Self-Correction Strategies for Frequency Domain Ringdown Analysis

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Abstract

This paper introduces a set of self-correction strategies for improved oscillation modal analysis of ringdown responses from synchrophasors in frequency domain analysis. It is shown that the modal estimates of frequency, damping ratio and mode shape using the improved frequency domain algorithms of the paper match well with other time-domain algorithms while preserving their inherent advantages of superior noise performance and computational speed. The new algorithms of the paper are especially suited for simultaneous ringdown oscillation analysis of large number of Phasor Measurement Unit (PMU) measurements efficiently.

Keywords: Power system stability, oscillations, Prony methods, damping estimation, power system dynamics.

1. Introduction

Large numbers of PMUs are being implemented in power systems all over the world as the cost of PMU devices has been declining in the past ten years. Hundreds of PMUs are already streaming real-time measurements into the control centers in the eastern American and western American power grids. Motivated by the availability of synchronized wide-area PMU measurements, many algorithms have been proposed in the recent few years for deriving system dynamic information from wide-area measurements.

Power systems are large-scale nonlinear dynamic systems where instability can arise from any of small-signal instability, voltage instability or angle instability [1]. Small-signal stability plays an important role in the safe operation of power grid. The main job in small signal stability analysis is to detect presence of any negatively damped or poorly damped oscillatory modes so that proper actions could be taken in time to damp out the oscillations before they become critical. As an example, the August 10, 1996 blackout in the western American power grid was caused by a negatively damped inter-area oscillation mode [2].

Oscillation monitoring of power systems related to small-signal instability using PMUs has been active area of research. The focus of this paper is on analysis of ringdown responses from the power system following disturbances such as from tripping of lines and generators [3]-[10]. There are several traditional model based modal analysis methods, for example, Prony [3], Eigensystem Realization Algorithm (ERA) [4], Matrix Pencil (MP) [5] and Hankel Total Least Square (HTLS) [6] for ringdown analysis of PMU measurements. Assuming linearity of power system responses, most of these algorithms provide excellent modal estimation results.

The common challenge in all the time-domain algorithms such as Prony, ERA, MP and HTLS is the large computational burden especially for processing large number of PMU signals simultaneously. Recent frequency domain algorithms proposed in [9],[10] address the scalability of ringdown analysis by tracking the trends of energy decay (or increase) for multiple oscillatory modes to estimate their damping ratios efficiently. This paper analyzes some of the fundamental limitations of the frequency domain algorithms in [7]–[10] and discusses how to overcome some of the limitations. By comparing the PMU measurements versus the signals as seen from the outputs of the modal estimation algorithms, a series of self-correcting strategies are introduced. It is shown that these strategies enable the estimated signals from the improved frequency domain ringdown algorithms to closely match the measured PMU responses while preserving the simplicity and efficiency of frequency domain analysis.

The paper is organized as follows. A brief introduction to the algorithm, Modal Energy Trending for Ringdown Analysis (METRA), from [9] is provided in Section 2. Limitations of METRA arising from frequency domain challenges are discussed in Section 3. Improvements to METRA are offered in Section 4 in the spirit of self-tuning corrections to METRA estimates. An algorithm with self-correction, denoted improved METRA (iMETRA), is outlined in Section 4. Comparisons of estimation results for
METRA and iMETRA from test cases and PMU data provided in Section 5 show the effectiveness of iMETRA as compared to METRA.

2. METRA algorithm

METRA is a fast oscillation detection and monitoring algorithm to do oscillatory stability analysis of power systems [9]. The first step in METRA is to find the system modes by identifying the local maxima in Power Spectrum Density (PSD) function in the spirit of Frequency Domain Decomposition (FDD) [11]. The next step is to track the change in oscillatory energy for each mode over time so that its damping level can be estimated.

