An Evaluation of Algorithms to Solve for Do-Not-Exceed Limits for Renewable Resources

Shaobo Zhang, Nikita G. Singhal, Kory W. Hedman, Vijay Vittal, Junshan Zhang
Arizona State University
{szhan144, nsinghal, kory.hedman, vijay.vittal, junshan.zhang}@asu.edu

Abstract
As power systems become more reliant on intermittent resources, system operators are faced with the fact that the availability of intermittent resources is beyond human control and largely unpredictable. Due to the lack of compliance from intermittent resources to follow strict dispatch instructions, the dispatch instruction for an intermittent resource is proposed to be a desired dispatch range or a do-not-exceed (DNE) limit. In this paper, a two-stage robust optimization problem determines the DNE limits, which define the region where the system is guaranteed to reliably handle any wind realization within the specified DNE limit range. Two algorithms are implemented in this paper, each of which can handle the bilinear sub-problem in this two-stage problem. The results show that local search algorithms do not perform well when compared to a big-M based MILP reformulation. All numerical results are based on the IEEE-118 bus test system.

Nomenclature

Indices
- \( g \) Generator.
- \( k \) Transmission line.
- \( t \) Time period.
- \( R \) Reference node.
- \( b \) Bus location.
- \( b(g) \) Bus location of generator \( g \).
- \( b(n) \) Bus location of wind generator \( n \).
- \( b(q) \) Bus location of demand \( q \).

Parameters
- \( p_{gt} \) Production of generator \( g \) in time \( t \) (solution obtained from unit commitment).
- \( \omega_{nt} \) Lower bound of wind generation at bus \( n \) in time \( t \).
- \( \omega_{nt}^{UB} \) Upper bound of wind generation at bus \( n \) in time \( t \).
- \( \omega_{nt} \) Wind generation at bus \( n \) in time \( t \).
- \( W \) Wind uncertainty set.
- \( z_{nt} \) Relates wind generation \( \omega_{nt} \) to the \( \omega_{nt}^{LB} \) or the \( \omega_{nt}^{UB} \).

Variables
- \( d_{qt} \) Forecasted load at bus \( q \) in time \( t \).
- \( p_{k}^{min} \) Minimum capacity rating of transmission line \( k \).
- \( p_{k}^{max} \) Maximum capacity rating of transmission line \( k \).
- \( p^{g}_{min} \) Minimum capacity of generator \( g \).
- \( p^{g}_{max} \) Maximum capacity of generator \( g \).
- \( PTDF_{b,k} \) Power transfer distribution factor.
- \( u_{gt} \) Unit commitment status of generator \( g \) in time \( t \) (solution obtained from unit commitment).
- \( R_{10}^{g} \) 10 minute ramp up/down rate for generator \( g \).
- \( M_{nt} \) Big-M value for wind generator \( n \) in time \( t \).

1. Introduction

The anticipated growth of intermittent resources, such as wind and solar, in the United States has raised concerns about how system operators will maintain energy balance between generator production and demand. In order to maintain a constant and reliable supply of electricity to the consumers, the system frequency needs to be maintained within a tolerable range of 60 Hz (or 50 Hz, depending on country/region). Control responses to energy imbalances take place in a particular hierarchy. The primary control response comes from system inertia
and governor response (< 20 seconds). The secondary control response comes from specific generation units (e.g., natural gas) on automatic generation control (AGC); these units provide regulation reserves in the ancillary markets. Regulation reserves are useful in handling small (net) load fluctuations, thereby ensuring energy balance between supply and demand, which is necessary for effective system control. AGC is independent of economic cost and dispatch signals are sent to the units every few seconds (four seconds for many systems). The tertiary control response comes from spinning and non-spinning reserves. These reserves are also referred to as contingency reserves as they are called upon when there is a disturbance, i.e., a generator or transmission outage. Tertiary reserves are also used when the change in load is too large for regulation reserve to compensate [1]. Spinning reserve is also used when it is necessary to replace activated regulation reserve such that it is possible to maintain the required amount of regulation reserve. The operating point of the generating units needs to be moved to a new set point for tertiary control [1]. Tertiary control signals, dispatch rate ($/MWh) or economic base point (MW), are usually derived from economic re-dispatch and they enable the power system to follow the load over a 24-hour period [1].

