Pricing Strategies in a Dual-channel Supply Chain with Local Advertising

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Abstract

With the rapid popularization of the Internet, nowadays manufacturers are increasingly adopting a dual-channel to sell their products, i.e., the traditional retail channel and online direct channel. Empirical studies have shown that retailer’s local advertising has significant effects on demand and pricing decision. However, there is scant literature addressing the decision on the local advertising and its impacts on the manufacturer and retailer’s pricing decisions. To fill this gap, we explore the optimal decisions of local advertising and pricing in a centralized and a decentralized dual-channel supply chain using the two-stage optimization technique and Stackelberg game, and analyze the impacts of local advertising and customers’ preference to the direct channel on the pricing strategies. The results show that local advertising strongly influences pricing strategies and profits of channel members. Numerical analysis reveals that customers’ preference to the direct channel has great effects on the local advertising and pricing decisions.

Keywords: Supply chain; Dual-channel; Local advertising; Pricing strategies; Stackelberg game

1. Introduction

With the rapid development of e-commerce and Internet technologies, an increasing number of manufacturers expand their product sale channel from traditional retail channel to dual channel, i.e., direct sale channel via the Internet and the retail channel, such as IBM, Hewlett-Packard, Nike, Mattel et al possess dual channel in the last decades. Nowadays, the existence of dual-channel supply chain from electronic industry, clothing industry, books industry to agricultural products industry. The dual channel is a new mode of business and manufacturers’ direct channel causes new channel competition with traditional retailers in supply chain. So, in order to survive, retailer needs to improve his level of local advertising to compete market with manufacturer. Retailer’s local advertising, with the passage of time, brings potential consumers to the stage of desire and action and gives an immediate reason to buy [1]. It becomes an effective way to resist the direct channel, especially the agricultural products in Walmart, Carrefour, Tesco et al, retailer’s local advertising has significant effects on demand, profit, customers’ channel choice and pricing strategy in a dual-channel supply chain.

Empirical studies have shown that demand volume is affected mainly by the retailer’s local advertising [1] and sale price [2, 3, 4]. It indicates that local advertising even goes beyond product price as one of the main reasons for customers’ purchasing behavior. This is why so many retailers (bricks-and-mortar), such as Walmart, Carrefour, Tesco, are striving to invest local advertising. So, we can conclude that local advertising play a significant role in a dual-channel supply chain.

Therefore, the following question is of great practical importance: How the local advertising influences the manufacturer and retailer’s pricing decisions and profits? To answer this question, we consider a manufacturer-retailer supply chain, which consists of a mix of the traditional retail channel and a direct channel. Customers can purchase products from either the traditional retailer channel or the direct channel. We present an analytical framework for local advertising and price decisions in a centralized and a decentralized dual-channel supply chain, provide a decision making aid for the manufacturer and the retailer, and analyze the impacts of local advertising and customers’ preference to the direct channel on the manufacturer’s and retailer’s pricing decisions. In a centralized dual-channel supply chain, the manufacturer controls all three decision variables: the traditional retail price, the direct channel price, and the local advertising. For a decentralized dual-channel supply chain, we formulate a Stackelberg game model, with the manufacturer as the leader, determining the direct channel price in the direct channel as well as the wholesale price, the retailer is represented as the follower, determining his local advertising and retail price given the manufacturer’s direct channel price and wholesale price.

This paper makes at least three important
contributions. Firstly, hardly previous literatures examined retailer’s local advertising decision and its impacts on the manufacturer’s and retailer’s pricing decisions, we fill this gap; Secondly, few papers consider the local advertising and price joint optimal decisions, we consider it; Thirdly, we obtain several new results different from our common knowledge, such as the direct channel price decreases with increasing customers’ preference to the direct channel under a certain condition in centralized supply chain.

The reminder of the paper is organized as follows. Section 2 provides a literature review. Section 3 details the notation and formulates the basic models. In Sections 4 and 5, we examine the price and local advertising decisions and analyze the impacts of local advertising and customers’ preference to the direct channel on the pricing decisions of a centralized and a decentralized supply chain, respectively. In Section 6, the results of numerical simulation carried out. Finally, section 7 conclude the work and provide future research directions.

