Modeling Correlated Effect and Random Effect of Advertising and Word-of-Mouth on Product Diffusion

Yang Cenying
Fudan University
13210690021@fudan.edu.cn

Zhang Cheng
Fudan University
zhangche@fudan.edu.cn

Abstract
We develop a non-homogeneous Poisson model to study how the relationship between advertising and WOM and the occurrence of unexpected events affect product diffusion. For correlated effect, we generate parameters of advertising and WOM with given correlation by Copula method. For random effect, we use a Markov chain to modulate correlation between two parameters as well as distributions of two parameters. Numerical studies show that correlation between advertising and WOM speeds up diffusion process, but when considering a reasonable level within substitution or complement, managers should balance between scale of diffusion and speed of diffusion. Consequently, managers might be able to use correlation wisely to hedge from uncertainties.

1. Introduction

Due to the rapid growth of social media like Twitter and Epinions.com, electronic word-of-mouth (e-WOM) has emerged as a dominating force for online marketing. Admittedly, WOM’s effectiveness in shaping consumers’ attitudes and behaviors has been corroborated [12, 17, 29]. A recent report also reveals that around 60% of people tend to acquire information through friends’ recommendations when considering buying cell phones [3]. In response to the changing trend of consumers’ behavior, companies are gradually shifting their budgets from traditional channels to digital channels in order to stimulate WOM. Expenditure on WOM marketing is anticipated to climb up to $3 billion by 2013 [38]. At the same time, from 2012 to 2013, pharmaceuticals backed away quickly from print advertising and direct-to-consumer advertising, for which spending fell 44% and 22%, respectively [19].

However, debates on WOM strategies versus advertising strategies have arisen when it comes to first purchase of consumers. Although WOM seems to be more convincing because its counterpart tends to exaggerate information [35], advertising still plays its role in attracting customers. WOM takes time to spread and its effectiveness depends on getting people to talk. Low involvement in WOM might be attributed to an inconsistency between incoming information and consumers’ current beliefs or expectations [46]. On the contrary, advertising can reach many customers quickly, provide information about qualities of products and increase customers’ awareness. So, it has long been regarded as an effective way to encourage consumers to buy [25, 15]. Therefore, it might be too early to assume the demise of advertising.

Then, a question arises naturally that how advertising and WOM could work together to affect product diffusion, i.e., the process of product adoption at aggregate level in a given time window [8]. However, the interplay between two strategies is blurred, reflecting in distinct marketing strategies in practice. For example, Krispy Kreme Doughnuts does not spend any money on advertising. Instead, it invests heavily in WOM. This practice leads to a substitutive relationship between advertising and WOM. In 2012, Data2Decisions and Thinkbox reported that TV sports advertising alone drive 51% of WOM, indicating a complementary correlation between two strategies [45]. Theoretically, both substitutive and complementary relationships have been identified under different contexts [11, 33]. While extant literature demonstrates what the relationship is between two factors, we pay more attention to how the relationship stimulates sales.

What further complicates the issue is the occurrence of unexpected events due to external uncertainties. Its influence on advertising and WOM is defined as random effect. First, unexpected events could impose an impact on advertising and WOM. For instance, product-harm crises make firms to invest more in advertisements to regain consumers’ trust [13]; regulation in advertisements force firms to adjust their advertising strategies [1]; accidental scandals are likely to trigger negative WOM among customers.
[14]. Second, unexpected events could affect the correlation between advertising and WOM. Consider celebrities who are embroiled in a scandal; their featured advertisements are thus not as credible as before. Exposed to those advertisements, customers have greater needs for WOM to learn about the quality of the products advertised. So, companies have to invest in WOM as well and this leads to a complementary relationship between advertising and WOM. Thus, it is worth exploring such random effect on advertising and WOM and further on product diffusion. To our knowledge, no work has yet incorporated the occurrence of unexpected events into a diffusion model and we would like to fill the gap.

