**Optimization of Customer Subscription Rates to Electric Utility Tariffs**

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**Abstract**

In 2006 a test of residential electricity pricing using multiple tariffs was performed using a mixture of fixed-prices, time-of-use prices and 5-minute real-time prices. Each tariff has advantages and disadvantages for both consumers and utilities. This paper examines a numerical optimization method to efficiently identify the optimal combination of customers to enroll under the available tariffs such that a utility can minimize the cost associated with both serving energy and the cost associated with the uncertainty of peak loads.

1. **Nomenclature**

   - \( a, b \) The exponential weighting shape values for the energy use data correction function.
   - \( c(t) \) The combined fixed, variable and uncertainty cost at the time \( t \) (in $).
   - \( c_F \) The fixed cost of power (in $).
   - \( c_P \) The variable cost of power (in $/kW).
   - \( c_U \) The uncertainty cost of power (in $/kW²).
   - \( C(\Omega) \) The cost of the subscription portfolio \( \Omega \) (in units of $).
   - \( C_P(\Omega, t) \) The variable cost of delivering energy at the time \( t \) given the subscription portfolio \( \Omega \) (in units of $/kWh).
   - \( C_U(\Omega, t) \) The uncertainty cost of delivery energy to \( N \) customers given the subscription portfolio \( \Omega \) (in units of $/kW²h).
   - \( \langle x \rangle \) The computed mean value of \( x \) (in units of \( x \)).
   - \( F(t) \) The load forecast at the time \( t \) (in units of kW).
   - \( G(t) \) The generation forecast at the time \( t \) (in units of kW).
   - \( \lambda^* \) The Lagrange multipliers for the equality constraint in the optimization problem.
   - \( L(t) \) The load at the time \( t \) (in units of kW).
   - \( \mu_x \) The observed mean energy consumption per unit time of the group \( x \) (in kW).
   - \( M(t) \) The energy cost vector for subscription portfolio \( \Omega \).
   - \( N \) The number of customers.
   - \( P(t) \) The power demand at the time \( t \) (in kW).
   - \( \hat{P}(t) \) The predicted power demand for the time \( t \) (in kW).
   - \( \rho_{xy} \) The covariance coefficient for subscriptions weights \( x \) and \( y \) (pu).
   - \( \sigma_x^2 \) The observed variance of energy consumption per unit time of the group \( x \) (in kW²).
   - \( \omega_x \) The fractional subscription rate of the tariff \( x \) (per unit \( N \)).
   - \( V \) The uncertainty cost covariance matrix.
   - \( \langle x^2 \rangle \) The computed variance of the value \( x \) (in units of \( x^2 \)).

2. **Introduction**

   The concept of efficient frontiers was pioneered by Harry Markowitz in 1952 as part of the development of the Capital Asset Pricing Model (CAPM) for portfolio theory [5]. His mathematical treatment of stocks and its contribution to the field of finance earned Markowitz the Nobel Prize in Economics in 1990. The application of Markowitz’s method to electricity pricing is depicted in Figure 1, which illustrates in the shaded area all the possible combinations of a pair of customer tariff subscription rates making up a utility tariff design. The vertical axis plots the energy cost of the customer portfolio and the horizontal axis plots the cost of the uncertainty associated with those customers’ loads. Any region outside the shaded area is unrealizable because either the energy costs or the load uncertainty is less than the minimum possible for the composition of customers. In addition, all combinations of customers in the shaded region are suboptimal because for any given load uncertainty and energy cost in this region, there is a combination of customer tariff subscription rates with either lower cost or lower uncertainty or both. Thus, points on the lower boundary of the shaded region are the optimal mixes a utility should seek.

   This “efficient” frontier can be used to examine the utilities...
obligation to deliver energy to retail customers. These utilities must determine their supply portfolio from a mixture of long term, short term and spot transactions, each with varying costs and uncertainties. This problem was addressed by Woo et al. [6] but the solution ignores the impact of demand response. At each moment in time, a utility must not only consider the uncertainty in the supply cost but also the uncertainty of demand itself, which may differ according to the tariff which customers choose. These costs depend on a fixed cost $c_F$, a variable cost $c_P$ for serving power $P$, and a cost $c_U$ associated with the variability in the power demand

$$c(t) = c_F + c_P P(t) + c_U [P(t) - \hat{P}(t)]^2,$$

where $c(t)$ is the total cost per unit time at time $t$, $P(t)$ is the measured power use at time $t$, and $\hat{P}(t)$ is the predicted power demand for time $t$. The relationship of these costs forms the energy-uncertainty cost curve shown in figure 1. For any given energy/uncertainty curve, there is only one point on the efficient frontier that will intersect a curve that is tangent to it. This point is the optimal mix of customer tariff subscriptions rates given the cost conditions expressed in equation (1).

