A Risk-averse Optimization Model for Unit Commitment Problems

Gabriela Martínez*, Lindsay Anderson*
*Department of Biological and Environmental Engineering
Cornell University, Ithaca, USA
Emails: {mgm256,cla28}@cornell.edu

Abstract—In this paper, we consider the unit commitment problem of a power system with high penetration of renewable energy. The optimal day-ahead scheduling of the system is formulated as a risk-averse stochastic optimization model in which the load balance of the system is satisfied with a high prescribed probability level. In order to handle the ambiguous joint probability distribution of the renewable generation, the feasible set of the optimization problem is approximated by a quantile-based uncertainty set. Results highlight the importance of large sample size in providing reliable solutions to the SCUC problems. The method is flexible in allowing a range of risk into the problem from higher-risk to robust solutions. The results of these comparisons show that the higher cost of robust methods may not be necessary or efficient. Numerical results on a test network show that the approach provides significant scalability for the stochastic problem, allowing the use of very large sample sets to represent uncertainty in a comprehensive way. This provides significant promise for scaling to larger networks because the separation between the stochastic and the mixed-integer problem avoids multiplicative scaling of the dimension that is prevalent in traditional two-stage stochastic programming methods.

Index Terms—renewable energy, chance constraints, order statistics, proximal bundle methods.

I. INTRODUCTION

An electrical power system is a highly complex system composed of many components whose complex interactions are not effectively computable. Coordinating its operation in a cost-effective, efficient, and reliable manner is a challenging problem, from both economical and mathematical points of view. The secure and reliable operation of an electrical power system is fundamental to the economy and quality of life because electric power systems are an integral part of modern society. As the power generation mix of the system shifts to include more renewable energy resources there is increased uncertainty in the generation output of the system, which has a significant impact on the operation of system. In particular, this variable and uncertain generation output creates the need for Independent System Operators and related entities to address this source of uncertainty in the day-ahead unit commitment (UC) problem, the main topic of this paper.

Other sources of uncertainty in the day-ahead UC problem, for example, the fluctuation of demand load throughout the day and unexpected generator outages, have been managed by setting system reserve margins to keep a secure and reliable operation of the power system [1]–[5]. However, the integration of intermittent sources of power, such as solar and wind power, will randomly change the generation levels of the system and impact the need for system reserves both in quantity and location, consequently increasing the operating cost of the system [6]–[8]. Therefore, to accommodate these intermittent sources of power, UC problems will benefit from an appropriate stochastic optimization approach to handle uncertainty. The stochastic optimization approach [9]–[11] takes into consideration the uncertainty of the inputs of an optimization model. In this approach, the random parameters are characterized by weighted scenarios representing the space of possible outcomes of the random variables.

The complex nature of the stochastic UC problem has attracted researchers from different fields, and there is an extensive literature on UC and stochastic UC models [12]–[19]. The numerical complexity of this large-scale mixed-integer optimization problem is well known. Typically, the stochastic unit commitment problem is solved as a two-stage problem, wherein first stage commitment
decisions are made to minimize expected cost of the second stage (dispatch) decisions. The second stage of the problem represents uncertainty through a set of possible future scenarios [12], [15], [20], [21]. The primary challenge of the SCUC problem is the scaling complexity with larger systems. Essentially the second stage problem is replicated with each scenario, and large networks create large mixed-integer programs that are challenging to solve. Therefore, most stochastic unit commitment formulations cannot be solved in reasonable time for realistic system size and a sufficient number of scenarios. One solution to this problem is decomposition through use of a progressive hedging algorithm, as described in [14], though this does not address the limitation on the size of the scenario set. The work presented here investigates the use of a chance-constrained formulation that enables separation of the mixed-integer problem from the stochastic sub-problem. This approach allows comprehensive representation of the uncertainty set, without replicating the (already complex) mixed-integer problem.