Shedding light on the above-mentioned questions relies on exploring the correlated effect between advertising and WOM and the random effect of unexpected events. Although various diffusion models have been developed to integrate two strategies or to capture the randomness of the external environment in past decades, the correlation is empirically corroborated rather than explicitly modeled and few works combine correlated effect and random effect. The research vacuum might be due to difficulties in describing correlation and randomness explicitly. This paper presents a non-homogeneous Poisson process with correlation and randomness modulated by a Markov chain. We consider first purchase behavior of customers without repetitive purchases and aggregate individual adoption into a diffusion process. We test the validity of our model through numerical studies in terms of average number of adopters in each time period (i.e. average scale of diffusion) and speed to reach the total potential adopters in each time period (i.e. average speed of diffusion). We attempt to answer the following research questions: (1) How does substitutive or complementary correlation between advertising and WOM affect product diffusion? (2) How does the occurrence of unexpected events affect product diffusion through advertising and WOM? A deeper understanding of those questions can help practitioners make wise investments in advertisements and WOM and better respond to unexpected events.

Our study generates several findings. First, correlation between advertising and WOM can help speed up the process of product diffusion, but substitution and complement exhibit different patterns in terms of average number of adopters in each time period. On average, a substitute relationship reduces the average number of adopters, while a complementary relationship adds more buyers on average, indicating the strength of high levels of both advertising and WOM in product diffusion. Another interesting finding is that as substitution becomes greater, the average number of customer drops and the time to reach all potential consumers shortens, while as complement becomes greater, the results show the opposite. Practitioners can infer that complementary correlation outperforms substitute correlation overall, but when considering a reasonable level within substitution or complement, a balance between scale of diffusion and speed of diffusion may exist.

Second, when random effect is neutral, the average number of customers in each time period drops as the correlation coefficient decreases and the frequency of unexpected events increases. Also, given a correlation coefficient, when advertising and WOM are substitutes, fewer people purchase products on average as unexpected events occur more frequently. However, when advertisements complement WOM, more people buy products on average as unexpected events occur more frequently. This provides managerial implications that firms might be able to create correlation wisely to hedge from uncertainties arising from the external environment.

Finally, when randomness generates a positive effect, the average number of buyers goes up as the frequency of unexpected events increases, but when randomness generates a negative effect, the average number of buyer declines as the frequency of unexpected events increases. This is consistent with common intuition because if an unexpected event favors diffusion, it could create more adopters on average.

This study contributes to the extant literature in several important ways. On one hand, we successfully add correlation between external and internal factors, usually considered deterministic rather than stochastic, to the diffusion model. The method allows us to address the correlated effect directly. On the other hand, to our knowledge, we are the first to incorporate unexpected events into diffusion of products. We believe that random effect can help us better understand what might happen after unexpected events and how companies can protect themselves from uncertainty by making use of the correlation between advertising and WOM.

We organize the rest of this paper as follows: Section 2 provides a literature review; in section 3, we give the description of our model, a non-homogeneous Poisson process with stochastic parameters modulated by Markov chain, and present our numeric results; finally, conclusions and discussions will be given.

2. Literature review

Advertising could acquire, retain and grow customers. Retention and growth are related to
post-sale stage in which companies make efforts to build consumers’ loyalty to brands [2]. Since our concern is on the first purchase, we will limit our discussion in acquisition aspect of advertising. Advertisements are usually used to persuade consumers to purchase a product by offering information about both its existence and its quality [26]. More recent studies have prioritized the effect of word-of-mouth, a notion first defined by [6] as an interpersonal communication through which people disseminate information regarding a product or service. A surge in social media generates a great deal of e-WOM, which has enriched our understanding of the impacts of both factors on the products diffusion. Broadly speaking, the literature on modeling two strategies can be classified into three categories: (1) studies that discuss the impact of advertising and WOM on product adoptions independently, (2) studies that deal with the correlation between advertising and WOM, and (3) studies that focus on unexpected events influencing advertisements and WOM.

2.1. Literature on modeling the two effects independently

Advertising can affect consumers’ purchasing behavior through both awareness and persuasion [4, 7]. Awareness refers to the situation in which people who are exposed to advertising obtain product- or service-related information. Such information is important when consumers decide whether to buy a product for the first time. Persuasion means that companies, through advertising, signal the quality of products or services and further convince consumers to buy them [27]. Various regression models [15, 25] and diffusion models [41] have been applied to assess the impact of traditional advertising on consumers’ adoptions. Recent empirical studies have provided further evidence of the effectiveness of advertisements [20, 43].