This interpretation of utility costs, as it relates to customer tariff subscriptions, can be applied to the problem of optimizing how many customers should ideally subscribe to a given electricity rate to achieve any given business objective. These subscription rates can then be used to determine incentives to offer customers who may be considering changing to new rates. This problem is posed as an optimization problem similar to that posed by Markowitz in the sense that a utility should maintain and continually adjust a portfolio of electricity rates with different numbers of customers subscribing to those rates. If the rates are designed with significantly different time-varying characteristics, such as fixed price, peak-time prices, and real-time prices, then depending on the number of customers subscribed to each rate and the cost of energy and uncertainty to serve those customers, the utility will incur varying total costs [3].

In 2006, a test of a multiple-rate utility operation with very diverse time-based electricity rates was conducted on the Olympic Peninsula of Washington State [4]. This test included the following three time-based rate structures:

1) Fixed pricing where the price of electricity remained constant over time;
2) Time-of-use (TOU) pricing where the price of electricity switched between two values, a high price for peak periods and low price for off-peak periods; and
3) Real-time pricing (RTP) where the price of electricity was determined by a 5-minute real-time double auction for the feeder’s capacity.

The primary purpose of the project was to design and demonstrate a true real-time retail market design, the utility control system that operates the market, and the distributed generation and building control systems that participated in the market. However, without pre-existing data or simulations available, the market designers were unable to recommend how many customers the utility should be solicite for each time-based electricity rate. Without a method to optimize a solution for the project’s objective to minimize peak power, the market designers determined to conduct the experiment with approximately 1/4 of the customers on each plan and 1/4 in a control group.

The problem of electric utility rate subscription portfolio design remained unsolved until an exhaustive search method was later implemented and tested on the Olympic Peninsula telemetry data [2]. It was determined that the 1/3 split (excluding the control group) was indeed very nearly optimal for minimizing the uncertainty of the peak load, as shown in Figure 2.

![Fig. 2. Olympic Peninsula project rate portfolio analysis for peak load uncertainty minimization [3] (Link to image)](image-url)
favor of an optimization strategy that would be perceived as more revenue neutral.

The exhaustive search method worked because of the small number of rates being considered. But for a non-trivial portfolio of rates, this method becomes quickly intractable because of the high dimensionality of the search space. Fortunately, a rigorous mathematical treatment of such a problem is possible based on how energy consumption and load uncertainty are correlated and combine as random variables.

Suppose we have a group of customers on rate $A$ whose behavior is a function of both the rate itself and other factors independent of the rate at which they pay for electricity. The group’s energy consumption per time interval is a random variable with mean $\mu_A$ and variance $\sigma_A^2$. Similarly, group $B$’s consumption has mean $\mu_B$ and variance $\sigma_B^2$. Because the behaviors of the two groups are partially correlated, the mean and variance of the combined consumption are

$$\mu_{AB} = \omega_A \mu_A + \omega_B \mu_B$$

and

$$\sigma_{AB}^2 = \omega_A^2 \sigma_A^2 + \omega_B^2 \sigma_B^2 + 2\omega_A \omega_B \rho_{AB} \sigma_A \sigma_B,$$  

where $\omega_A$ and $\omega_B$ are the weighting factors (which must sum to unity) that describe how many customers from group $A$ and $B$ are contributing to the overall combination, and $\rho_{AB}$ is the correlation coefficient of each group’s consumption with respect to the other. The objective of a utility’s customer subscription optimization program is to determine the combination of $\omega_A$ and $\omega_B$ that minimizes the cost of serving those customers.

Suppose that $P(t)$ is the expected value of $P(t)$ at time $t$, and there are $N$ groups of customers. From equation (1), the expected value of the cost function $c(t)$ is

$$\langle c(t) \rangle = c_F + c_P \langle P(t) \rangle + c_U \langle P(t)^2 \rangle,$$  

where $P(t) = \sum_{r=1}^{N} \omega_r P_r(t)$, $\langle x \rangle$ is the expected value of $x$ and $\langle x^2 \rangle$ is the variance of $x$. Because the time step is fixed, the energy consumption at time $t$ is a scalar multiple of $P(t)$. The cost $C_P$ of delivering power at the time $t$ to $N$ groups of customers subscribing to $N$ rates is then

$$C_P(\Omega, t) = c_P \langle P(t) \rangle \Delta t = M^T(\Omega) \Omega,$$  

where $M(t) = c_P [\mu_1(t) \mu_2(t) \cdots \mu_N(t)]^T$ is the energy cost vector with $\mu_r(t) = \langle P_r(t) \rangle \Delta t$, and $\Omega = [\omega_1 \omega_2 \cdots \omega_N]^T$ is the group subscription weighting vector with $\omega_r = n_r/n$, where $n_r$ is the number of customers subscribing the rate $r$ and $n$ is the total number of customers.