Regional transmission organizations (RTOs) and independent system operators (ISOs) in the United States manage the real-time dispatch using security constrained economic dispatch (SCED) [1]. SCED sends dispatch instructions, i.e., desired dispatch points (DDPs), to conventional generators, AGC units, and dispatchable load; SCED also provides the resulting market clearing prices, e.g., locational marginal prices (LMP). Intermittent resources are assumed to produce at the forecasted levels in traditional SCED problems. However, due to their intermittent characteristic, renewable resources, such as wind and solar, usually fail to follow these stiff DDPs. Thus, the dispatch instruction for an intermittent resource is proposed to be a desired dispatch range or a do-not-exceed (DNE) limit instead [2]-[4]. Here, a DNE limit is defined as the intermittent resource’s maximum output range (lower bound and upper bound) that the power system can handle without compromising system reliability. However, it should be noted that, with adequate levels of storage and the choice to spill wind, intermittent resources have the ability to produce at DDPs. Although, this comes with the costs of purchasing and maintaining storage equipment [17] as well as wasting the potential wind energy when spillage occurs. There is, thus, a tradeoff between forcing wind farms to follow strict DDP instructions versus allowing them to produce within a DNE limit range. The DNE limit range imposes uncertainty on system-wide operations. By requiring renewable resources to always match a DDP, such a strict policy is rather costly as then the uncertainty is required to be handled locally. Instead, with reserve sharing, it is more efficient to handle (at least part of) the uncertainty at the system-wide level.

This paper focuses on determining the DNE limits for wind generators on a nodal basis. The proposed solution framework is also applicable to other kinds of renewable resources. The wind generators are expected to produce within their DNE limits. Any violation of the DNE limit would result in penalties for the wind generators [2]. In [2], the authors determine the DNE limits for wind generators using three approaches: 1) an affine policy approach with fixed participation factor, 2) an affine policy approach with optimal participation factor, and 3) a fully adaptive approach. Although the affine policy approach with fixed participation factors was the easiest to implement, it was not the least conservative as that occurred with the fully adaptive approach. Reference [2] also indicates the use of ancillary services, which can provide 10-minute ramping products, to compensate for deviations in wind generators from their forecasted levels. These products are usually available in the 10-minute dispatch window (as required by SCED). In this paper, we investigate two different algorithms that are commonly used to solve the bilinear sub-problem in a two-stage robust optimization problem: a mountain climbing method [5]-[7] and a mixed integer linear program (MIP) formulation [8]-[10] by using big-M values to formulate disjunctive constraints. Here, the two algorithms to solve the bilinear sub-problem were implemented for comparison purposes. An approach, which is similar to the fully adaptive approach in [2], is proposed to determine the DNE limits for wind generators, given what the system can accommodate in terms of ramping capability.

The paper is organized as follows: Section 2 describes the do-not-exceed limit problem formulation. Section 3 discusses the solution methodology for the do-not-exceed limit problem. Section 4 presents the computational results and Section 5 concludes with potential future work.

2. Do-Not-Exceed limit problem

The structure of the DNE limit problem is such that it does not fit the conventional definition of a standard robust optimization problem. Consider the case of a standard two-stage adaptive robust unit commitment (UC) model [11] for security constrained unit commitment (SCUC) in the presence of wind uncertainty. In this case, the uncertainty set is determined beforehand. In other words, given the wind
uncertainty set, the objective of the standard two-stage adaptive robust UC model is to design the power system such that it can accommodate the worst-case wind scenario within the wind uncertainty set. However, in the case of a DNE limit problem, the objective of the two-stage adaptive robust optimization problem is to determine the largest wind uncertainty set that the system can accommodate, given what the system is capable of handling (in terms of 10-minute ramp capability of the AGC units) in order to operate reliably. In other words, the goal is to determine the worst-case wind uncertainty set.

The DNE limit problem is extensively studied in [2]. The two-stage adaptive robust optimization problem to determine the DNE limits is given in (1)-(8). The first stage of the problem is to determine the largest wind uncertainty set, $W = \prod_{t} [w_{nt}^{LB}, w_{nt}^{UB}]$, while the second stage of the problem is to determine the worst-case cost or the worst-case realization of wind that the system can accommodate with the associated wind uncertainty set obtained in the previous stage. To simplify the problem, a linear function of the DNE limits is used to measure the size of the wind uncertainty set $W$.