As an alternative to the two scenario-based stochastic programs, the chance-constrained formulations have been receiving increased attention. This class of optimization problem, also known as probabilistically-constrained optimization, requires performance of the solution with high probability [22]. The advantage of the chance-constrained formulation is the flexibility of the solution: providing compromise between the risk of scenario-based methods based on minimizing expected cost, and the most expensive robust optimization methods that ignore the specific distribution of the uncertainty and provide a commitment schedule for the worst-case bounding scenario. In the chance-constrained formulation, the operator can tune the risk-level of the solution through manipulation of the probability level. Recently, chance-constrained optimization methods have been applied to power system operations and planning in [23]–[25]. Specifically, in [23], the optimal power flow problem is solved with a probabilistic constraint on power flows in the network, while [24] considered the unit commitment problem with a probabilistic constraint on the load balance in the second stage of the problem. Both of these solutions use the convexification proposed by [26], which is a special case of the “near robust” solution; the chance-constraint with probability close to one. Typically the solution of chance-constrained formulations require specific assumptions about the distribution of underlying uncertainty; for example, [25] places a probabilistic constraint on the system load balance, and assumes that underlying load uncertainty is multivariate normal. In this work, we do not assume a specific distributional form of the uncertainty, but focus on a data-driven approximation model for a joint chance constrained optimization model applied to a UC problem, since the distribution of uncertainty associated with renewable generation is not known. The formulation proposed here also allows separation of the stochastic sub-problem from the mixed-integer commitment problem, thereby reducing the multiplicative scaling of model complexity. In addition, the convexification used in this work is a tighter relaxation than those used in [22]–[25].

In Section II we further describe the chance constrained UC model considered and also describe the quantile-based approximation of its feasible set. In Section III we describe the test system and numerical results, while discussion and conclusions are provided in Section IV.

**Notation.** Boldface letters represent random variables; $\mathbb{R}^n$ and $\mathbb{R}_+^n$ are the space of $n$ vectors and the non-negative orthant, respectively; $x \preceq y, x, y \in \mathbb{R}^n$ denotes the entry-wise inequality between two vectors; $\mathbb{P}(A)$ denotes the probability of an event $A$; and the ceiling function of a number $x$ is denoted by $\lceil x \rceil$.

II. UNIT COMMITMENT FORMULATION

The UC problem is a primary operational function of power systems and markets, wherein the system operator aims to determine the most effective use of generation resources. Specifically defining when each generating unit in the system is to be switched on or off, and how much power each unit should produce in order to meet the power load and reserve requirements at minimal operating cost over a specific time horizon. In this paper, we consider an chance-constrained approach to the UC problem that addresses the computational tractability of the problem by avoids the non-linear dimensional scaling of existing stochastic programming approaches to unit commitment.

The chance-constrained formulation of the UC problem is a minimization problem that determines the optimal commitment schedule and dispatch levels to guarantee an acceptable level of reliability risk, and is given in equation 1.

The UC problem for a hybrid power system composed of $N$ thermal generating units and $I$ renewable generating units, for example wind farms, and a scheduling horizon determined by $T$ periods. Let $\mathcal{N} = \{1, \ldots, N\}$, $\mathcal{I} = \{1, \ldots, I\}$, and $T = \{1, \ldots, T\}$ denote the index set of the thermal generating units, renewable generating
units, and time scheduling periods, respectively. The set of nodes of the transmission grid is denoted by \( \mathcal{K} := \{1, \ldots, K\} \). \( N_k \) corresponds to the indices of the generators which are located at node \( k \in \mathcal{K} \), \( \mathcal{I}_k \) collects the indices of renewable generators which are located at node \( k \), and \( \mathcal{B} \) is the set of lines connecting nodes \( k \) and \( j \). The state of each thermal generating unit \( n \), at time period \( t \), is represented by the binary variable \( u^t_n \). Denote by \( p^t_g \) the power produced by the \( n \)-th thermal generating unit, \( p^t_r \) the power produced by the \( i \)-th renewable generating unit, at time period \( t \). Let \( p^t_{r_k} \) denote the aggregated renewable generation at time \( t \) at node \( k \), i.e., \( p^t_{r_k} := \sum_{i \in \mathcal{I}_k} p^t_i \). \( L^T_k \) denotes the \( k \)-nodal demand of the system at time \( t \), and define \( p_g := [p^1_{g_1}, \ldots, p^T_{g_N}]^\top \), \( u^t_g := [u^t_{g_1}, \ldots, u^t_{g_N}] \), \( p_r := [p^1_r, \ldots, p^T_r]^\top \), \( L := [L^1, \ldots, L^T] \). The mathematical formulation of the chance-constrained UC problem is as follows:

\[
\begin{align}
\min_{p_r, u_g} & \quad \sum_{n \in \mathcal{N}} \sum_{t \in \mathcal{T}} c_n(p^t_g, u^t_g) + S^t_n(u^t_g, u^{t-1}_g) \\
\text{s.t.} & \quad (p^t_g, u^t_g) \in \mathcal{E}^\text{gen} \cap \mathcal{E}^\text{stat}, \\
& \quad \mathbb{P}(\sum_{n \in \mathcal{N}_k} p^t_{g_n} + p^t_r - \sum_{i \in \mathcal{B}} B^t_{ij}(\delta^t_k - \delta^t_i) \geq L^t_k, k \in \mathcal{K}, t \in \mathcal{T}) \geq \pi, \\
& \quad |B_{ij}(\delta^t_k - \delta^t_i)| \leq F_{kj}, k \in \mathcal{K} \end{align}
\]

where \( c_n(\cdot) \) denotes fuel cost, and \( S^t_n(\cdot) \) is the start-up costs, for the thermal generating unit \( n \) at time \( t \). The line connecting the nodes \( k \) and \( j \) is denoted as \( l_{kj} \in \mathcal{B} \), \( B^t_{kj} \) is the negative of the susceptance on line \( l_{kj} \), and \( \delta^t_k \) denotes the voltage angle at node \( k \). The set \( \mathcal{E}^\text{gen} \) comprises of the dynamic operating rules for each thermal unit, which couples the variables of the same unit along time: minimum on and off times, start-up and shutdown times, ramping constraints. The set \( \mathcal{E}^\text{stat} \) gathers the static operating rules of the system such as reserve constraints and generation limit bounds of the units. We refer the reader to [3], [4], [8], [27] for a detailed description of these constraints.

The probabilistic constraint (1c) guarantees that the commitment schedule will serve system load with with a high probability (\( \pi \)) and can be an intuitive representation of the problem faced by the ISO: demand should be met with very high probability, though the possibility of constraint violation exists in extreme scenarios.

The formulation presented in equation 1 requires a decomposition method to provide computational tractability. Our solution approach combines a data-driven relaxation of (1c), Lagrangian relaxation to decompose the problem (1), and a proximal bundle method, [28], [29], is used for solving the dual. In what follows, we will describe the quantile-based uncertainty set used to relax (1) and the decomposition solution approach utilized.

A. Chance Constraint Relaxation

Among the difficulties that arise when solving chance-constrained optimization problems are the non-convex structure of the feasible region, and verifying the feasibility of a given candidate solution is extremely difficult. To handle these issues, convex relaxations of chance-constrained problems have been proposed in the literature. In [30] a convex relaxation is developed in order to obtain a deterministic lower bound of the optimal value. In [31] integrated chance constraints and their relation with traditional chance constraints are analyzed, in [26] deterministic convex relaxations, known as “Berstein approximations”, are constructed. Scenario approximations are proposed in [32], [33]. As in these related works, we will exploit the particular structure of (1c) to construct a numerically tractable relaxation of the feasible set described by (1c).

To simplify the exposition and introduce the relaxation procedure proposed in this work, we will aggregate the balance constraint, i.e., we will ignore the DC network constraints in this section. The quantile-based set described below can be easily adapted to incorporate the DC network constraints, and also the dual decomposition scheme is analogous when DC network constraints are considered, to be illustrated in the numerical results presented in SectionIII.

