1) and the valuation function for the case of multiple regions (equation 10), which immediately implies that \( V(\bar{Y}) \) is also increasing and concave in \( \bar{Y} \). Indeed, the single region case can be viewed as the same as a multiple-region case in which \( M = 1 \).

Because the valuation function is only a function of \( Y_1, \ldots, Y_M \) through \( \bar{Y} \), the task of undertaking multiple procurements in multiple regions is essentially the same task as undertaking a single procurement in one sector in which the bandwidth capacity is procured from several hotspots in different regions. In other words, we are dealing with a variable quantity procurement auction with multiple winners. In the first stage, the cellular network’s optimization problem is characterized as a direct revelation game in which hotspots announce their types and truthful revelation is a Bayes-Nash equilibrium. We adopt the notational convention of writing \( \theta_{i-} = (\theta_1, \ldots, \theta_{i-1}, \theta_{i+1}, \ldots, \theta_n) \). The optimal allocation for the cellular network can be implemented via a direct revelation mechanism where

- The cellular network announces a payment-bandwidth schedule \( P_i(\theta_i, \theta_{i-}) \), and a bandwidth allocation schedule \( q_i = Q(\theta_i, \theta_{i-}) \),
- Hotspot \( i \) reports the private cost parameter \( \theta_i \) given \( P_i(\theta_i, \theta_{i-}) \) and \( Q(\theta_i, \theta_{i-}) \),
- Hotspot \( i \) provides WiFi Capacity \( q_i = Q(\theta_i, \theta_{i-}) \) to the cellular network and its payment is \( P_i = P_i(\theta_i, \theta_{i-}) \).

The optimal mechanism \( \left( P_i^*(\theta_i, \theta_{i-}), Q^*(\theta_i, \theta_{i-}) \right) \) for the cellular network is given by the following proposition:

**Proposition 2** (Multiple Regions). In the optimal direct revelation mechanism, all hotspots truthfully announce their cost parameters \( \theta \). The optimal bandwidth allocation schedule \( q_i = Q^*(\theta_i, \theta_{i-}) \), for \( i = 1, 2, \ldots, n \) is given by:
\[
V'(\sum_{i=1}^n q_i) = c(q_1, \theta_1) + c(q_i, \theta_i)H(\theta_i),
\]

where
\[
V(\bar{Y}) = V'(\sum_{i=1}^n q_i) = M \int_0^1 C_0(\bar{X})d\bar{G}(\bar{X}) - M \int_0^1 C_0(\bar{X} - \bar{Y})d\bar{G}(\bar{X}) \tag{10}
\]

The optimal payment schedule \( P_i = P_i^*(\theta_i, \theta_{i-}) \), for \( i = 1, 2, \ldots, n \) is given by:
\[
P_i^*(\theta_i, \theta_{i-}) = C(Q^*(\theta_i, \theta_{i-}), \theta_i) + \int_{\theta_i}^{\theta_{i+}} C_0(Q^*(\theta_i, \theta_{i-}), \theta)\,d\theta.
\]

The cellular network’s expected profit is
\[
E[\mathcal{V}(\sum_{i=1}^{n} Q^*(\theta_i, \theta_{-i})) - \\
\sum_{i=1}^{n} C(Q^*(\theta_i, \theta_{-i}), \theta_i) - \\
\sum_{i=1}^{n} C_0(Q^*(\theta_i, \theta_{-i}), \theta_i) H(\theta_i)].
\]

Under asymmetric information, this is the highest expected profit for the cellular network when it can procure capacity from multiple hotspots in different regions (second best).

In the direct revelation game, hotspot i announces its cost parameter \(\theta_i\). The capacity it needs to provide is \(q_i = Q^*(\theta_i, \theta_{-i})\), and its payment is \(P_i = P_i^*(\theta_i, \theta_{-i})\). This optimal mechanism is a global auction including all hotspots from different regions. Note that launching separate auctions within each region is not optimal. The intuition is that procuring more WiFi resources in one region frees up more cellular resources, and the cellular network can allocate the cellular resources to other regions. In equilibrium, the virtual marginal costs are equalized across hotspots in different regions.

The following steps describe the procedure of computing the optimal procurement auction.

- Define the map \(q: \Theta^n \rightarrow R^n\) as follows:
  - For each \(i = 1, 2, \ldots, n\) and \(x \geq 0\), let \(\phi_i(x)\) be the implicit function satisfying the following equation
    \[c(\phi_i(x), \theta_i) + c_0(\phi_i(x), \theta) H(\theta_i) = x.\]
  - From equation 10, \(V(q)\) can be written as
    \[V(q) = \int Y c_0(X - Y) dG(X) = \int_{\theta/M}^{1} c_0(X - q/M) dG(X).\]

Let \(q^*\) be the solution to the following equation:
\[
\sum_{i=1}^{n} \phi_i(V(q)) = q.
\]

Again, because the left-hand-side is decreasing in \(q\), we can easily solve for \(q^*\) using bisection in the interval \([0, M]\).