$$\min_{w_{nt}^{LB}, w_{nt}^{UB}, p_{gt}} \left( \sum_{n,t} (w_{nt}^{UB} - w_{nt}^{LB}) + \sum_t \max_{w_{n,t}} 0 \right) \tag{1}$$

s.t.:

$$w_{nt}^{min} \leq w_{nt}^{LB} \leq w_{nt} \leq w_{nt}^{UB} \leq w_{nt}^{max} \tag{2}$$

$$\sum_g p_{gt} + \sum_n w_{nt} = \sum_t d_{gt}, \forall t \tag{3}$$

$$-p_{k}^{min} \leq \sum_g P_{TD}D_{D(n)}kP_{gt} + \sum_n P_{TD}F_{b(n)}k_{wt} - \sum_t P_{TD}D_{D(n)}k_{wt} \leq p_{k}^{max}, \forall k \tag{4}$$

$$p_{gt} \leq \mu_{gt}^{max}, \forall g, t \tag{5}$$

$$p_{gt} \leq \gamma^{10} + p_{gt}, \forall g, t \tag{6}$$

$$-p_{gt} \leq \gamma^{10} - p_{gt}, \forall g, t \tag{7}$$

$$-p_{gt} \leq \gamma^{10} - p_{gt}, \forall g, t \tag{8}$$

In the above formulation, $w_{nt}^{LB}$ and $w_{nt}^{UB}$ represent the lower bound and the upper bound of the wind uncertainty set. Equation (1) represents the objective of the DNE limit problem and has two terms, which reflects the two-stage nature of decision [11]. Equation (2) represents wind uncertainty. Equation (3) represents the energy balance constraint that equates the system level supply and demand at each time period. Equation (4) is the transmission flow constraint. Equations (5)-(6) are the generator output limit constraints for a fixed commitment status. Equations (7)-(8) represent the speed at which a unit can ramp up and ramp down its production levels, i.e., the ramp rate constraints. It is important to note that, in this case study, deviations in wind generators from their forecasted levels are considered to be compensated by units that can provide 10-minute reserve products (spinning and non-spinning reserves). Thus, the 10-minute ramp rate, $R_{n}^{10}$, is considered in the ramp rate constraints.

It is important to note that the fixed commitment status and dispatch decisions, needed in (5)-(8), are obtained from the SCUC and the SCED problems respectively. Consider a real-time economic dispatch problem. The dispatch decision is made 10-minute ahead (i.e., at $t = 0$) of the actual uncertain wind realization. The fixed dispatch decision cannot guarantee energy balance at $t = 10$ since the wind is uncertain. Thus, the AGC units are required to adjust their output within the 10-minute dispatch window to maintain the energy balance. The 10-minute ramp capability of the generators is limited; hence, the generator production levels cannot deviate too much from their initial dispatch points (obtained from SCED). Thus, the unit status and the dispatch level of the units are fixed in the DNE limit problem. In other words, the DNE limits are obtained from a post-processing algorithm [2]. In order to show the impact of different operating points on the DNE limits, the DNE limits are calculated for the initial 10 minutes of every hour in a 24 hour period. The DNE limit problems for each hour are solved simultaneously for simplicity reasons, although they are actually independent from each other.

The above problem is separated into two stages: the outer level master problem, which determines the largest uncertainty set, $W$, and the inner max-min problem, which determines the worst-case wind realization, $w \in W$, while ensuring the system can accommodate the realization. It is evident from (1)-(8) that the master problem is to

$$\min_{w_{nt}^{LB}, w_{nt}^{UB}, \sum_{n,t} (w_{nt}^{UB} - w_{nt}^{LB}) \text{ subject to constraint } (2)} \tag{9}$$

while the inner max-min problem is to

$$\min_{p_{gt}, \sum_{n,t} \max_{w_{n,t}} 0} \text{ subject to constraints } (3)-(8).$$

It is important to note that the inner problem to determine the worst-case cost or worst-case wind realization has a max-min term in the objective function [11]. Also, it was noticed that for any $w_{nt}$ in the uncertainty interval, $W$, $w_{nt}$ can be expressed as a linear combination of the lower bound, $w_{nt}^{LB}$ and the upper bound, $w_{nt}^{UB}$ with a continuous parameter $z_{nt} \in [0,1]$ as follows [2]:

$$w_{nt}(z_{nt}, w_{nt}^{LB}, w_{nt}^{UB}) = z_{nt}w_{nt}^{UB} + (1 - z_{nt})w_{nt}^{LB}, \forall z_{nt} \in [0,1]. \tag{9}$$

Slack variables $s_{n}^{+}, s_{n}^{-}, z_{kt}^{+}$ and $z_{kt}^{-}$ are included in the energy balance and the transmission flow constraints respectively to ensure the feasibility of the inner level max-min problem [11]. It is useful to
separate the above formulation into two stages: 1) an outer level master problem and 2) an inner level max-min problem as follows:

Outer level problem:
\[
\begin{aligned}
& \min_{w_{nt}} \varphi_{lb}(u_{nt}, w_{nt}) + \sum_{k} \max_{z_{nt}(0,1)} Q_t(w_{nt}, z_{nt}, w_{nt}, l_{lb}, u_{nt})) \\
& \text{s.t.:} \\
& w_{nt}^{min} \leq w_{nt}^{lb} \leq w_{nt} \leq w_{nt}^{max} \\
& Q_t(w_{nt}(z_{nt}, w_{nt}, l_{lb}, u_{nt})) = \min_{\alpha_{nt}^+ \geq \alpha_{nt}^- \geq 0} (C(t_s + \alpha_{nt} + \sum_k(z_{kt} + \alpha_{kt}))) \\
& \text{s.t.:} \\
& \sum_g P_{gt} + s_t = \sum_g d_{gt} - \sum_n w_{nt}(z_{nt}, w_{nt}^{lb}, w_{nt}^{ub}), \forall t \\
& -\sum_g \varphi_{lb}(u_{nt}, w_{nt}) - \sum_k PTDF_{b(g),k} d_{gt} - z_{kt} \leq \sum_k PTDF_{b(g),k} d_{gt} + p_{kt}^{max}, \forall k, t \\
& -\sum_n w_{nt}(z_{nt}, w_{nt}^{lb}, w_{nt}^{ub}) + \sum_k PTDF_{b(g),k} d_{gt} + p_{kt}^{max}, \forall k, t \\
& -p_{gt} \leq -p_{gt}^{min}, \forall g, t \\
& p_{gt} \leq p_{gt}^{max}, \forall g, t \\
& -p_{gt} \leq R_{gt}^{10} - \bar{P}_{gt}, \forall g, t \\
& -p_{gt} \leq \bar{R}_{gt}^{10} - \bar{P}_{gt}, \forall g, t
\end{aligned}
\]

where $C$ represents the penalty cost to violate the energy balance and transmission flow constraints. In this study, $C$ is an approximate value, $5500/\text{MWh}$ [11], which is assumed to be the value of lost load (VOLL). This was done to ensure feasibility of the inner level max-min problem. $Q_t(w_{nt}(z_{nt}, w_{nt}^{lb}, w_{nt}^{ub}))$ denotes the cost of the inner level problem at each time period. According to [11], the inner level max-min sub-problem can be reformulated into a single level optimization problem by taking the dual of the inner level problem as follows:

\[
\begin{aligned}
& \max_{\alpha_{nt}^+ \geq \alpha_{nt}^- \geq 0} -F_{kt}^+ - F_{kt}^- (\sum_q d_{qt} - \sum_n w_{nt}(1 - z_{nt}(1 - z_{nt}^{lb}) + z_{nt}^{ub})) + \sum_k PTDF_{b(g),k} d_{gt} + p_{kt}^{max} F_{kt}^+ + \sum_n w_{nt}(z_{nt}, w_{nt}^{lb}, w_{nt}^{ub}) + \sum_k PTDF_{b(g),k} d_{gt} + p_{kt}^{max} F_{kt}^- \\
& \text{s.t.:} \\
& \lambda_t \leq C, \forall t \\
& -\lambda_t \leq C, \forall t
\end{aligned}
\]

where (28)-(33) define a bounded polyhedron. It is important to note that the objective function (27), in this case, is non-linear due to the presence of the non-concave bilinear terms $w_{nt}^{lb} \lambda_t$, $w_{nt} F_{kt}^+$ and $w_{nt} F_{kt}^-$. Global optimization techniques are usually used to solve bilinear programs; however, the solution time to obtain an optimal solution to bilinear programs may be prohibitively long, since a bilinear program is NP-hard [12]. It can be shown that if the polyhedron set defined by the linear constraints (28)-(33) is bounded then there is an optimal solution for the bilinear programming problem such that the optimal solution lies at an extreme point. Also, in this case, the optimal solution of the worst-case wind realization, $w_{nt}^*$, is an extreme point of the wind uncertainty set $W$ (i.e., $w_{nt}^{lb}$ or $w_{nt}^{ub}$), which in turn is obtained from the outer-level master problem. There are numerous extreme points of both the polyhedron set defined by the (28)-(33) and the wind uncertainty set $W$ and not all extreme points need to be modeled in order to determine whether an optimal solution is obtained. With conventional Benders’ decomposition, when a single level problem is broken into a master problem and a slave problem, there is no limitation on the choice of the master problem (i.e., the master problem can be non-convex).
however, the slave problem is required to be a convex problem. In the case of the two-level DNE limit problem, Benders’ decomposition is an exact algorithm when the inner level problem is solved to optimality when using the MIP reformulation as presented in Section 3.3.2; note that there are a finite number of extreme points for this MIP reformulation of the original bilinear optimization problem for the inner level. When the inner level problem is solved by a heuristic, which is proposed in Section 3.3.1, the final solution is an approximate solution.