In order to introduce the relaxation procedure used, we will first define the \( \pi \)-level set of the time-aggregated joint probability function:

\[
\mathcal{W}_\pi = \{w \in \mathbb{R}^T : \mathbb{P}(L - p_r \leq w) \geq \pi\},
\]

then, (1c) can be equivalently formulated as follows:

\[
\sum_{n \in \mathcal{N}} p^t_{g_n} = w^t, t \in \mathcal{T},
\]

\[
w \in \mathcal{W}_\pi.
\]

Therefore, problem (1) can be reformulated as shown
Here, the set $\mathcal{W}_\tau$ describes the feasible set of the chance constraint (3). Interested readers are referred to [10] for a comprehensive treatment of chance-constrained problems.

**Relaxation Procedure.** The main ideas of our relaxation approach are to construct vectors $v \in \mathbb{R}^T$ whose coordinates $v^t$ represent a quantile, $\rho_t \geq \pi$, of the $t$-marginal distributions $\mathbf{L}^t - \mathbf{p}^t$, and to distribute these risk-levels $\rho_t$ throughout the scheduling horizon such that $v \in \mathcal{W}_\tau$, i.e., $\mathbb{P}(\mathbf{L} - \mathbf{p} \preceq v) \geq \pi$. For our application problem (4), a closed form of $\mathbf{l}$-marginal distribution of $\mathbf{L}^t - \mathbf{p}^t$ is not known. Therefore, we will utilize historical data or generated realizations to estimate the extreme values of the $l$-marginal distributions.

In what follows, we will drop the index $t$ to simplify the notation and describe the quantile estimator used in our approach. Let $V := \mathbf{L} - \mathbf{p}$, and let $\rho \geq \pi$. A natural estimator for $\rho$-th quantile of $V$ is given by the $\rho$-th sample quantile, which will be denoted as $\hat{v}_\rho$. In more details, given an i.i.d equally-likely sample $V_1, \ldots, V_M$, and let $V_{[1]} \leq V_{[2]} \leq \cdots \leq V_{[M]}$ be the ordered sample, then $\hat{v}_\rho = V_{[M\rho]}$. The construction of this estimator is quite simple, the difficulty arises when we wish to construct confidence intervals. We will use the jackknife quantile estimator, see [34], to construct the vectors given below:

$$
\hat{v}_\rho = \left\{ \begin{array}{ll} 
\hat{v}_\rho^t & t \neq \tau, \\
\hat{v}_\pi^t & t = \tau,
\end{array} \right.
$$

where $\tau \in T$. The quantity $\hat{v}_\rho^t$ is the jackknife $\pi$-th estimator of the $t$-marginal sample $\mathbf{L}^t - \mathbf{p}^t$, the quantities $\hat{v}_\rho^t$ are jackknife estimators, of the $t$-marginal sample, representing a user-specified risk-level $\rho_t \geq \pi$. The selection of the risk-levels $\rho_t$ will reflect how the user would like to distribute the overall risk. For example, a risk-conservative user could select values $\rho_t = 1$. Let us remark that we kept one of the coordinates of the vectors $\hat{v}_\rho^t$ to represent a risk-level $\pi$ because we are interested to obtain vectors $\hat{v}_\rho^t$ which are likely to be in $\mathcal{W}_\pi$.

Therefore, our quantile-based relaxation of (4) for a sample of size $M$ is given by:

$$
\min p_{g_{\pi0}} \sum_{n \in \mathcal{N}} \sum_{t \in \mathcal{T}} c_n(p_{g_{\pi0}}^t, u_{g_{\pi0}}^t) + S_n(u_{g_{\pi0}}^t, u_{g_{\pi0}}^{t-1}) 
\text{s.t.} \mathbf{p}_{g_{\pi0}} \in \mathcal{E}_\text{gen} \cap \mathcal{E}_\text{stat},
\sum_{n \in \mathcal{N}} p_{g_{\pi0}}^t = w^t, t \in \mathcal{T},
$$

where $\mathcal{W}_\pi := \text{co}\{\hat{v}_\rho^t, \tau \in T\} + \mathbb{R}^T_+$, where co{$A$} stands for the convex hull of a set $A$.