In the METRA algorithm, to get the total energy value from the multiple PMU signals for each dominant mode, Single Value Decomposition (SVD) is applied to the PSD matrix. Unlike in time-domain algorithms such as ERA, MP, HTLS, the size of the matrix requiring SVD is small, and it does not slow down the estimation. Then to track the energy trend, the power spectrum magnitude of the mode of interest from multiple smaller windows is calculated separately. By fitting all these values in a least square sense, the damping ratio which describes the change of oscillatory mode energy can be obtained [10].

As described in [1], power system with small disturbance can be linearized around its equilibrium points. The linear combination of exponential terms of all system oscillatory modes can be used to describe system response at the disturbance period. Suppose the post-disturbance ringdown response of a power system measurement $y_i$ is represented by

$$y_i(t) = \sum_{j=1}^{m} A_j e^{-\sigma_j} \cos(w_j t + \phi_j)$$  \hspace{1cm} (1)

where $m$ is the number of modes; $A_j$ and $\phi_j$ represents the amplitude and phase angle of mode j respectively; $w_j$ is the mode frequency; $\sigma_j$ is the damping factor.

It is assumed that $y_i$ is sampled at $F_s$ samples per second. Complex Fourier Transform of the signal $y_i(t)$ over the time window from $n_0$ to $n_0 + T$:

$$F(w)\bigl|_{n_0}^{n_0+T} = \sum_{n=n_0}^{n_0+T} y(n) e^{-j2\pi w_n}$$  \hspace{1cm} (2)

The frequency spectrum will have $m$ peaks, each of which represents the frequency component of an oscillatory mode. However, the “peaks” associated with well-damped modes will be small compared to those of poorly damped modes. In other words, only a few of the “dominant” oscillatory modes can be observed in the FFT spectrum of real measurements [9] and our interest is to estimate the properties of such modes. It is assumed that each dominant mode has little impact on other dominant modes of the system (that is, the dominant modes are reasonably far apart). Fourier Transform of the signal $y_i(t)$ over the time window from $n_0$ to $n_0 + T$ can then be expressed [9] as

$$F_i(w) = \frac{\sum_{n=n_0}^{n_0+T} A_i e^{-\sigma_j} \cos(w_j n + \phi_j)}{\sqrt{\sum_{n=n_0}^{n_0+T} e^{-j2\pi w_n}}}$$  \hspace{1cm} (3)

It should be noticed that the time window $T$ is chosen large enough so that the energy of each mode $w_j$ can be found reliably. For one dimensional system, by multiplying the complex Fourier Transform $F_i(w)$ at each mode frequency by its complex conjugate, the Power Density Spectrum $S(w)$ can be calculated:

$$S(w) = F_i(w) F_i(w)^*$$  \hspace{1cm} (4)

In the power system modal estimation, analysis of PMU information from many channels is expected to provide insight on the nature and source of oscillations by including all the signals in mode shape analysis. Accordingly, METRA is designed to process many signals simultaneously in a multi-dimensional algorithm. To do the multi-dimensional analysis, Power Spectrum Density (PSD) matrix is introduced at each mode frequency $w_j$:

$$S(w_j) = \begin{bmatrix} F_1(w_j) & F_2(w_j) & \cdots & F_m(w_j) \\ F_2(w_j) & F_1(w_j) & \cdots & F_m(w_j) \\ \vdots & \vdots & \ddots & \vdots \\ F_m(w_j) & F_2(w_j) & \cdots & F_1(w_j) \end{bmatrix}$$  \hspace{1cm} (5)

Where $F_i(w_j)$ is the Fourier Transform of signal $y_i(t)$ at $w_j$. Next, PSD matrix is decomposed into the following SVD form:

$$S(w_j) = U_j (w_j) S_j (w_j) V_j (w_j)^*$$  \hspace{1cm} (6)