2. Literature review

Literature related to this paper focuses mainly on pricing in a dual-channel supply chain and advertising. For the pricing in a dual-channel supply chain, most papers concentrated on pricing with other factors and analyzed their impacts. Chiang et al [5] analyzed the influence on retailer’s pricing when manufacturer adds online direct channel. Hua et al [6] showed that the difference between the demand transfer ratios in the two channels with respect to lead time and direct sale price and customers’ acceptance of the direct channel have great effects on the lead time and pricing decisions. Huang et al [7] developed a two-period pricing and production decision model in a dual-channel supply chain, and found that the optimal pricing decisions are affected by customers’ preference to the direct channel and the market scale change in both centralized and decentralized dual-channel supply chains. Dumrongsrir et al [8] showed that demand variability has major influences on the equilibrium prices and manufacturer’s motivation for opening a direct channel. Yao and Liu [9] established a price competition model based on Bertrand game and Stackelberg game in a dual-channel supply chain, respectively, and proposed an appropriate strategy for the manufacturer to adopt when adding an e-tail channel. Liu et al [10] depicted optimal production and pricing strategies and used a principle-agent method to find some interesting insights. Dan et al [11] examined the optimal decisions on retail services and prices in a dual-channel supply chain using the two-stage optimization technique and Stackelberg game.

As an important factor that influences the market demand, the advertising in supply chain has been researched since last century. Berger [12] used an optimization approach to study cooperative advertising in a marketing channel firstly. In the last decade, advertising was introduced into supply chain, the typical researchers include Huang and Li [13], Karray and Zaccour [14], Xie and Wei [15] et al. Xie and Wei [15] assumed that the market demand is only influenced by advertising expenditures but not by retail price, studied optimal cooperative advertising strategies and equilibrium pricing in a two-member distribution channel. Szmerekovsky and Zhang [16] ignored this assumption and analyzed the optimal advertising and pricing strategies of supply chain members based on Stackelberg game. Xie and Neyret [17] and SeyedEsfahani et al [18] also ignored this assumption and researched on advertising and pricing decisions based on four power structures: Nash, Stackelberg retailer and Stackelberg manufacturer and cooperative game, but in the setting of Nash, add a more hypotheses: the manufacturer and the retailer respective margins are equal. Austand and Buscher [19] based on the formers, ignored this assumption and not limited the ratio between manufacturer’s and retailer’s margin, analyzed advertising and pricing decisions under different power structures in the vertical supply chain.

Our work is also related to advertising in a dual-channel. The research is relatively scantly in previously, only a few researchers pay attention to it. Yan et al [2] and Wang [3] studied equilibrium pricing and cooperative advertising policies in a dual channel under two different competitive types: Bertrand and Stackelberg, but not see the advertising as decision variable and not consider cross-price effect. Wang and Zhou [4] considered market demand is influenced by price and advertising simultaneously, explored optimal policies. Liu et al [20] studied retailer’s cooperative advertising in a traditional retail channel and a dual channel under Stackelberg manufacturer, but it ignored some factors that influence the demand of product, such as service and price. Huang et al [21] as similar as Wang and Zhou [4].

So far, researches on dual-channel supply chain mainly concentrate on pricing of channel members and advertising literatures mainly focus on advertising and pricing policies in a traditional vertical supply chain.

3. Basic Assumption and Model

We consider a simple supply chain made up of one manufacturer and one retailer (see Fig. 1). The
manufacturer is the only supplier of the retailer and they sell the same product. Customers can purchase product either through the retail channel or through the manufacturer’s direct channel. We assume that both the manufacturer and the retailer choose their own decision variables to maximize their respective profits.

Following Hua et al [6], Dan et al [11], Yue and Liu [22], Huang and Swaminathan [23], we assume that the demand functions are linear in self- and cross-price effects. Since the retailer’s local advertising significantly affects customers’ preference to the retail channel, it should be incorporated into the demand functions. Obviously, the higher the retail advertising a is, the more customers will switch from the direct channel to the retail channel. The demand is a concave function with a and has the property that is consistent with the commonly observed “advertising saturation effect”, i.e., additional advertising spending generates continuously diminishing returns [15]. so we assume that demands are linear in $\sqrt{a}$, i.e., our demand functions are assumed as:

$$D_d = \theta b - \alpha_1 p_d + \beta p_r - k_d \sqrt{a}$$

(1)

$$D_r = (1-\theta) b - \alpha_2 p_r + \beta p_d + k_r \sqrt{a}$$

(2)