With the popularity of online platforms where buyers can post ratings or reviews about products or services, attention has shifted to e-WOM’s role in consumers’ purchase behavior. The mechanisms under which WOM shapes consumers’ attitudes and behavior are twofold. The WOM valence (whether the opinion is positive or negative) can change customer expectations and valuation of the product [12, 34]. WOM volume (the amount of information available) can increase the degree of awareness about products [17, 29]. Using statistical regression models, scholars have reached a consensus that WOM is a decisive factor in promoting sales. However, the roles played by valence and volume are inconsistent [17, 21, 22]. The reason might be that characteristics of certain products, such as perceived risk, ambiguity, and uncertainty, may affect the degree to which WOM influences adoption decisions [5].

Therefore, two major methods in this stream of literature are statistical regression and diffusion models. Empirical studies have shown that both advertisements and WOM are powerful in stimulating product adoptions.

2.2. Literature on modeling the correlation between the two strategies

Treating advertising and WOM as independent factors ignores that they might be endogenously determined rather than exogenously given [30]. Consider a customer who buys a product or service as a result of advertising and then shares his opinion or experience online. As initial adopters accumulate, WOM has more chance to spread. On the other hand, firms tend to adjust their advertising strategy in response to e-WOM provided on a third-party platform [11]. As a result, growing interest has manifested in the interdependence between the two factors. There are two conceptually different effects of one on the other. First, the interaction is substitute when an increase (decrease) in one aspect lowers (raises) consumers’ needs for the other. When customers are exposed to a great deal of advertising, their needs to acquire additional information through WOM are reduced due to a high level of awareness. Consequently, companies do not have to invest too much in WOM. Second, the relationship is complementary when an increase (decrease) in one aspect leads to an increase (decrease) in the other. Higher exposure to advertising might stimulate consumers to learn more about the product or service via communication with others. In this case, it might be necessary for companies to make efforts in creating buzz among customers.

The major approach under this stream of literature employs a statistical model, such as value at risk (VAR) or customer life-long value (CLV), while only a few studies have adopted diffusion models. The results are mixed in different contexts. For high-quality products, WOM can serve as a substitute for advertising so firms can spend fewer resources to promote products [33]. Based on a study on printers and running shoes, [11] detects both substitutive and complementary effects between third-party reviews and advertising, depending on the penetration rate of the review. Again, the inconsistency arises from the distinct features of the product itself [5]. When taking dynamics into consideration, studies reveal that advertising and WOM can serve as substitutes for
each other over time. According to [32], people tend to put different weights on the effects of advertising and interpersonal communication over time. Initially, customers rely more on advertising because it is easily accessible and not much WOM is available. As adopters and pertinent reviews gradually accumulate, WOM becomes the driving force because customers can learn about a product or service through others’ opinions or experiences. Consequently, it has been found that advertising is more effective in an early stage in promoting sales, while WOM plays a bigger role in later stages [10, 22, 26, 44].

Therefore, correlation between advertising and WOM does exist. However, correlation is empirically corroborated rather than explicitly modeled. Consequently, little literature directly addresses the correlated effect on product diffusion and we attempt to fill this gap.

2.3. Literature on modeling random effect on advertising and e-WOM

Unexpected events are among the reasons why advertising and WOM change over time. Certain advertisements are regulated. For example, advertisements for tobacco products and alcoholic beverages are at the top of the agenda for regulators [1]. Such regulatory policies are unpredictable and regulated firms must reduce their investment to accommodate a sudden conversion [31]. Another uncertainty arises from product-harm crises. A recent notable example is Toyota’s world-wide recall of more than seven million cars due to technical problems; advertising is an effective communication device to regain customers’ lost trust after such a reputation disaster [13]. In that case, unexpected events might increase the investment in advertising. With respect to WOM, accidental crises like scandals can be seen as a form of inappropriate behavior that can lead to anger, which in turn can trigger negative WOM [14]. Unsatisfied consumers tend to spread unfavorable information about products. The increased use of the internet makes such negative e-WOM easier to disseminate [42]. Consequently, an unexpected event might suddenly change customers’ overall attitudes toward products.