where matrix $U_j = [u_1, u_2, \ldots, u_m]$ is a unitary matrix of the left singular vectors. Matrix $S_j (w_j)$ is a diagonal singular value matrix, where the singular values are denoted by $s_1 (w_j), s_2 (w_j), \ldots, s_m (w_j)$ respectively. With the assumption that every dominant frequency has little effect on other modes, the total energy at the frequency $w_j$ from all signals can be well represented by the
largest singular value \( s_1(w_j) \). Then, the Complex Mode Indication Function (CMIF) \( \hat{S} \) can be defined [11] as
\[
\hat{S}(w_j) = s_1(w_j)
\]  
(7)
where \( s_1(w_j) \) comes from the largest singular value of SVD of the Power Spectrum Density matrix. The dominant oscillatory modes can be searched from the peaks of CMIF. After the dominant frequency \( w_j \) which represents one mode is selected from the CMIF peak, the first singular vector \( u_1(w_j) \) corresponding to the peak frequency \( w_j \) is the estimation of mode shape for that mode. \( u_1(w_j) \) can be rewritten in the polar form:
\[
u_1(w_j) = A_j \angle \phi_j
\]  
(8)
where \( A_j, \phi_j \) are the magnitude and phase angle of the mode shape for mode \( w_j \) respectively.

To calculate the damping factor \( \sigma_j \) of the mode \( w_j \), we take a smaller time window \( N \) from the big time window \( T \), where \( N \) starts from \( n_0 \) and ends at \( n_0 + N \). The CMIF \( \hat{S}(w_j) \) over the smaller time window can be calculated from Fourier Transform and Power Density Spectrum SVD of signal \( y(t) \) at the same time window. Similarly, \( \hat{S}(w_j) \) can be obtained after Fourier Transform and Power Density Spectrum SVD of signal \( y(t) \) over a later time window
\[
[n_0 + kG, n_0 + N + kG] \quad (n_0 + N + kG < T)
\]
where \( G \) is the step size of the sliding window. Normally, \( G \) is taken as \( 1/2f_s \). Then, the damping factor \( \sigma_j \) can be determined from the change of magnitude of the elements in the power spectrum measure set
\[
\{ \hat{S}(w_j)_{n_0+N}, \hat{S}(w_j)_{n_0+N+G}, \hat{S}(w_j)_{n_0+2G}, \ldots \}
\]
Let vector \( Y \) contains all magnitudes of power spectrum measure of signal \( y(t) \) at different sliding time windows, and vector \( X \) contains all the end times of the sliding time window. That is,
\[
Y = [\hat{S}(w_j)_{n_0+N}, \hat{S}(w_j)_{n_0+N+G}, \hat{S}(w_j)_{n_0+2G}, \ldots]
\]  
(9)
\[
X = [n_0 + N, n_0 + N + G, n_0 + N + 2G, \ldots]/P
\]  
(10)

Taking the Logarithmic value of all the magnitude of power spectrum measure in \( Y \), if the original time domain signal \( y(t) \) is noiseless, the Logarithmic magnitude in the power spectrum measure would fit a straight line [7],[8]. Therefore, all the measures can be described by an over determined system:
\[
X^i\sigma_i + b = \ln(Y_i), \quad i = 1,2,3,\ldots,k
\]  
(11)
where \( k \) is the number of sliding windows and \( b \) is the power spectrum measure magnitude of \( y(t) \) over time window \([-T,0] \). The optimum damping factor \( \sigma_i \) that best fits data in hand can be attained by solving the Least Square Fit problem which minimize the error \( E(X,Y) \) [9],[10] :
\[
E(X,Y) = \sum_{r=1}^{k} W_i[(\ln(Y_i) - (X_i\sigma_i + b)]^2
\]  
(12)
where the weights \( W_i \) come from PMU data window with good quality which give pre-specified emphasis on some spectrum measures over other windows. The over-determined system with all \( k \) sliding time window measures can be written as:
\[
\begin{bmatrix}
\ln(Y_i) \\
\vdots \\
\ln(Y_i)
\end{bmatrix} = 
\begin{bmatrix}
X_1 & 1 & \beta \\
\vdots & \vdots & \vdots \\
X_k & 1 & b
\end{bmatrix}
\]  
(13)

The least square estimate of \( \beta \) is given as
\[
\beta = (X^TWX)^{-1}X^TW\tilde{Y}
\]  
(14)
The damping ratio can then calculated from
\[
\zeta_k = \frac{-\sigma_k}{4\pi f_k}
\]  
(15)

Additional details of the algorithm and examples can be seen in [9].