Which show that demand in the direct channel, represented by $D_d$, and demand in the traditional channel, represented by $D_r$, depend on the direct sale price $p_d$, as well as the retail price in the traditional channel $p_r$. $p_d$ represents the customer pays $p_d$ for a unit product if he patronizes the direct channel. $b$ represents the base level of product demand or demand rate (i.e., potential demand/demand rate if the goods are free of charge). The share of this demand going to the direct channel is $\theta(0 \leq \theta \leq 1)$, and $1 - \theta$ goes to the traditional channel when $p_d$ and $p_r$ are zero. $\theta$ reflects customers’ preference to the direct channel when the goods are free of charge. $\alpha_1$ and $\alpha_2$ are the coefficients of price elasticity of $D_d$ and $D_r$, respectively. The cross-price sensitivities $\beta$ reflects the degree to which the goods sale via the two channels are substitutes. The parameters $k_d$ and $k_r$ are the retailer’s local advertising sensitivities of the demand in the direct channel and the retail channel, respectively. It means that if the local advertising increases by one unit, $k_r \sqrt{a}$ units of the demand will increase in the retail channel, and $k_d \sqrt{a}$ units of that will be lost in the direct channel.

In order to analytical feasibility, we have several assumptions listed following:

i. To maintain analytical tractability, following Yue and Liu [22], and Hua et al [6], we assume that the cross-price effects are symmetric.

ii. Since the total demand of the two channels should be downward sloping in the retailer’s price, direct channel price, we assume that $\beta < \alpha_i$ for $i = 1, 2$, i.e., own price effects are greater than cross-price effects as similar as Hua et al [6].

iii. We have $k_r > k_d$ because retailer’s local advertising will promote the whole demand of the dual-channel supply chain.

iv. Following Xie and Wei[15], we assume that the manufacturer’s unit production cost and the retailer’s unit handling cost are constants, thus they can be normalized to zero for simplicity of the expressions.

v. Obviously, all the parameters should be positive.

We denote by $w$ the manufacturer’s wholesale price to the retailer. Then, the manufacturer’s profit is determined by:

$$\Pi_m = w D_r + p_d D_d$$

(3)

and the retailer’s profit is determined by:

$$\Pi_r = (p_r - w) D_r - a$$

(4)

Here the subscript “m” , “r” means the parameters corresponding to the manufacturer, the retailer; in the following, the subscript “c” means the whole system, the superscript “c” , “d” means the parameters corresponding to the centralized and decentralized system.

4. Centralized supply chain

In this section, we consider a centralized supply chain, in which the manufacturer and the retailer are vertically integrated. The profit of the centralized dual-channel supply chain is:

$$\Pi_c = \Pi_m + \Pi_r = p_r D_r + p_d D_d - a$$

(5)

Substituting (1) and (2) into (5), we have
\( \Pi_c = \Pi_m + \Pi_r \)
\[ = p_r(1 - \theta)b + \alpha_p r + \beta_p d + k_r \sqrt{\alpha} \]
\[ + p_d(\theta - \theta_r p + \beta_p r - k_d \sqrt{\alpha}) \]  
(6)

In order to maximize \( \Pi_c \), we examine a proposition regarding \( \Pi_c \).

**Proposition 1:** The profit of the centralized dual-channel supply chain \( \Pi_c \) is jointly concave in \( p_r \) and \( p_d \), concave in \( a \), but not jointly concave in \( p_r \), \( p_d \) and \( a \).

Proposition 1 indicates that we cannot find the optimal values of \( p_d \), \( p_r \) and \( a \) by using only the first-order optimal conditions. However we can deal with it using the two-stage optimization technique, i.e., we first find the optimal value of \( \Pi_c \) for a given \( a \) (see Proposition 2), and then find the optimal \( a \) to maximize \( \Pi_c \).

**Proposition 2:** For any given local advertising \( a \), the optimal retail price \( p_r \) and the optimal direct channel price \( p_d \) are given by
\[ p_r^*(a) = \frac{1}{2} (N b + T \sqrt{\alpha}) \]  
(7)
\[ p_d^*(a) = \frac{1}{2} (M b + S \sqrt{\alpha}) \]  
(8)

Where
\[ M = \frac{\alpha_2 \theta + \beta (1 - \theta)}{\alpha_1 \alpha_2 - \beta^2}, \quad N = \frac{\beta \theta + \alpha_2 (1 - \theta)}{\alpha_1 \alpha_2 - \beta^2} \]
\[ S = \frac{\beta k_r - \alpha_2 k_d}{\alpha_1 \alpha_2 - \beta^2}, \quad T = \frac{\alpha_1 k_r - \beta k_d}{\alpha_1 \alpha_2 - \beta^2} \]