\[U(Y_1) = C_0(\bar{X} - X_B) - C_0(\bar{X} - X_B - Y_1).\]

\[U(Y_1)\] is strictly increasing and strictly concave, and
\[
U'(Y_1) = C'_0(\bar{X} - X_B - Y_1).
\]

The optimal contingent auction can be implemented via a quantity auction, where:

- The cellular network announces a contingent payment-bandwidth schedule \(B = B(Q, \bar{X})\);
- Each hotspot chooses a contingent bandwidth it wants to sell, \(Q(\theta, \bar{X})\), given \(B(Q, \bar{X})\);
6. Simulation

Applying our model to the network data from one of the largest U.S. service providers, we address the following question in this section: As compared with the standard VCG auction, how much can our optimal procurement auction improve the cellular network's expected payoff? The Monte Carlo simulation results demonstrate that, as compared with the standard VCG auction, our contingent procurement auction significantly improves the cellular network's expected payoff.

Note that the VCG auction is both truth-telling and socially efficient by standard arguments. All hotspots bid their cost parameters truthfully, irrespective of other hotspots’ bids. The VCG mechanism guarantees the minimum total cost. However, it leads to an overpayment to hotspots.

Figure 2. Area map of a typical cell sector

In our simulations, we consider a typical urban neighborhood in New York City, NY, USA, as shown in Figure 2. We define a cell sector as the range of the cell tower. Our dataset consists of the location information of 14,576 cell towers from a large cellular provider in the U.S. In our simulation study, we pick a cell tower in New York City from the full list of cell towers and simulate the mobile data demand in this sector. In Figure 2, T represents the cell tower, and others are 69 WiFi hotspots in the given cell sector.

Following [13], we set the communication range for a cell tower as 250m, and set the communication range for Wi-Fi as 100m. The following steps describe the procedure of simulations:

Generating traffic demands in the given cell sector: To gain a sense of the population density in the coverage area of the cell tower, we use 2010 census data, which contains the land area coverage and population density of each zip code. Combining the market share of this service provider for the first
quarter 2013, we estimate the number of users in the given cell sector.

Generating WiFi regions in the cell sector: Dong et al. [13] showed that the appropriate number of WiFi regions in a cell sector is six. Following their approach, we generate six WiFi regions by clustering the WiFi hotspots using k-means. In Figure 2, Region A through Region F indicate which region each WiFi hotspot belongs to.

Generating traffic demands in each WiFi region: We use two different methods to place users in the cell sector and assign them to the corresponding WiFi regions according to their locations. (1) All users are randomly placed in the cell sector. (2) All users are placed according to the densities of the hotspots.

After placing all the users, a nearest hotspot is calculated for each user location. If the distance between the nearest hotspot found and the user location is less than the hotspot range (100m), the user is counted as one of the regional population according to the WiFi region; otherwise, the user is considered as in the region with no hotspots (region 0). We run 1,000 simulations to generate traffic demands in each WiFi region.

Generating cell tower capacity: The cell tower capacity is set to three carriers, that is, three times 3.84 MHz [13]. Data spectral efficiency varies across towers from 0.5 to 2 bps/Hz. We set spectral efficiency to be 1 by default and then vary the spectral efficiency to evaluate its impact. Note that when the user demand for mobile data is below 80% of the cell tower capacity, the cellular service provider faces no congestion cost.

Using the algorithms in Section 4, we conduct a variety of simulations and compute the corresponding allocation under different auction rules. We set the parameter values: \( C_0(x) = 2x^2 \), and \( C(x, \theta_i) = \left( \frac{1}{x} + \theta_i \right)x^2 \). \( \theta_i \) is drawn from a uniform distribution [0,1] for 1,000 times. The choice of parameter values follows from previous literature [17]. The simulation result is shown in Figure 3. In the left panel, the users are randomly placed in the cell sector. In the right panel, the users are placed according to the densities of the hotspots. The two panels show similar results: our non-contingent procurement auction significantly outperforms the VCG mechanism in terms of the expected net gain of the cellular service provider (the expected net gain = the reduction of congestion cost - the payment to hotspots). The contingent arrangements can further improve the expected gain of the cellular service provider. Note that both of the panels suggest that the VCG mechanism leads to an overpayment to hotspots. Our contingent mechanism reduces procurement cost by 57.7% in the left panel and by 55.4% in the right panel compared to the VCG mechanism.

Data spectral efficiency varies across cell towers using different wireless technologies. An increase in spectral efficiency significantly contributes to tower capacity. Figure 4 evaluates the impact of spectral efficiency (cell tower capacity) on the performance difference. We find that as the cellular capacity increases, the advantage of our contingent procurement auction (CPA), in comparison with the VCG mechanism, decreases. The service provider is less willing to purchase WiFi resources when it owns a relatively large cellular capacity, and the overpayment problem in the VCG mechanism is thus less detrimental to the service provider's expected gain.