3. LIMITATIONS OF METRA

The original METRA algorithm from [9] is designed with emphasis on speed, though it has some weaknesses and limitations. The limitations are mostly due to typical short time windows (less than 30 seconds) of PMU data used in ringdown analysis. It is well-known from signal theory that such short time windows are problematic in extracting the frequency domain features of the PMU responses owing to leakage issues [12]. In our context, short time windows can lead to small errors in the estimation of mode frequencies, and consequently in the estimation of their damping ratios and mode shapes. This section discusses two main effects caused by the short time windows on METRA algorithm. All the tests in the
paper have been implemented in a laptop with a 3 GHz Intel quad-core processor and 32 GB memory. The algorithms have been coded in C sharp using Intel Math library for accelerating all the vector and matrix related numerical calculations.

A. Frequency spacing

For taking the FFT of the windowed signal, number of samples must be zero padded to a number which should be a power of 2. Increasing this number decreases the frequency spacing in the “Power Spectrum Density” or “PSD” and produces a better looking plot, although it does not change the frequency resolution. Consequently, zero padding with a larger number of points can result in better frequency estimation [12].

The problem with zero padding with large number of points is that it proportionally increases the time needed for FFT calculation which slows down the algorithm and off-sets the original advantage of METRA.

On the other hand, decreasing the number of points for zero padding leads to larger frequency spacing which may cause two problems. First, the estimated frequency for each peak value in CMIF may not be close to the actual value, and second, when two adjacent modes are close to each other, it is nearly impossible to find two separate peaks.

B. Short time window effects

For all ringdown algorithms, a time window of about 15 to 20 seconds (450 to 600 samples assuming 30 Hz PMU sampling rate) is typically chosen for the analysis. Short time windowing of an infinite length signal in the time domain, especially when using rectangular window, can cause leakage effect in frequency domain [12]. When the signal has a mode at frequency \( \omega = \omega_j \), there must be a sharp peak in the power spectrum density of the signal at this particular frequency, but, as a consequence of windowing using a short time rectangular window, the power spectrum density will have the shape of a smeared impulse, with a main lobe roughly centred at \( \omega = \omega_j \). For a purely sinusoidal signal which contains only one frequency, the side lobes have a height proportional to the height of the main lobe (about -13 dB) [12]. These side lobes can negatively affect frequency algorithms such as METRA in two ways; first, the algorithm might find a spurious mode for each side lobe. Also, if there are two modes in the system that are close to each other, instead of having two distinct peaks for each mode, because of the width of main lobes, we may see one smeared peak. Therefore, the algorithm might miss one of the modes in this case.

C. Missing local modes

METRA uses CMIF for finding peak frequencies. When there is a local mode in some channels and the energy of the mode is not high, there is always the possibility that this mode would have a low peak in the CMIF and so the algorithm may not be able to find this mode. Because, when using a rectangular time window, there must be a threshold to distinguish between spurious peaks like side lobes and the real peaks; and when the mode is local, the height of its peak in CMIF might be lower than the threshold, so the algorithm may consider the peak to be a spurious one.

D. Error in the estimation of mode shapes

The error in the estimation of mode shapes is directly dependent on the error in the estimated mode frequency. Especially for the phase estimation of mode shapes, because of the relatively sharp slopes of the phase plots near the mode frequency, even a slight change in the estimation of mode frequency can change the phase significantly.

E. Example: Western system test case

To show the limitations of short time windowing on the estimations produced by METRA, a test case from a simulation of a 179 bus western American power system model is adopted. The test system has three dominant oscillatory modes, namely at 0.28 Hz, 0.66 Hz and 0.74 Hz. For this case, an event was simulated using TSAT [13] where a system disturbance caused the system modes to get excited in an oscillatory response. Figure 1(a) shows the responses of the bus voltage magnitudes at buses 1 to 4. Fig. 1(b) shows the respective values after subtracting the mean values from each signal and after normalizing. A time window of 20 seconds starting from \( t = 15 \) seconds was used for the ringdown analysis with 5 different engines. SVD thresholds for Matrix Pencil, HTLS and ERA are set to be 10% and Prony model order is 125. The signals in the time window are zero padded to 2048 points for METRA.