3. Solution framework for the DNE limit problem

In this paper, Benders’ decomposition is used to solve the two-level problem discussed in the previous section. Benders’ optimality cuts are applied to the outer-level problem based on the solution from the inner level problem. Two algorithms are investigated to solve the non-convex inner level problem, one of which is a heuristic that cannot guarantee global optimality of the inner-level problem and, thus, the resulting solution from the two-level problem is also not guaranteed to be a global optimal solution.

3.1. Compact matrix formulation

For notation brevity, a compact matrix formulation of the DNE limit problem is formulated as follows:

\[
\begin{align*}
\min_{W, y} & \{ A(W) + \max_{w \in W} b^T y(w) \} \\
\text{s.t.:} & \\
F(W) & \leq f \quad (34) \\
Hy(w) & \leq h, \forall w \in W \quad (35) \\
l_w y(w) & = w, \forall w \in W \quad (36)
\end{align*}
\]

where \( W \) represents the first stage decision variable (i.e., the wind uncertainty set) associated with each wind generator. \( A(W) \) is a measurement for the size of the uncertain set \( W \). In this case study, \( A(W) \) is a linear function of the lower and upper limits of the wind farm generation. Constraint (35) represents the linear constraints of the uncertainty set’s upper and lower limits. Constraint (36) represents the linear second-stage dispatch problem, in which \( y(w) \) represents the decision variables that are functions of wind uncertainty realization \( w \). Constraint (37) models the injections at buses where wind farms are located.

3.2. Outer level: Benders’ decomposition

The DNE limit problem has a two-stage structure. The bi-level optimization required in the slave problem can be converted into a single level bilinear program. Consider the dual of the second stage recourse problem with \( w \) fixed to some value:

\[
\begin{align*}
\max_{\varphi, \eta} & \quad \varphi^T h + \eta^T w \\
\text{s.t.:} & \\
\varphi^T H + \eta^T L_u & = 0 \\
\varphi & \leq 0, \eta \text{ free}
\end{align*}
\]

where \( \varphi \) and \( \eta \) are the dual variables for constraint (36) and (37) respectively.

Now, combining with the bi-level maximization problem, the slave problem can be converted into the following bilinear programming (BP) problem:

\[
\begin{align*}
\max_{W, \varphi, \eta} & \quad \varphi^T h + \eta^T w \\
\text{s.t.:} & \\
\varphi^T H + \eta^T L_u & = 0 \\
\varphi & \leq 0, \eta \text{ free}, w \in W
\end{align*}
\]

It is important to note that, a bilinear problem with separable linear feasible set has a finite set of extreme points. Now, consider a general formulation for the bilinear problem [6] with separable feasibility set:

\[
\max_{x \in X, y \in Y} f(x, y) = a^T x + x^T Q y + b^T y
\]

Here, sets \( X \) and \( Y \) are bounded polyhedrons defined by linear constraints. Defining \( V(X) \) and \( V(Y) \) as the set of extreme points of sets \( X \) and \( Y \) respectively, it can be shown that if \( X \) and \( Y \) are bounded then there is an optimal solution for the BP problem, \( (x^*, y^*) \), such that \( x^* \in V(X) \) and \( y^* \in V(Y) \), i.e., the optimal solution to the separation problem will exist at an extreme point [6]. Thus, the bilinear problem can be converted into the following equivalent form:

\[
\max_{x \in V(X), y \in V(Y)} f(x, y) = a^T x + x^T Q y + b^T y
\]

There are many extreme points, \( V(X) \) and \( V(Y) \), and most of them need not to be modeled. We could use the slave problem to find the extreme points that need to be modeled, which are then passed to the outer level problem and represented by an optimality cut as is the case with Benders’ decomposition.

With the Benders’ decomposition algorithm, the set of extreme points in the slave problem is independent of the first stage decision variables. Since the feasible set of \( w \) is \( W \) in the DNE limit problem, it is apparent that \( W \) will be dependent only on the first stage decision variables. In order to overcome this
problem, a new variable $z_{nt}$, as mentioned in (9), was introduced to reformulate the problem into an equivalent form as shown below:

\[
\min_{w, y} (A(W) + \max_{z \in z} b^T y(w)) \quad \text{(46)}
\]

\[
s.t.: \quad F(W) \leq f
\]

\[
H y(w) \leq h, \forall z \in Z \quad \text{(47)}
\]

\[
l_i y(w) = w, \forall z \in Z \quad \text{(48)}
\]

where $z = \{z_{nt}, \forall n, t\}, w = w(z, W)$ as defined in (9), which is a linear function and $Z = \{z_{nt}, \forall n, t | z_{nt} \in [0,1]\}$, which is independent of $W$. Let the inverse function of (9) with fixed $W$ be: $z(w, W) = w^{-1}(z, W)$.