The cost of uncertainty $C_U$ for $N$ groups of customers subscribing to $N$ rates can be computed as

$$C_U(\Omega, t) = c_U \langle (P(t) \Delta t)^2 \rangle = \Omega^T V(t) \Omega,$$  

where $V(t)$ is the uncertainty cost covariance matrix at the time $t$, i.e.,

$$V(t) = c_U \begin{bmatrix}
\sigma_1^2(t) & \cdots & \rho_{1N}(t) \sigma_1(t) \sigma_N(t) \\
\vdots & \ddots & \vdots \\
\rho_{1N}(t) \sigma_1(t) \sigma_N(t) & \cdots & \sigma_N^2(t)
\end{bmatrix}.$$  

a symmetric positive semi-definite matrix.

It is the utility’s primary business objective to design rates that minimize the sum of these two costs given a particular customer portfolio $\Omega$, such that

$$C(\Omega) = \Omega^T V \Omega + M^T \Omega + constant$$  

where $t$ is omitted for notational convenience. This is initially accomplished by designing tariffs for every group of customers that support all anticipated long-term costs including both energy and uncertainty. The tariffs are submitted to the regulatory bodies that must approve it. Long lead times, lengthy reviews and approval delays lead to infrequent changes to the tariffs, which prevents utilities from optimizing their financial performance in the short-run. But short-term cost reductions can also be achieved by giving incentives for customers to change which rate they subscribe to. A more cost-effective mix of customer subscriptions $\Omega$ could be identified and an incentive provided to move the utility toward that new lower cost mix. For a multiple time interval problem, this objective is

$$\minimize f(\Omega) = \sum_{t=0}^{T} C(\Omega, t)$$  

subject to

$$\sum_{t=1}^{N} \omega_r = 1$$

where $T$ is the number of time intervals over which the cost is to be minimized. The problem is readily solved for any single time interval as

$$\Omega^* = (2V)^{-1}[A^T \lambda^* - M]$$  

where $A = 1_{(N \times 1)}$ and $\lambda^*$ is the Lagrange multiplier from the equality constraint, i.e.,

$$\lambda^* = [A(2V)^{-1}A^T]^{-1}[A(2V)^{-1}M + 1].$$

Utilities will find the solution in equation (9) is useful for single time interval studies such as peak hour studies where the values of $M$ and $V$ are computed for all the peak hours over a single month. However, for multiple time intervals the problem must be solved in the form presented in equation (8) using appropriate standard numerical methods [1].

In equation (8), it is essential to verify that the matrix $V$ is positive definite so that the solution is truly a global minimizer. In the case of the Olympic Peninsula project, the covariance matrix $V$ is positive definite and the value $\Omega^*$ is a global minimizer given the constraints. However, if some of the loads are anti-correlated in some significant way, or the variances of the loads are relatively small with respect to the correlations between the loads, the covariance matrix $V$ may only be positive semi-definite. In addition, constraints on the subscription rates, such that they remain in the interval $[0, 1]$, can be added to ensure that the solutions are feasible.

3. Test Data and Results

The high-resolution data used to solve this problem comes from utility metering systems such as the new smart meters now being widely deployed. However, the data must be processed before it can be used in equation (8) or equation (9). For example, the concurrent raw metering data for the Olympic Peninsula covers 11 months and is shown in Figure 3.
There is very strong correlation in the seasonal load shapes and the high diurnal variability present in all the customer groups, as well as the subtle differences between them attributable to both the natural randomness of customers and the differences associated with the rates to which they subscribe. The actual customer subscription rates did change slightly during the course of the project, but the overall fractions of customer subscribing to each rate remained relatively steady and close the desired 1/4 split during the course of the project, as shown in Figure 4.