Table I shows a comparison of estimation results from different engines. In Table I, the estimation results of MP, HTLS and ERA all match well with each other for all three modes. While the Prony results are a little bit off, the differences in estimation results are somewhat lesser in the case of METRA. In terms of computational times shown in the second column, METRA is the fastest in Table I while Prony is the slowest. The last row in Table I shows estimation results for the improved iMETRA algorithm that will be introduced in Section 4.
Table I. Estimation results (f in Hz and \( \frac{\varepsilon}{g} \) in %)

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Time (sec)</th>
<th>Mode 1</th>
<th>Mode 2</th>
<th>Mode 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prony</td>
<td>1.170</td>
<td>0.272</td>
<td>7.98</td>
<td>0.664</td>
</tr>
<tr>
<td>MP</td>
<td>0.103</td>
<td>0.274</td>
<td>7.89</td>
<td>0.664</td>
</tr>
<tr>
<td>HTLS</td>
<td>0.093</td>
<td>0.274</td>
<td>7.64</td>
<td>0.664</td>
</tr>
<tr>
<td>ERA</td>
<td>0.101</td>
<td>0.274</td>
<td>7.89</td>
<td>0.664</td>
</tr>
<tr>
<td>METRA</td>
<td>0.042</td>
<td>0.278</td>
<td>7.55</td>
<td>0.674</td>
</tr>
<tr>
<td>iMETRA</td>
<td>0.054</td>
<td>0.271</td>
<td>7.87</td>
<td>0.656</td>
</tr>
</tbody>
</table>

shows the CMIF for the zero padded signals. In Fig. 2(a), it can be seen that there is one dominant mode between 0.2 and 0.3 Hz and two adjacent modes between 0.6 and 0.8 Hz. Because of the large frequency spacing the estimation for the dominant mode has some error reflected in the results of Table I. Also, because of the short length of the time window, it is hard to distinguish two peaks between 0.6 and 0.8 Hz. Fig. 2(b) shows the CMIF for the same system when zero padded with 8192 points. Comparing the two figures, peak frequency estimation from the second plot would be closer to the actual value. Because of the short time windowing effect, it can be seen that it is hard to distinguish the two peaks even with zero padding to higher number of FFT points.

Fig. 3 shows that the mode shapes produced from METRA algorithm differ somewhat from the mode shape from HTLS both in magnitude and phase. Mode shape results for iMETRA in Fig. 3 will be discussed later in Section 4.F.

4. RECOMMENDATIONS FOR IMPROVING METRA

In this section, improvements that have been made to METRA for reducing the estimation errors in mode frequencies and mode shapes are discussed.

A. Polynomial interpolation in frequency domain

As it has been discussed in the previous section, using larger number of points for zero padding will make smaller frequency spacing in power spectrum density calculation, and can result in possibly better estimation of the peak frequencies. However, the FFT calculation for a long signal is time consuming, and it can reduce the speed of the algorithm. Another approach is to use smaller number of points for zero padding and then doing an interpolation on the resulting FFT for having the desired frequency spacing. The interpolation is done by a polynomial fit of a certain degree. The main advantage of this approach over zero padding is the smaller number of points for FFT calculation. Moreover, since most of the dominant modes in a power system signal are typically within a certain frequency range, there is no need to do the interpolation for the whole frequency range. The
appropriate range for frequency axis in which the interpolation should be done, can be simply determined by a search for the peaks in the short FFT. Interpolation can be applied only for points near the peak frequencies from short time FFT to refine the exact location of the peak frequencies.