Store the solution as $w_k, \varphi_k, \eta_k$. Store $z_k = z(w_k, W_k)$. Set $UB_{BD,k} = A(W_k) + \varphi_k^T h_k + \eta_k^T w_k$. If $UB_{BD,k}$ is lower than the lowest upper bound, $UB_{BD}$, then set $UB_{BD} = UB_{BD,k}$.

Step 3) Form optimality cut $k$ as follows and apply to the master problem:

\[
q \geq \varphi_k^T h_k + \eta_k^T w(z_k, W)
\]

Step 4) Check for convergence. If not, return to step 2.

Note that the sub-problem (the slave) is always feasible and, thus, feasibility cuts are not needed for this particular problem.

3.3. Inner level bilinear problem

As seen in the previous section, the bilinear problem is NP hard in general. In this case study, two different algorithms were investigated: a mountain climbing method and a mixed integer linear program (MIP) formulation by using big-M values to formulate disjunctive constraints. This paper implements both the algorithms for comparison purposes.

3.3.1. Mountain climbing algorithm. The mountain climbing algorithm is an example of a local search method. It is a heuristic approach to reach a local optimal solution [5]-[7]. The following two linear programs are solved iteratively:

\[
Q(w) = \max_{\varphi, \eta} \varphi^T h + \eta^T w \quad \text{(57)}
\]

\[
s.t.: \quad \varphi^T H + \eta^T l_i = 0
\]

\[
\varphi \leq 0, \eta \text{ free, } w \in W \quad \text{(58)}
\]

\[
Q(\varphi, \eta) = \max_w \varphi^T h + \eta^T w \quad \text{(60)}
\]

\[
s.t.: \quad w \in W. \quad \text{(61)}
\]

Initialization: Find an initial value $w_0 \in W$.

Step 1) Solve BP1, i.e. $Q(w_{i-1})$. Store the solution as $\varphi_i$ and $\eta_i$.

Step 2) Solve BP2, i.e. $Q(\varphi_i, \eta_i)$. Store the solution as $w_i$.

Step 3) If $Q(w_{i-1}) = Q(\varphi_i, \eta_i)$, then stop the algorithm. Otherwise let $i = i+1$, go to Step 1.

3.3.2. MIP formulation by the big-M method. The mountain climbing method can only guarantee a local optimal solution. The quality of the cut generated by the slave problem affects the convergence rate of the decomposition algorithm. Thus, the BP problem is
reformulated into an MIP that represents the extreme points of the uncertainty set. This reformulation is able to guarantee a global solution since it is known that the solution to the BP problem will be at one of these extreme points. By reformulating the problem into an MIP, big-M values (large multipliers) are used to form disjunctive constraints; the primary setback of this approach is that it is well-known that such formulations have poor relaxations. Therefore, the solution time may be prohibitively long. However, it is possible to guarantee robustness of the final solution by using a tolerable optimality gap, which helps reduce the computational burden.

In the DNE limit problem, the extreme points of the uncertainty set can be easily obtained. They are the binary combinations of each uncertain wind farm’s output, which, in turn, is either the wind farm’s upper limit or its lower limit.

A new continuous variable, \( \hat{\mu}_{nt} \), is used to substitute the bilinear terms in the objective function of the sub-problem. Binary variables \( X_{nt} \) were introduced to represent the extreme points of the uncertainty set. The following constraints were added to the model,

\[
\mu_{nt} - w_{nt}^{UB} \eta_{nt} + M_{nt} X_{nt} \geq 0 \quad (62)
\]

\[
\mu_{nt} - w_{nt}^{LB} \eta_{nt} - M_{nt} X_{nt} \leq 0 \quad (63)
\]

\[
\mu_{nt} - w_{nt}^{LB} \eta_{nt} + M_{nt} (1 - X_{nt}) \geq 0 \quad (64)
\]

\[
\mu_{nt} - w_{nt}^{UB} \eta_{nt} - M_{nt} (1 - X_{nt}) \leq 0 \quad (65)
\]