Substituting (7) and (8) into (5), we have the total profit \( \Pi_c(a) \) as a function of \( a \) is given by
\[ \Pi_c(a) = \frac{2N + 2(M - N) - \alpha_1 M^2 - \alpha_2 N^2 + 2\beta MN \sqrt{\alpha}}{4} \]
\[ + \frac{T + \theta(S - T) - \alpha_1 MS - \alpha_2 NT + \beta(MT + NS) + k_r N - k_d M}{b} \]
\[ \frac{1}{2} + \frac{2 \beta ST - \alpha_1 S^2 - \alpha_2 T^2 + 2k_r T - 2k_d S - 4}{4} \]
\[ \frac{\alpha}{a} \]  
(9)

To find the optimal \( a \) to maximize \( \Pi_c(a) \), we differentiate \( \Pi_c(a) \) with respect to \( a \), which yields the first-order condition:
\[ \frac{d \Pi_c(a)}{da} = \frac{T + \theta(S - T) - \alpha_1 MS - \alpha_2 NT + \beta(MT + NS) + k_r N - k_d M}{b} \]
\[ \frac{1}{4} + \frac{2 \beta ST - \alpha_1 S^2 - \alpha_2 T^2 + 2k_r T - 2k_d S - 4}{4} = 0, \]
We have
\[ a^c = \left( \frac{T + \theta(S - T) - \alpha_1 MS - \alpha_2 NT + \beta(MT + NS) + k_r N - k_d M}{2 \beta ST - \alpha_1 S^2 - \alpha_2 T^2 + 2k_r T - 2k_d S - 4} \right)^2 \]

Substituting (9) into (7) and (8), we obtain the optimal prices in the two channels.
\[ p_r^*(a) = \frac{1}{2} \left( \frac{N + T(T + \theta(S - T) - \alpha_1 MS - \alpha_2 NT + \beta(MT + NS) + k_r N - k_d M)}{2 \beta ST - \alpha_1 S^2 - \alpha_2 T^2 + 2k_r T - 2k_d S - 4} \right) b \]
\[ p_d^*(a) = \frac{1}{2} \left( \frac{M + (S + \theta(S - T) - \alpha_1 MS - \alpha_2 NT + \beta(MT + NS) + k_r N - k_d M)}{2 \beta ST - \alpha_1 S^2 - \alpha_2 T^2 + 2k_r T - 2k_d S - 4} \right) b \]

**Corollary 1:**
(i) For any given \( a \), the optimal retail price \( p_r^*(a) \) is decreasing in \( \theta \), and the optimal direct channel price \( p_d^*(a) \) is increasing in \( \theta \);

(ii) The optimal retail price \( p_r^*(a) \) increases with increasing \( a \) and the rate of change of \( p_r^*(a) \) with respect to \( a \) is more than that of \( p_d^*(a) \);

(iii) When \( \lambda = \beta k_r - \alpha_2 k_d > 0 \), the optimal direct channel price \( p_d^*(a) \) will increase with increasing \( a \); when \( \lambda = 0 \), \( p_d^*(a) \) is a constant independent of \( a \); and when \( \lambda < 0 \), \( p_d^*(a) \) will decrease with increasing \( a \).

Corollary 1(i) shows that the optimal retail price \( p_r^*(a) \) will decrease and the optimal direct channel price \( p_d^*(a) \) will increase with increasing \( \theta \), which is reasonable because, intuitively, if the demand in one channel is large, then the price in that channel should be set high. Corollary 1(ii) shows that \( p_r^*(a) \) increases with increasing \( a \). This is straightforward because an increase in \( a \) intuitively means an increase in local advertising investment, so the retail price would increase. Corollary 1(iii) also shows that the rate of change of \( p_r^*(a) \) with respect to local advertising is more than that of \( p_d^*(a) \), it is an important result to provide strategic aid for manufacturer and retailer. Corollary 1(iii) indicates that \( \lambda \) determines the manufacturer’s pricing behaviors. Therefore, \( \lambda \) is very important for the integrated supply chain or the two players to make decisions. Then, what does \( \lambda \) represent? From the formula of \( \lambda \), we can see that \( \lambda \) reflect the competitive intensity between the direct channel and the retail channel. Corollary 1(iii) shows that if \( \lambda \) is positive, in other words, if the competitive intensity is relative fierce, then the optimal direct channel price increases with the local advertising increasing. Correspondingly, if the \( \lambda \) is negative, i.e., the competitive intensity is relative weak, then the optimal
The direct channel price decreases with the local advertising increasing. And if $\lambda$ is zero, the optimal direct channel price is a constant independent of the local advertising.