Scholars have modeled uncertainties by regarding the whole process as stochastic [9, 23, 36] or by adding stochasticity into coefficients of external and internal factors [18]. But those diffusion models cannot reflect how unexpected events affect levels of advertising and WOM and the correlation between the two. To address the issue, we combine two approaches and use a Markov chain to modulate the frequency of unexpected events and its impact on advertising and WOM and further on product diffusion.

3. Model development and numerical results

3.1 Model development

We develop a non-homogeneous Poisson process. For correlated effect, we regard coefficients of advertising and WOM as random variables and generate them through Copula method given their correlation. For random effect, we use a Markov chain to modulate correlation between two factors as well as distributions of two factors. The Markov chain is assumed to be ergodic and irreducible. Its transition probability matrix captures the frequency of unexpected events.

Consider a non-homogeneous Poisson process, \( \{N(t), t \geq 0\} \), and a market with fixed size \( m \), then the Poisson process has a finite state space \( \{0, 1, \ldots, m\} \). If a customer has not bought the product by time \( t \), the chance that she adopts the product during the time interval \( (t, t+\Delta t) \) is influenced by external factors, like advertising, and internal factors, like WOM. For external factors, we suppose that the external impact on the probability of individual adoption is given by

\[
\alpha \times \Delta t + o(\Delta t)
\]

where \( \alpha \) is assumed to be a random variable which is uniformly distributed. For internal factor, if one person adopts the product at time \( t \), then her impact on the probability of a non-adopter buying the product is given by

\[
\beta \times \Delta t + o(\Delta t)
\]

where \( \beta \) is assumed to be a random variable which is uniformly distributed. Suppose that at time \( t \), there are \( N(t) \) number of adopters, then the chance that one potential customer buys the product at time \( t \) can be expressed as

\[
\phi(t) = (\alpha + \beta \times N(t)) \times \Delta t + o(\Delta t)
\]

Multiplying \( \phi(t) \) by the number of potential customers at time \( t \), which is \( m - N(t) \), gives us the adoption rate during the time period \( (t, t+\Delta t) \)

\[
\lambda(t) = \alpha \times (m - N(t)) + \beta \times N(t)(m - N(t))
\]

which is dependent on the number of adopters at the beginning of the time period. For \( \{N(t), t \geq 0\} \) to be a Poisson process, we assume that the random variable \( N(t) \) is Poisson distributed with mean \( m(t) = \int_0^t \lambda(s) \, ds \) and \( \lambda(t) \) is the intensity function of Poisson process.

In order to study the correlated effect on product diffusion, we apply Copula method to generate parameters of advertising and WOM with specified correlation. Assume that parameters \( \alpha \) and \( \beta \) are uniform random variables with probability density functions \( f(x) \) and \( f(y) \) respectively. Also, we
denote the Pearson correlation coefficient between \( \alpha \) and \( \beta \) as \( \rho_{\alpha\beta} \), which can be calculated as \( \rho_{\alpha\beta} = \text{Cov}(\alpha, \beta) / \sigma_\alpha \sigma_\beta \). Then, it is obvious that the negative correlation coefficient represents substitute relationship, while the positive one represents complementary relationship.

For \( m \) uniform random variables, \( U_1, \ldots, U_m \), Copula function is defined as
\[
C(u_1, \ldots, u_m) = \text{Pr}(U_1 \leq u_1, \ldots, U_m \leq u_m)
\]