It can be seen that when \( X_{nt} \) equals to 0, the first two constraints will enforce \( \mu_{nt} = w_{nt}^{LB} \eta_{nt} \); when \( X_{nt} \) equals to 1, the latter two constraints will enforce \( \mu_{nt} = w_{nt}^{UB} \eta_{nt} \). The value of \( M_{nt} \) should be chosen to be big enough in order to make sure that, when one set of the constraints are binding, the value of \( \mu_{nt} \) will not violate the other set of constraints (no matter what value the dual variables \( \eta_{nt} \) take) [15]. When \( X_{nt} = 0 \), from (62-63), \( \mu_{nt} = w_{nt}^{LB} \eta_{nt} \). Also from (64-65),

\[
-M_{nt} \leq (w_{nt}^{LB} - w_{nt}^{UB}) \eta_{nt} \leq M_{nt} \quad (66)
\]

From (21-24), we have

\[
|\eta_{nt}| \leq -\lambda_t + \sum_k PTDF_{b(n),k} (F_{kt}^- - F_{kt}^+)
\]

\[
\leq |\lambda_t| + \sum_k |PTDF_{b(n),k}| (|F_{kt}| + |F_{kt}^+|)
\]

\[
\leq (1 + 2 \sum_k |PTDF_{b(n),k}|) C
\]

Since constraint (64-65) should not be binding when \( X_{nt} = 0 \), we have \( |\eta_{nt} (w_{nt}^{LB} - w_{nt}^{UB})| \leq (1 + 2 \sum_k |PTDF_{b(n),k}|) (w_{nt}^{max} - w_{nt}^{min}) C \). A similar result can be obtained when \( X_{nt} = 1 \). Thus, the lower bound of \( M_{nt} \) is found to be \( (1 + 2 \sum_k |PTDF_{b(n),k}|) (w_{nt}^{max} - w_{nt}^{min}) C \).

However, if the value of \( M_{nt} \) is too large, it will cause trickle flow and numerical stability problems when solved by MILP solvers [16]. Reference [13] mentions that, the indicator constraint in CPLEX can avoid potential numerical problems caused by an improper value of \( M_{nt} \). This requires further investigation that is beyond the scope of this paper.

4. Computational results

The proposed two-stage robust optimization DNE limit problem was implemented on the IEEE 118-bus test system, see UW (2010) [14]. It is important to note that the standard IEEE 118-bus test system was modified for the purpose of this study. The test system consists of 118 buses, 186 transmission lines, 71 conventional generators with a total generation capacity of 6871 MW, and 9 wind generators with a total generation capacity of 1350 MW (150 MW each). The peak load of the system is 4004 MW. Generator information from the reliability test system-1996 [14] was used to create the generator data for the test system. The maximum 10-minute ramp down and ramp up capability in the test system is 894 MW and 431 MW respectively. The proposed algorithm was written in C++ and solved with CPLEX version 12.5.1. All simulations were run on a PC with an Intel(R) Core(TM) i5-3320M 2.60 GHz CPU and 8 GB RAM. The convergence tolerance for the outer level Benders’ decomposition algorithm was \( e = 100 \) and the convergence tolerance for the inner level bilinear problem, using the mountain climbing approach, was \( e = 100 \). The MIP gap for the inner level bilinear problem using the big-M formulation was \( 10^{-4} \).

We compare the performance of the two algorithms based on: 1) system level DNE limits, 2) individual wind farm DNE limits, 3) factors affecting ramp up limits from DNE limit, and 4) factors affecting ramp down limits from DNE limit.

4.1. Big-M reformulation results

Since the big-M formulation can guarantee a global optimal solution to the slave problem, the result of the big-M formulation is used as the final result of the DNE limits. The results of the big-M formulation are shown in Figure 2 to Figure 5. The total wind generation DNE limit for the test system is shown in Figure 2. We can see that both the upper and the lower limits are quite far away from the predicted wind generation values except for a few time periods, which
indicates that the test system under study is quite strong in accommodating intermittent wind generation.

Figure 2. Total wind generation DNE limit for the test system, big-M formulation.

Figure 3. Wind generator 1 DNE limit, big-M formulation.