With the relaxation of the chance constraint, a proximal bundle method is used to decompose (6), similar to the approach described in [28], [29]. This approach provides an $\epsilon$-optimal solution to the dual problem, and an augmented Lagrangian technique is used to recover the primal-feasible commitment and dispatch schedule from the solution of the dual problem. Note that this primal-feasible schedule can be recovered in various ways, but the numerical results that follow are based on the linearized augmented Lagrangian technique proposed in [3].

### III. Results

In this section we conduct numerical tests that illustrate the flexibility and performance of the proposed solution method through consideration of system cost, solution reliability level, and the sample size used to represent uncertainty. The algorithms were implemented in Python, the optimization problems were modeled with the Python-based modeling package Pyomo [35], and solved with the IBM Academic Initiative Cplex 12.5 optimizer. The computations were conducted on a Linux laptop equipped with Intel®Core i5 processor and 4 GB RAM.

Before presenting our results, let us remark how the quantile-based set is constructed when network constraints are considered. If DC power flow constraints are used, the chance constraint has the following form:

$$
\sum_{n \in \mathcal{N}_k} p_{g_{\pi0}}^t - \sum_{l \in \mathcal{B}} B_{l_{\pi0}}^t (\delta_l^t - \delta_{i}^t) = w_{k}^t, k \in \mathcal{K}, t \in \mathcal{T}, 
$$

where $\mathcal{W}_\pi = \{w \in \mathbb{R}^{TK} : \mathbb{P}(\mathbf{L}^t - \mathbf{p}^t \preceq w_{\pi}^t, k \in \mathcal{K}, t \in \mathcal{T}) \geq \pi\}$. Therefore, to construct the estimators (5), it is required to construct jackknife quantile estimators, see [34], [36] for a review of jackknife estimators, corresponding to each node $k \in \mathcal{K}$ and each time $t \in \mathcal{T}$. 


A. Relative Performance of Stochastic Sample Size

When planning system decisions under uncertainty, it is critical to consider the size of the stochastic sample used to represent the uncertainty. Stochastic programming models are capable of making decisions that are optimal only for the set of scenarios used in the model, therefore larger scenario sets will result in more effective decisions. Figure 1 illustrates the trade-off between solution reliability, cost to serve load, and the impact of the sample size used to represent uncertainty.

In Figure 1, the numerical results provide a comparison of the results of this data-driven approach on sample sizes of $10^3$ and $10^6$. Recall that the probabilistic constraint is imposed spatially and temporally, so that the x-axis shows the combined probability, to allow representation at lower dimension. For example, the blue line ($C_\pi$) begins with a load-balance constraint that must hold with probability equal to 0.8 for all nodes and time periods. Increasing (to the right) this probability on selected time periods, shows an increase in cost to serve load. Note that this is essentially a tuning parameter that the system operator could use to force higher reliability at more critical times in the day, or at more important nodes in the network. Conversely, the line $C_r$ represents the robust solution where the probability on the chance constraint is 1 everywhere. The lines $C_\rho$ and $C_{\pi,r}$ show risk-levels between these bounding levels.

As shown in Figure 1, the cost of the optimal solution increases with the reliability required. While it is intuitive that more reliability translates to higher cost, the costs do not uniformly follow this pattern in Figure 1a, which shows cases where the less reliable solution $C_\pi$ is less costly than the more reliable solution $C_\rho$. This anomaly results from the fact that in 1a, a sample of size $M=10^3$ is used, and Figure 1b is based on $10^6$ samples. This illustrates the danger of insufficient representation of uncertainty in stochastic models.

B. Robustness

It is also useful to compare the solutions obtained for the spectrum of robustness. The standard robust solution requires that the optimization solution is secure for every possible outcome in the scenario set. This is often achieved through the use of the most risky, and often very unlikely, scenario in the uncertainty set similar to the approach used in [37]. However, this can result in two primary problems: an excessively costly solution, and protection against only the scenarios in the uncertainty set. This puts significant confidence in the representation of uncertainty, and only providing very robust solutions when a large set of scenarios are used.