The optimization method of equation (9) was tested on the Olympic Peninsula customer metering data using the energy cost \( C_P = 0.022 \) $/kWh and the uncertainty cost \( C_U = 0.371 \) $/kW$^2$h. The means of the observed energy costs per unit time for the FIXED, TOU and RTP tariff groups are

\[
M = \begin{bmatrix} 0.31 & 0.24 & 0.30 \end{bmatrix}^T,
\]

respectively. The uncertainty costs covariance matrix the three groups (in the same tariff order) is

\[
V = \begin{bmatrix} 1.63 & 0.24 & 0.11 \\ 0.24 & 1.56 & 0.07 \\ 0.11 & 0.07 & 1.36 \end{bmatrix}.
\]

and with eigenvalues \([1.32, 1.36, 1.87]\), \(V\) is positive-definite. Equation (9) was applied to the FIXED, TOU and RTP and a cost surface for TOU vs RTP subscription rates is given in Figure 5, which shows that the optimal subscription rate is yielded.

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4. Discussion

A flaw in the control system for auxiliary home heating under real-time price was discovered during the analysis of the Olympic Peninsula Demonstration Project results [4]. The defective thermostats turned on auxiliary resistance heating (with \(COP \approx 1.0\)) instead of heat pump (with \(COP \approx 3.0\)) when prices decreased enough to raise the setpoint to more than two degrees above the current temperature. This defect did not affect FIXED and TOU customers but increased RTP group energy usage significantly more than expected when price volatility was high and therefore decreased the optimal number of RTP customers in this result. The net effect of the defect is a significant shift in the optimal number of customers away from the RTP tariff to the FIXED tariff and the TOU tariff, which provided good on peak demand response and did not use more energy than expected.

To confirm the hypothesis about the impact of this thermostat defect, the energy use data collected from the RTP group...
was rescaled after the experiment was completed. We used an exponential weighting scheme (specifically, \(1 - \frac{2a}{1 + e^{a(b - 1)}}\) with \(a = 2/3\) and \(b = 5\), where \(a\) and \(b\) are shape parameters) based on magnitude. The larger magnitude values were scaled down more than the smaller magnitude values to account for the fact that heat is roughly 1/2 of the total load on peak and that it was roughly 3 times greater than it should be when significant pre-heating and post-curtailment recovery occurred.

The corrected value of the RTP energy peak usage was 2/3 of the original value. This correction procedure resulted in a shift of the optimal customer subscriptions to values much more in accordance with expected values, as shown in Table 1.

As a result of this post-processing correction, a number of important observations are made. First, the optimal subscription rate to both FIXED and RTP tariffs are reduced while the overall subscription rate to RTP is increased. Second, the overall cost of serving these customers is decreased by 7%.

While some of the decrease is attributable to the decreased energy consumption of the RTP group itself, about 10% of the overall subscription rate to RTP is increased. Second, the important observations are made. First, the optimal subscription rate to both FIXED and RTP tariffs are reduced while the overall subscription rate to RTP is increased. Second, the overall cost of serving these customers is decreased by 7%.

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4.1. Other Objectives

Not all useful objective functions can be expected to offer closed-form solutions like equation (9). The objective function used will depend on the utility’s choice of business objective, as shown by the examples in Table 2. These were not examined in detail in this paper and will be the subject of future papers.

4.2. Load uncertainty

Load uncertainty is the difference between the forecasted load and the actual load. Utilities must schedule their load in advance so that the system operator can schedule generating units to be dispatched efficiently and securely. Any discrepancy between the load forecast and actual load requires generation resources to be added or withdrawn after they are scheduled and increases the cost of electricity for that period.

This premium energy is typically charged to the utilities contributing to the imbalance based on their forecast error. The price is based on an imbalance market and is usually higher than the price of load scheduled in advance. Thus, the utility’s objective is to minimize this error and thus minimize their costs.

This insight suggests that an N-rate design problem to minimize forecast error can be treated as a least-squares optimization problem, which is readily treatable using the Gauss-Newton method [1]. Such an optimization process requires knowledge of both the load forecast and actual load requires generation resources to be added or withdrawn after they are scheduled and increases the cost of electricity for that period.

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5. Conclusions

An easy-to-use solution has been found to an emerging smart-grid cost reduction opportunity for utilities wishing to take advantage of smart-meter technology and new tariffs made possible by recent innovation in smart-grid technology, such as those demonstrated in the Olympic Peninsula project. Utilities can use this method to determine the optimal mix of customer tariff subscriptions for a variety of business objectives using high-resolution metering data collected from smart meters. In the case of the cost-minimization objective, we have shown that the problem is convex when considering both energy cost and the cost of serving an uncertain load peak. Thus, the problem is solved very efficiently in closed form and amenable to very large utility operations without significant computational burden.

Future work includes developing numerical treatments for feasibility constraints and non-convex cost-minimization situations as well as other more complex utility business objectives that are either not convex or have more complex constraints.

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7. References