However, the results for individual wind farms are not satisfying. The DNE limit for wind farm 1 is plotted in Figure 3. In some time slots, the lower limit is as low as the minimal generation capacity of the wind farm, i.e., 0 MW in our study. However, there are some time slots in which the lower limit of the DNE limit is equal to the predicted output value for the wind farm, which means that the wind farm should generate at least the predicted value. Similarly, it can be observed in Figure 2 that when \( t = 3 \), the upper limit of the DNE limit for the test system is equal to the predicted output value for all the wind farms combined together, which means that the wind farms can generate at the most the predicted values. This phenomenon is caused due to the structure of the master problem. In this case study, we put great efforts in solving the bilinear problem; however, a very basic formulation was used to solve the first stage problem (master problem). Also, in this case study, the sum of the DNE intervals for each wind farm is the objective of the master problem. This means the master problem is a linear programming problem and there is nothing constraining the master problem from giving such unrealistic limits for individual wind farms as its purpose is to maximize the interval for all wind farms collectively. Further investigation regarding the form of the objective for the master problem should be considered; furthermore, future work is needed to develop constraint sets that ensure adequate DNE limits are obtained for each wind farm individually.

Figure 4. Factors affecting ramp up limit from DNE limit, big-M formulation.

Figure 5. Factors affecting ramp down limit from DNE limit, big-M formulation.

It is of interest to analyze what factors are affecting the DNE limit. There are three factors that may have impact on the DNE limit: the wind generation max/min capacity limit, 10-min reserve ramp up/down limit, and transmission network congestion. The former two factors are easy to analyze; their impact on the DNE limit can be directly calculated. The latter factor, i.e., network congestion constraint, cannot be calculated explicitly. This constraint also makes the slave problem much harder to solve. To analyze these factors, the wind generation capacity limit and the 10-min reserve ramp limit are plotted together with the ramp up/down limits from the DNE limit. The ramp up limit is defined by the upper limit of DNE limit minus the predicted value of wind generation. The ramp down
limit is similarly defined. From Figure 4, we can see that the ramp up limit is constrained by the minimal value of wind generation max capacity limit and the 10-min reserve ramp down limit (notice that when wind generation goes up, the other generators should ramp down to balance the load). It is interesting to note that, due to congestion in the system, the DNE limit is affected by the network congestion constraint at frequent intervals. In this case, the impact of congestion, for example, was seen in time periods 2, 3, 4, 5, 12, 13, 14, 15, 17, 21, 23, and 24. With increased congestion, we expect the impact of DNE limits to be more severe. The impact of network congestion constraints can be easily analyzed with the formulation used in this work.

Figure 5 shows similar results as Figure 4; the only difference is that, throughout the time periods, the binding constraint is always the 10-min reserve ramp up limit.

4.2. Mountain climbing results

The mountain climbing algorithm is a local search method that can only guarantee local optimality. Thus, the DNE limits, obtained from mountain climbing, are not always robust; it is possible that some wind realizations within the DNE limit solution cannot be accommodated by the system. Figure 6 to Figure 9 show the corresponding results. It can be observed from Figure 6 and Figure 7 that the upper limit of the DNE limit is the maximum generation capacity of the wind farms, which is different from the results obtained from the big-M formulation.

Also, Figure 8 shows that the ramp up limit obtained from the DNE limit is different from the ramp up limit obtained when using the big-M formulation. It was seen earlier that, due to network congestion, the ramp up limit was less than both the wind generation max capacity limit and the 10-min reserve ramp down limit in certain intervals. However, with the mountain climbing algorithm, the limiting factor was the wind generation max capacity limit for almost all intervals. The fact that there is no network congestion in the one particular scenario searched by the mountain climbing method does not imply that there will not be network congestion in all other possible scenarios. The results obtained from the mountain climbing method are substantially different in comparison to the results from the big-M reformulation, which indicates that the mountain climbing method is not preferred.
5. Conclusions

In this work, the problem of determining the DNE limits for wind generation is studied. The problem is formulated as a two-stage problem and Benders’ decomposition is used to solve the problem. Due to the bilinear term in the objective, the slave problem is no longer a LP problem. Two different algorithms, to solve the bilinear sub-problem, are discussed and implemented in this paper for comparison purposes. The algorithms that are presented are: a mountain climbing method and a big-M reformulation for disjunctive constraints. The mountain climbing method can only guarantee local optimality whereas the big-M formulation can guarantee global optimality. One major concern for the big-M formulation is that improper values of M in the equations have the potential of resulting in numerical problems. It is also known to cause poor relaxation during the branch and bound process, thereby resulting in a poor computational performance when searching the solution space. Hence, further study is required regarding how to determine appropriate big-M values for the big-M formulation. Also, further study is required to improve the model formulation such that the DNE limits are more evenly distributed across different wind farms. Future work should also focus on whether the DNE limits should be determined with a nodal model of the electric power grid or with a zonal model. Solving this class of optimization problems will be very challenging for large-scale power grid models and it is crucial to determine the right level of modeling complexity versus solution quality.

References