What we consider here is a bivariate copula function \( C(u_1, u_2, \rho) \) for standard uniform random variables \( U_1 \) and \( U_2 \), defined on the area \( \{(u_1, u_2) | 0 \leq u_1 \leq 1, 0 \leq u_2 \leq 1\} \). Among common Copula functions, we apply Gaussian Copula which can be expressed as
\[
C(u_1, u_2, \rho) = \Phi'(\Phi^{-1}(u_1), \Phi^{-1}(u_2), \rho)
\]
where \( \Phi' \) is the bivariate normal distribution function with correlation coefficient \( \rho \) and \( \Phi^{-1} \) is the inverse of a univariate normal distribution function. Note that the \( \rho \) here does not equal the desired Pearson correlation coefficient \( \rho_{\alpha\beta} \). It is common to use Spearman’s Rho to approximate the original Pearson correlation coefficient. In accordance with our notations here, we have
\[
\rho = 2 \sin(\rho_{\alpha\beta} \pi / 6).
\]

Once we get a pair of \((u_1, u_2)\) with given \( \rho \), it is trivial to transform them into \((\alpha, \beta)\) whose density functions are \( f(x) \) and \( f(y) \) respectively. In summary, the algorithm for generating a pair of \((\alpha, \beta)\) with given \( \rho_{\alpha\beta} \) can be stated as:
1. Compute \( \rho \) from the desired correlation coefficient \( \rho_{\alpha\beta} \) using equation \( \rho = 2 \sin(\rho_{\alpha\beta} \pi / 6) \).
2. Generate a pair of \((X_1, X_2)\) from bivariate normal distribution with \( \rho \).
3. Generate a pair of \((U_1, U_2)\) using Gaussian Copula, in which \( U_1 \) and \( U_2 \) are standard uniform random variables.
4. Transform \( U_1(U_2) \) into \( \alpha(\beta) \) whose density function is \( f(x)(f(y)) \).
5. Repeat (2) to (4) for \( n \) times.

Note that step (4) does not change the approximate correlation coefficient \( \rho \) because it is a linear transformation of random variables. Furthermore, since we use Spearman’s Rho as approximation of Pearson correlation coefficient, there exists a gap between the simulating correlation coefficient and the desired correlation coefficient. The variation becomes smaller as the number of sampling becomes large.

In order to study the random effect on product diffusion, we apply Markov modulated process (MMP) to modulate correlation coefficient \( \rho_{\alpha\beta} \) as well as distributions of parameters \( \alpha \) and \( \beta \). We consider a two-state, 0 and 1, Markov chain with transition probability \( \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \). The Markov chain is assumed to be ergodic and irreducible.

Without loss of generality, 0 and 1 represent normal and abnormal conditions respectively. Parameter \( x \) represents the probability that conditions remain normal, while parameter \( y \) represents the probability that conditions change from abnormality to normality. For simplification, we let \( x \) be equal to \( y \) and adjust the value of \( x(y) \) to determine the frequency of unexpected events and further explore the random effect on adoption pattern. We consider here three scenarios of random effect: (1) neutral effect, in which distributions of \( \alpha \) and \( \beta \) remain unchanged under two states; (2) positive effect, in which the means of random variables \( \alpha \) and \( \beta \) are bigger under abnormal state than under normal state; (3) negative effect, in which the means of random variables \( \alpha \) and \( \beta \) are smaller under abnormal state than under normal state.

To test the validity of our model, we conduct a numerical study. We fix the number of potential buyers and assess performance of product diffusion in terms of average number of adopters in each time period (i.e. scale of diffusion) and speed to reach the total customer base (i.e. speed of diffusion). We apply “thinning” method to simulate the timing of each adoption. First, compute \( \lambda^* \), the upper bound of \( \lambda(t) \). Since \( N(t) \) is finite and bounded below and above, \( \lambda(t) \) can be seen as a parabola going downwards in terms of \( N(t) \). So, the intensity function, \( \lambda(t) \), is bounded above and there exists a \( \lambda^* > 0 \) such that \( \lambda(t) \leq \lambda^* \). Second, sequentially generate independently and identically distributed inter-arrival times with exponential rate \( \lambda^* \). Use the recursion, \( t_n = t_{n-1} + \frac{-1}{\lambda^*} \ln(U_0) \), to obtain the arrival times which are denoted as \( t_n \). Third, for each arrival time \( t_n \), we accept it with probability \( p_n = \lambda(t_{n-1}) / \lambda^* \), and reject it with probability \( 1 - p_n \). The resulting sequence of accepted times, \( \{t_n\} \), forms our desired non-homogeneous Poisson process. For each accepted \( t_n \), we also record the corresponding \( \lambda(t_{n-1}) \). So, the average number of adopters is calculated as the average of accepted adoption rate and the speed is calculated as the timing of last adoption. The thinning algorithm for generating non-homogeneous Poisson process can be stated as:
1. \( t=0, N=0 \)
2. Generate a \( U' \) and compute \( t = t - \ln U' / \lambda^* \). If \( t > T \), then stop.
3. Generate a \( U'' \). If \( U'' < \lambda(t)/\lambda^* \), then set \( N=N+1, t_n=t \), and \( \lambda = \lambda(t_{n-1}) \).
4. Go back to (2).