Figure 4 shows the CMIF for the test case of Section 3.E, where a second order polynomial interpolation is applied to the entire frequency range of CMIF of Fig. 2(a) which had 2048 points. The interpolation is done with a second order polynomial to create three additional points between each two points of the non-interpolated CMIF. Comparing Fig. 4 with Fig. 2(b), it can be seen that the polynomial interpolation leads to almost the same CMIF as from zero padding with 8192 points. In summary, this approach yields the same results as in the case of longer zero padding but with significantly lower computational time. The average computational time of the 8192 point FFT plus peak searching for all 4 channels and using a regular sequential for loop is 0.0739 seconds (with a standard deviation (STD) of 0.0115 seconds) from 20 simulations. For the other approach using frequency domain polynomial interpolation, the average computational time of the 2048 point FFT calculation with second order interpolation for finding the peak frequencies is 0.0203 seconds (with STD of 0.002 seconds) from 20 simulations. Clearly the polynomial interpolation with shorter FFT window is faster than the longer zero padded FFT.

![Fig. 4. CMIF with 2048 FFT points and interpolation](image)

B. Using different functions for windowing

It is well-known that unwindowed FFT or classical FFT results in leakage issues in estimation of FFT and PSD. Multi-taper method (MTM) [14] was used in [11] for effective estimation of PSD in the context of ambient frequency domain analysis. Two different windowing functions are compared in this section for truncating the signals and the results are shown.

First approach is to use the MTM technique with averaging from two Slepian functions [14]. While the Slepian functions in theory have important signal properties [14], they are defined implicitly which makes their computation tedious for multiple time window lengths of ringdown analysis. Accordingly, we have approximated the first two Slepian functions of [14] with the sinusoidal functions (16) which serve the smoothing objectives of MTM effectively.

\[
W_1[n] = 0.059 + \sin\left(\frac{0.05n}{N\pi}\right),
\]

\[
W_2[n] = 0.059 + \sin\left(\frac{0.05n}{2.5N\pi + \pi/4}\right),
\]

\[0 \leq n \leq L - 1\]  \hspace{1cm} (16)

where L is the number of actual data points in a window and N is the number of points for zero padded signals. Fig. 5(a) shows the resulting CMIF for the same 179-bus test case as in Figs. 2 and 4 with the use of approximated Slepian functions in MTM method. For the sake of comparison, in Fig. 5, all the CMIF functions are scaled so that they have the same peak as for the rectangular window.

The second approach is the use of well-known Hamming window instead of a simple rectangular window. Hamming window is known to have the effect of reducing the height of side lobes [12]. So, it may help reduce the energy threshold needed for avoiding spurious peaks.

\[
W[n] = 0.54 - 0.46\cos\left(\frac{2\pi n}{L-1}\right), \hspace{1cm} 0 \leq n \leq L - 1 \hspace{1cm} (17)
\]

Fig. 5(b) shows CMIF with Hamming window. While it has the desired effect on the dominant mode at 0.28 Hz mode, the Hamming windowed FFT also affects the CMIF shapes around second and third dominant modes at 0.66 Hz and 0.74 Hz respectively, making them less distinct from each other.

![5(a) CMIF with MTM](image)

![5(b) CMIF with Hamming window based FFT](image)
C. Weighted averaging for mode frequency estimation

Even with the use of interpolation and MTM/Hamming windowing techniques, the estimation for the value of peak frequency has error when using just the CMIF. In addition, as mentioned before, by ignoring PSD of each channel, there is always the possibility of missing local modes which may not have high peak values in singular value decomposition (6).

In the new method, first, the dominant mode is estimated with the use of the peak value of CMIF. Then, for each channel, frequency of the closest peak to this estimation is chosen. The final estimation for the dominant mode is a weighted average of peak frequencies from local PSD estimates. The weighting coefficients are the energy estimates of the mode from each channel in the spirit of Distributed Frequency Domain Optimization (DFDO) algorithm from [15]. An illustration of the CMIF for individual channels is presented in Fig. 6 for the same test case as in Fig. 5 for easy comparison of mode peaks. In the improved method, every channel is contributing in the estimation of the dominant mode based on the dominancy of this particular mode in the channel.