Managerial Implications

Our automated auction system is a vivid illustration of the power of Cyber-Physical Systems (CPS). CPS are integrations of computation with physical processes. In our context, embedded computers and networks monitor and control the data offloading processes. The literature on CPS mainly focused on the feedback loop where physical processes affect computation and vice
versa [18]. The economic incentives of different entities have been overlooked in the design of CPS. Our automated auction system consists of multiple self-interested WiFi hotspots each operating according to its own objectives, and the strategic behaviors of these hotspots may make predictable and reliable real-time performance difficult. We address this issue by using economic theory to design an incentive compatible procurement mechanism. The conventional data offloading is on the basis of the access network discovery and selection function (ANDSF) that processes static WiFi offload policies. Recently, the intelligent mobile solution company, Tekelec, Inc., has developed its Mobile Policy Gateway (MPG) to implement complex WiFi offload policies. The Tekelec MPG enables support for our smart data offloading based on the real-time auction approach.

The key to our proposed implementation is the combination of an auction framework and distributed computing. The optimal auction design achieves efficiency through facilitating inter-region competitions among WiFi hotspots in geographically dispersed regions. More specifically, we design a system where sensors are attached to capacity-supplying units, i.e., WiFi servers, to assist a central control center with the auction and data offloading process, administered by the cellular service provider.

8. Conclusion

In this paper, we designed an optimal procurement auction with contingent contracts for mobile data offloading. The integration of both cellular and WiFi resources significantly improves mobile bandwidth availability. We characterize the Bayesian-Nash equilibrium of the auction and compute the corresponding contingent contract. The simulation results show that our procurement auction significantly outperforms the standard VCG auction.

In the telecommunications industry, consumers, especially business users, are concerned about mobile QoS because the effects of congestion are costly to them. For simplicity, we abstract away the consumer side in our model. A direction for future research is to study the procurement auction when consumers form rational expectations of the network congestion. It allows the cellular network to consider various types of QoS warranties — that is, when a severe congestion occurs, the cellular network compensates business users through monetary payments, or other forms of goodwill. The provision of warranties may serve as signals of QoS for cellular networks.

In our procurement auction, we assume one cellular network and many WiFi Hotspots. In many geographical markets, one cellular network may dominate and operate as a monopoly [11]. In this case, a procurement auction framework with one cellular network is appropriate. However, an intense duopoly competition has arisen between Verizon and AT&T in some other areas. Thus, an important direction for future research is to extend our model to a setting with

Appendix: Proof

Proposition 1. The hotspot with the lowest $\theta$ always wins the auction, so the sum of the expected profits of the cellular network and the hotspots is

$$\sum_{i=1}^{n} (1 - F(\theta))^n - 1 F'(\theta) [V(Q) - C(\theta)]d\theta.$$ 

Thus, the cellular network’s expected profit is given by

$$\sum_{i=1}^{n} (1 - F(\theta))^n - 1 F'(\theta) [V(Q) - C(\theta)]d\theta.$$ 

Let $Q^*(\theta)$ be determined by the following first-order condition:

$$V'(Q^*(\theta)) = C_0(Q^*(\theta), \theta) + C_0(Q, \theta) H(\theta).$$ 

$Q^*(\theta)$ clearly maximizes the cellular network’s expected profit. ■

Proposition 2. This proof is similar to the proof of Proposition 1. For a multiple region auction, the cellular network’s expected profit is

$$E[V(Q)] - \Sigma_{i=1}^{n} C(q_i, \theta_i) - \Sigma_{i=1}^{n} C_0(q_i, \theta_i) H(\theta_i).$$ 

Because the hotspots’ congestion cost functions are convex, the virtual marginal costs are equalized across hotspots:

$$V'(\Sigma_{i=1}^{n} q_i) = c(q_i, \theta_i) + C_0(q_i, \theta_i) H(\theta_i).$$ 

for $i = 1, 2, \ldots n$. The bandwidth allocation schedule $q_i = Q(\theta_i, X_i)$ satisfying this equation maximizes the cellular network’s expected profit. ■

Proposition 3. Let $U_\delta(Z_\delta) = C_0(X_\delta) - C_0(X_\delta - Z_\delta)$. If $X_\delta \neq \bar{X}$, the cellular network should pretend that the demand is $X_\delta$: Under the payment-bandwidth schedule $B$, the bidding strategy for a hotspot should be $Q^*(\theta, X_\delta)$, which is given by:

$$U_\delta(Q) = C_0(Q, \theta) + C_0(Q, \theta) H(\theta).$$ 

However, the cellular network’s expected profit is

$$\sum_{i=1}^{n} (1 - F(\theta))^n - 1 F'(\theta) [U(Q) - C(\theta)]d\theta.$$ 

The bidding strategy $Q^*(\theta, X)$ maximizes the cellular network’s expected profit. Because $X_\delta \neq \bar{X}$, and $Q^*(\theta, X_i)$ is strictly increasing in $X_\delta$, $Q^*(\theta, X_\delta)$ cannot maximize the cellular network’s expected profit. ■
Proposition 4. Straightforward from the proof of Proposition 3. ■

Proposition 5. Straightforward from the proof of Proposition 2. ■

References