3.2 Numerical results

We set the number of potential buyers, \( m \), at 320.
In the benchmark model, correlation coefficient \( \rho_{ab} \) is 0, \( \alpha \) is uniformly distributed on \([0,1,0.3]\) with mean 0.2, and \( \beta \) on \([0.00115625,0.00315625]\) with mean 0.00215625. Those values are chosen according to regression results using Bass model. We find that, for the benchmark model, average number of adopters in each time period is 71.78 and time used to reach all potential customers is 9.01.

First, we study correlated relationship between advertising and WOM without considering randomness. Negative correlation coefficient represents substitution and positive one represents complement. Correlation coefficient \( \rho_{ab} \) is set at -0.8, -0.5, -0.2, 0.8, 0.5 and 0.2, representing for highly substitutive, intermediately substitutive, slightly substitutive, highly complementary, intermediately complementary and slightly complementary.

Second, we take unexpected events into account. We use an ergodic and irreducible Markov chain to modulate correlation between advertising and WOM and adjust the frequency of unexpected events. The Markov chain has two states, representing normal condition and abnormal condition respectively. Under normal condition, assumptions about \( \rho_{ab} \), \( \alpha \) and \( \beta \) are the same as those in benchmark model. For abnormal conditions, we consider three scenarios: neutral effect, positive effect and negative effect. Table 2 summarizes the choice of parameters of \( \rho_{ab} \), \( \alpha \) and \( \beta \) of those scenarios under abnormal situations.

### Table 1. Parameters under abnormal conditions

<table>
<thead>
<tr>
<th></th>
<th>Neutral random effect</th>
<th>Positive random effect</th>
<th>Negative random effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_{ab} )</td>
<td>-0.8, -0.5, -0.2, 0.8, 0.5, 0.2</td>
<td>-0.8, -0.5, -0.2, 0.8, 0.5, 0.2</td>
<td>-0.8, -0.5, -0.2, 0.8, 0.5, 0.2</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>U[0.00115625,0.00315625]</td>
<td>U[0.00215625,0.00415625]</td>
<td>U[0.00015625,0.00215625]</td>
</tr>
<tr>
<td>( \beta )</td>
<td>U[0.00115625,0.00315625]</td>
<td>U[0.00215625,0.00415625]</td>
<td>U[0.00015625,0.00215625]</td>
</tr>
</tbody>
</table>

For the transition probability matrix, \((x \quad 1-x \quad y \quad 1-y)\), we let \( x \) be equal to \( y \), so the parameter \( x \) represents the frequency of unexpected events. We set \( x \) at 0.9, 0.7, and 0.5. When \( x \) is 0.9, unexpected events seldom occur, but when \( x \) is 0.5, the unexpected events occur more frequently. So, we define frequency of unexpected events (or randomness) as \( 1-x \). This gives us 0.1, 0.3, and 0.5, representing slightly random, intermediately random, and highly random, respectively.

#### 3.2.1. Correlated effect

Compared to benchmark model, substitute relationship lowers the number of adopters and shortens the time (see Figure 1(a)). The average change is -1.36% and -2.63% respectively. As substitution increases (i.e. correlation coefficient decreases), the number of buyers in each time period on average declines, but the speed to reach all potential customers becomes higher. The rationale behind this might be that people feel comfortable with moderate amount of information and adopt the product despite few others actually doing the same.