D. Self-correction for estimating mode frequencies

It can be seen from Figs. 2 and 5 that accurate estimation of the second and third peak frequencies in the CMIF is a challenging task and can introduce errors in estimation results. Fundamentally, any FFT based analysis is bound to have such errors owing to the short time windows used in ringdown analysis. Therefore, a more reasonable approach is to seek improvement in the estimation by some kind of a time-domain formulation. The correction strategy is proposed as follows: for each mode, there is inherent uncertainty in the location of the peak frequency. Let us suppose that a better frequency estimate may be slightly off from the estimated frequency of METRA. By considering a fixed number of frequency values near the METRA estimated frequency, the damping ratio and mode shapes are estimated at each of these frequency values in parallel. PMU responses are then compared with their respective reconstructed signals based on the modal estimates. The algorithm will then choose that specific frequency estimate which provides the lowest norm of the error between the estimated and actual signals. After finding the suitable frequency for each mode using this process, a full estimation is carried out at the end using the chosen frequency values for each of the modes and the damping ratios and mode shapes are estimated for the final estimation. It is important to point out that all the calculations in this process are mutually decoupled so that the computation is friendly towards parallel implementation.

E. Calibrating the mode shape magnitude

The last part of the improved algorithm is a self-calibration strategy introduced for the mode shape magnitudes. Based on experiments, it has been seen that mode shape magnitude estimations are prone to errors from the leakage issues associated with short time window lengths. To correct this error, the mode shape magnitudes are adjusted (scaled) by time-domain comparison of reconstructed and actual signals. The reconstructed signal for each channel is formed based on the estimated values for modes and mode shapes. Then, for each channel, the algorithm searches for the biggest peaks in the actual data and the reconstructed signal and scales all the mode shapes in that particular channel with the ratio of the two peak values.

F. Improved METRA (iMETRA algorithm)

Improved METRA is then derived from METRA by implementing all the improvements discussed above in Sections 4.A through 4.E. iMETRA algorithm is illustrated for mode estimation in the 179-bus system test case in Fig. 1 and Table I. The last row of Table I states the estimated values of frequency and damping for the three modes. Table I shows that iMETRA estimates match better with ERA, HTLS, and MP modal engines compared to METRA results in Table I. The strength of iMETRA versus METRA is nicely captured in the time-domain signal comparison. Fig. 7 shows the reconstructed signals for all four channels using estimated values of modes and mode shapes from both METRA and iMETRA algorithms along with actual signals from the test system. All the self-correction features of iMETRA have been implemented in the form of multi-threaded tasks so that the computational time for iMETRA is still comparable to METRA in Table I. To quantify the estimation error, let us define a measure Total Error (TE) to be the sum of square of the error between reconstructed and actual signals across all time instants and all signals. TE values are 11.77, 31.77, and 18.03 respectively for HTLS, METRA and iMETRA for this case study.
5. Test Cases

Two test cases are analysed in this section to verify the improvements in modal estimations from the proposed algorithm over METRA. First case is another event simulated in the 179-bus western system model, and the second case consists of archived PMU data from a recent western power system event.

A. 179-bus system simulation event

For this case, another event was simulated in the 179-bus test system using TSAT. In this event, a disturbance caused a Northern part of the system to separate so that the resulting system had poorly damped oscillations. Fig. 8 shows the oscillations in bus voltages during this event. The proposed algorithm was applied to detect the modes of the system using an analysis window of length \( T = 20 \) seconds, highlighted in Fig. 8 by the dashed box. Fig. 9 shows the PSD plots for all four channels produced by the improved METRA algorithm. As it can be seen in the plots, the system has a dominant mode between 0.3 Hz and 0.4 Hz and there is also a local mode between 0.65 Hz and 0.75 Hz detected in PMU 1.