Next we compare the daily dispatch patterns that result from the chance-constrained solution for various levels of risk.

Figure 2 shows the hourly dispatch (aggregated across all nodes for presentation) for risk-levels ($\pi$ for all nodes and time periods) ranging from 0.8 to 1, showing that the dispatch of generation is much higher for the ‘nearly-robust’ solution ($w=0.999$), than the dispatch resulting from requiring load-balance constraint to be met in 99% of cases. Recalling the significant size of the scenario sets used for these results, the use of $w=0.99$ is a relatively risk-averse solution, with significantly lower cost.

Figure 3 illustrates the generation level obtained when the overall risk is distributed differently throughout the nodes and hours. For example, in figure 3a the generation
schedule \( PG_\pi \) represents the schedule obtained with a dispatch generated from estimators with probability level equal to 0.8 (0.9 in figure 3b) for all nodes and hours, \( PG_r \) is obtained with a robust dispatch representing risk-level 1 for all nodes and hours. Finally, \( PG_\rho \) and \( PG_{\rho,r} \) display schedules obtained with estimators between the bounding levels of \( PG_\pi \) and \( PG_r \).

It is also important to test the solution performance both in and out of sample. In Section III-C, we compare the reliability of the solutions obtained using in and out-of-sample scenarios.

C. In and Out of Sample Performance

In this section we will verify first if the solution obtained by the quantile-based approximation is feasible for (1) with respect to the sample utilized to generate the estimators. Table I collects the joint probability level of the solutions obtained, as illustrated in figure 3. As we can observe, the quantile-based solution performs well in sample.

To verify the performance of the solutions, the dispatch was tested with an outer sample of size \( 10^7 \). Table II displays the out-of-sample performance of the solution obtained. It is expected that solutions obtained with small-size samples perform poorly out-of-sample since large samples are required to estimate extreme risk values [34], [38]. We can observe that distributing the same level of risk for all nodes and hours, \( PG_\pi \), shows less reliable performance out of sample. Whereas, the solutions with distributed risk have a better performance.

Fig. 2: Dispatch levels, \( \pi = 0.8 \) to \( \pi = 1 \), sample size \( M = 10^6 \)

(a) Distributed risk from 0.8 to 1

(b) Distributed risk from 0.9 to 1

Fig. 3: Comparison of generation level for various reliability levels

IV. CONCLUSIONS

In this paper, a risk-averse UC problem was considered. The unit commitment decision is formulated as a chance constrained optimization problem with an ambiguous joint probability distribution function. We used a data-driven representation of uncertainty induced by variable renewable generation, including both wind and solar. Our solution method uses a combination of a convex relaxation of the probabilistic-feasible set and an inexact bundle method to accelerate solution of the dual form of the problem. We implement a small-scale network as a case study, to examine the impact and limitations on the sample size used to represent uncertainty. Case studies corroborate the potential of this approximation to be transferred to large-scale UC instances via separation of the stochastic representation.
from the network scale of the problem, showing significant potential in scaling.

The advantages of this method include: 1) the ability to use uncertainty sets that are much larger than those utilized in other approaches, resulting in excellent out-of-sample performance, 2) flexibility for the system operator to customize the riskiness of the solution across space (nodal) and time, to incorporate system-specific experience and knowledge and 3) the flexibility to adjust the overall risk-aversion in the solution from a more risking or ‘expected value’ case, to the most robust scenarios in one framework.

Some interesting research directions show promise for improving our approach. In the case when there is limited data to provide an accurate statistical model to generate realizations of the renewable generation, we could explore ways to describe our quantile-based approximation for small-size data sets by using the techniques described in [39]. The primary extension of this work will be to ensure scalability to larger networks. To this end, strength mixed-integer formulations for the thermal subproblem will be explored to reduce the computational time on the thermal sub-problem, and the model proposed in [40] shows significant potential for implementation within the framework implemented in this paper. Other heuristic techniques could be explored to accelerate primal-feasible solutions, for example, the Lagrangian-based techniques proposed in [41] could be modified for our model.
REFERENCES