Compared to benchmark model, complementary relationship adds more buyers on average and shortens the time (see Figure 1(b)), suggesting that advertising complementing WOM favors product diffusion as a whole. Furthermore, as complement increases (i.e. correlation coefficient increases), average number of consumers goes up, but it also takes longer time to persuade potential customers into buying. The reason might be that with more information available from both advertising and friends’ recommendations, customers are more likely to find them redundant and to respond slowly.

Comparing substitution with complement, we find that complementary correlation between advertising and WOM leads to higher average number of adopters, suggesting strength of both factors in product diffusion. The results also show that introducing correlation between advertising and WOM could help speed up diffusion process, but such an improvement might not be obtained without any expenses. Substitution shortens the time at the expense of fewer buyers on average, implying that there is a balance between average scale of diffusion and speed of diffusion when firms invest heavily in either advertisements or WOM.

#### 3.2.2. Neutral random effect

When unexpected events do not affect the distribution of external and internal factors (i.e. distributions of \( \alpha \) and \( \beta \) remain the same as in the normal situation), the average number of customers in each time period drops as correlation coefficient decreases and the frequency of unexpected events increases (See Figure 2.(a)). But the speed of attracting all potential consumers varies greatly along two dimensions (See Figure 2.(b)). Compared to benchmark model, the change of average number of purchases is around 0.23%, and the change of total time is around -1.41%. Another interesting finding is that given a correlation coefficient, when advertising and WOM are substitutes, fewer people...
purchase products on average as unexpected events occur more frequently. However, when advertisements complement WOM, more people buy products on average as unexpected events occur more frequently. The results offer managerial implications that, in terms of product diffusion, firms might be able to take advantage of uncertainty arisen from external environment. They can expose customers to both high level of advertising and WOM.

3.2.3. Positive random effect

Unexpected events sometimes stimulate advertising by firms and customers’ eagerness to talk, reflecting in higher mean of uniformly distributed random variables $\alpha$ and $\beta$. For instance, in April this year, Chinese government banned some American dramas, including *The Big Bang Theory*. However, the action spurred anger among viewers and created far more e-WOM about those dramas online. Under this circumstance, the results show that, given a correlation coefficient, average number of buyers goes up as frequency of unexpected events increases, a finding in accordance with common intuition (See Figure 3. (a)). Since randomness brings about greater investment in advertisements and more buzz among people, unexpected events actually favor product diffusion and could attract more consumers in each time period on average. With regard to speed, on the whole, less time is needed to persuade all potential customers into buying products as the randomness increases (See Figure 3. (b)). Except that when correlation coefficient is between -0.5 and 0.5, which means from slightly substitutive to slightly complementary, time and frequency relationship exhibits a U shape. This result indicates that only when substitution or complement between advertising and WOM is high can companies successfully reach all potential customers in a shorter time.

3.2.4. Negative random effect

Unexpected events might also reduce advertising level by firms and customers’ eagerness to talk, reflecting in lower mean of uniformly distributed random variables $\alpha$ and $\beta$. In fact, this is more common in our daily life. For example, *Chinese Idol* cut down its agenda for competition because of regulatory policies, leading to a reduced exposure to viewers by both advertising and online buzz. Under this case, we discover that given a correlation coefficient, average number of adopters declines despite some minor local fluctuations (See Figure 4. (a)). In addition, it takes longer time to reach all potential customers as unexpected events happen more often (See Figure 4. (b)). The results suggest that if unexpected events incur a negative impact on advertising and WOM, customers have more difficulties receiving information about products. Consequently, they have less incentive to
4. Conclusions and discussions

With its ever-increasing power, word-of-mouth has greatly shaped consumers’ behavior. In the meantime, advertising still maintains its strength. However, the relationship between two strategies is not yet clear with regard to the diffusion of products. In addition, even less work has dealt with unexpected events under a product diffusion framework. This paper addresses those crucial questions by using a non-homogeneous Poisson process modulated by a Markov chain. The model is original because the correlation between advertising and WOM is explicitly modeled and can be adjusted in the case of unexpected events through MMP. Therefore, we are able to unveil correlated effect and random effect on product diffusion. Several insights from our numerical analysis deepen our understanding in this domain.