The estimates for the two modes are presented in Table II for the proposed algorithm and all other five modal analysis methods. It can be seen in Table II that, comparing to METRA, in iMETRA, the estimates for the frequency and damping for 0.34 Hz mode are more consistent with other modal engines. The second column of Table II shows the computational time needed for each algorithm. It can be seen that, although the new improvements have led to a larger computational time compared to METRA, iMETRA in serial implementation is still faster than time domain algorithms. Fig. 12 shows the reconstructed signals for both METRA and iMETRA along with the actual signal in three channels. It can be seen that the reconstructed signals for iMETRA well match the actual signal. PMU 4 is located in the islanded portion of the system and is not shown in Fig. 10. Unlike the other three channels, bus voltage 4 does not show the oscillations. The 2-norms of the error between reconstructed and actual signals are 3.28, 117.05 and 5.94 for HTLS, METRA and iMETRA respectively.

The PSD CMIF plots for different channels in Fig. 9 shows that the 0.7 Hz mode is seen in only channel 1, and this mode has much lower energy compared to the 0.35 Hz mode. Because of the lower energy associated with the 0.7 Hz mode, the mode is not well observable in the signals which in turn leads to variation in damping estimates for the mode among different algorithms in Table II. The estimation results from iMETRA are still quite good as can be seen from the plots of reconstructed signals in Fig. 10.

![Fig. 7. Comparison of actual signals with reconstructed signals from METRA and from iMETRA.](image-url)
B. Western system event

Recorded PMU data from a recent western American power system is analysed. Fig. 11 shows the ringdown responses of bus voltage phase angles from 20 PMUs during the event. Fig. 12 shows the PSD CMIF plots for all 20 channels using a window of length $T = 13$ seconds. As it can be seen in the plots, the system has two modes between 0.2 Hz and 0.4 Hz.

The estimates for the two modes are presented in Table III for the proposed algorithm among other modal analysis methods. Table III shows excellent consistency between the estimates produced by iMETRA and estimations from HTLS, Matrix Pencil and ERA. The second column of Table III shows the computational time needed for each algorithm. Fig. 13 shows the reconstructed signals for METRA and iMETRA along with the actual signals for a random choice of signals say from channels 10 and 12. Comparison plots for other channels are not shown to save space. Fig. 13 shows that the reconstructed signals match very well with recorded PMU signals for the modal estimates provided by iMETRA. The error 2-norm measures between reconstructed and actual signals are 53.62, 279.1, and 111.26 for HTLS, METRA and iMETRA respectively.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Time (sec)</th>
<th>Mode 1 $f$ (Hz)</th>
<th>$\xi$ (%)</th>
<th>Mode 2 $f$ (Hz)</th>
<th>$\xi$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prony</td>
<td>0.6274</td>
<td>0.233</td>
<td>15.22</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>MP</td>
<td>0.1109</td>
<td>0.236</td>
<td>10.29</td>
<td>0.388</td>
<td>9.77</td>
</tr>
<tr>
<td>HTLS</td>
<td>0.1094</td>
<td>0.236</td>
<td>10.34</td>
<td>0.388</td>
<td>9.77</td>
</tr>
<tr>
<td>ERA</td>
<td>0.1163</td>
<td>0.236</td>
<td>10.29</td>
<td>0.388</td>
<td>9.77</td>
</tr>
<tr>
<td>METRA</td>
<td>0.0584</td>
<td>0.220</td>
<td>7.86</td>
<td>0.381</td>
<td>9.89</td>
</tr>
<tr>
<td>iMETRA</td>
<td>0.0725</td>
<td>0.234</td>
<td>12.02</td>
<td>0.380</td>
<td>10.04</td>
</tr>
</tbody>
</table>
6. Conclusion

The paper presents an improved methodology for frequency domain ringdown analysis by using a set of self-correction and self-calibration procedures. The estimation results are shown to result in excellent match between actual signals and reconstructed signals. Future formulations which formalize the self-correction methodology within ringdown modal analysis should be developed. Recent paper [16] has proposed formulating ringdown analysis into a nonlinear optimization problem which can be combined with model based estimation framework of this paper towards developing powerful hybrid formulations of the future.

7. Acknowledgements

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8. References