First, rather than merely identifying what the relationship is between two factors [11, 30], we take a step further to investigate how the relationship affects product diffusion. We reveal that correlation between advertising and WOM can help speed up the process of product diffusion, but substitution and complement exhibit different patterns in scale of diffusion. Complementary relationships add more buyers on average, indicating the strength of both high levels of advertising and WOM in product diffusion. Studies have demonstrated a carry-over effect of advertising, meaning that more advertising can create more WOM [10, 24]. Thus, companies’ huge investment in advertisements can be justified. In fact, in 2012, global product placement spending increased 11.7% to $8.25 billion [39].

Second, using the Copula method to generate two correlated random variables, we study the impacts of the correlations at different levels. Scale and speed of diffusion are two main concerns for firms [28]. Our results enrich our understanding of the interplay between advertising and WOM with regard to those two aspects. We find that as substitution becomes greater, the average number of customer drops and the time to reach all potential consumers shortens, while as complement becomes greater, the results show the opposite. With more information available from both advertising and friends’ recommendations, customers are more likely to find the information redundant and to respond slowly due to information overload [37]. So, when considering a reasonable level within substitution or complement, there might be a balance between scale of diffusion and speed of diffusion.

Third, since unexpected events affect advertising and WOM [13, 14] as well as product diffusion, modeling random effect of those events opens up a new theoretical venue. We successfully achieve our goal through a Markov modulated
process. When random effect is neutral, we find that the average number of customers in each time period drops as the correlation coefficient decreases and the frequency of unexpected events increases. This might explain the high level of both advertising and WOM in the high-tech industry, which is increasingly fast-changing. In 2012, tech companies grew their advertising spending significantly, with Apple and Google being among world’s biggest advertising spenders [40]. Tech companies also actively engage in e-WOM, with 39% of companies using social media to create buzz [16]. Furthermore, given a correlation coefficient, when advertising and WOM are substitutes, fewer people purchase products as unexpected events occur more frequently. However, when advertisements complement WOM, more people buy products as unexpected events occur more frequently.

Our major theoretical contributions are twofold. On one hand, we successfully add correlation between external and internal factors, which are usually considered as deterministic rather than stochastic, into the diffusion model. This allows us to address the correlated effect directly. On the other hand, to our knowledge, we are the first to incorporate unexpected events into diffusion of products. We believe that random effect can help us better understand what might happen after unexpected events and how companies can protect themselves from uncertainties by making use of the correlation between advertising and WOM.

Our research also provides managerial implications in marketing strategies. Practitioners should first clarify what the priority is, scale of diffusion or speed of diffusion? In a relatively stable industry, if the goal is to expand market size, it is necessary to introduce a high degree of complementary correlation. Companies could invest heavily in both domains or concentrate on advertising and let its carry-over effect to create subsequent WOM. In a relatively fast-changing industry, companies could create complementary relationship to add more buyers on average so as to hedge uncertainties arising from the external environment. If the goal is to speed up diffusion process and the unexpected events turn out to favor the diffusion, both substitutive and complementary relationship at intermediate level would be fine. Put another way, companies should avoid putting equal money in two domains or focusing on one and totally ignoring the other. Instead, firms should assign resources in advertising and WOM with an emphasis in either one.

Although this research extends our current knowledge, several directions may be worth pursuing in the future. First, empirical data are needed to further verify our model. However, obtaining an appropriate data set could be a challenge. Second, we do not distinguish WOM volume and valence. Modeling those two aspects into a diffusion model will provide additional insights into their respective role in spreading information. Third, our assumptions are rather simple based on the probability that a non-adopter buys the product, disregarding the heterogeneity of individuals and network structure. We believe it will be a worthwhile endeavor to incorporate network characteristics into a diffusion model.

Acknowledgement
The research was supported by National Natural Science Fund of China (71372112) and Innovation Program of Shanghai Municipal Education Commission (12ZS012).

Reference