## 5. Decentralized supply chain

In this section, we consider a decentralized dual-channel supply chain. The manufacturer, as the Stackelberg leader, determines the wholesale price $w$ and the direct channel price $p_d$ firstly, then the retailer as the follower sets his optimal retail price $p_r$ and local advertising $a$ based on the manufacturer’s decisions. We first give the retailer’s best response functions, then decide the Stackelberg equilibrium strategies of the two players.

### 5.1. Retailer’s best response

The retailer’s best response to wholesale price $w$, and direct channel price $p_d$ set by the manufacturer is given by the following proposition.

**Proposition 3:** For given $p_d, w$ and $a$, the retailer’s best pricing strategy $p_r$ is given by

$$p_r^d(p_d, w, a) = \frac{(1-\theta)b + \beta p_d + \alpha_2 w + k_r \sqrt{a}}{2\alpha_2} \tag{11}$$

And the retailer’s optimal profit $\Pi_r^d$ as a function of $p_d, w$ and $a$ is given by

$$\Pi_r^d(p_d, w, a) = \frac{[(1-\theta)b + \beta p_d - \alpha_2 w + k_r \sqrt{a}]^2}{4\alpha_2} - a \tag{12}$$

To examine the impacts of $p_d$ and $w$ on the retailer’s best pricing strategy and profit, we take the first-order partial derivatives of $p_r^d(p_d, w, a)$ and $\Pi_r^d(p_d, w, a)$ with respect to $w$ and $p_d$, respectively, and obtain the following corollary.

**Corollary 2:**

(i) The retailer’s best response price $p_r^d(p_d, w, a)$ decreases with decreasing $p_d$ and $w$, respectively.

(ii) The retailer’s profit $\Pi_r^d(p_d, w, a)$ increases with increasing $p_d$, and decreases with increasing $w$;

(iii) The impacts of wholesale price $w$ to $p_r^d(p_d, w, a)$ and $\Pi_r^d(p_d, w, a)$ is much more than $p_d$.

Since $p_r^d(p_d, w, a)$ decreases with decreasing $p_d$ and $w$. Similar to Chiang et al [5], the manufacturer may control the retail price by adding a direct channel, and setting its wholesale price $w$ and the direct channel price $p_d$. The results of Corollary 2(ii) and Corollary 2(iii) which is intuitively evident because increasing $p_d$ may force some customers or demands in the direct channel to switch to the retail channel, and these new customers and demands will increase the retailer’s profit, however the retailer’s marginal profit will decrease with increasing $w$, so the retailer’s profit would decrease.

### 5.2. Manufacturer’s pricing strategy

Substituting (11) into (3) and simplifying, we get

$$\Pi_m = \frac{\alpha_2 w + \beta p_d}{2\alpha_2}((1-\theta)b + \beta p_d + k_r \sqrt{a} + p_d(\theta b - \alpha p_d - k_d \sqrt{a} + \beta w) - \frac{\alpha_2 w^2}{2} \tag{13}$$

We use the two-stage optimization method to maximize the manufacturer’s profit $\Pi_m$. In the first stage, we derive the optimal wholesale price, direct channel price, and the manufacturer’s optimal profit for any given local advertising $a$. The results are summarized in the following proposition.

**Proposition 4:** The manufacturer’s profit $\Pi_m$ is jointly concave in $p_d$ and $w$. For any given local advertising $a$, the manufacturer’s optimal wholesale price and the optimal direct channel price are given as follows:

$$p_r^d(a) = \frac{1}{2}(Mb + S\sqrt{a}) \tag{14}$$

$$w^d(a) = \frac{1}{2}(Nb + T\sqrt{a}) \tag{15}$$

Proposition 4 indicates that for the same local advertising, the formulations of the optimal wholesale price and the direct channel price are identical to the optimal retail price and the direct channel price in the centralized supply chain. The result may be surprising, however, intuitively, compared with the pricing strategies in the centralized supply chain, the manufacturer should consider the retailer as an end customer and set the wholesale price equal to the retail price in the centralized supply chain, and keep the direct channel price unchanged. The extreme case is that the manufacturer sells all his products through the direct channel. From Propositions 3 and 4, we can derive the following proposition.

**Proposition 5:** For any given local advertising, the retailer’s optimal retail price and optimal profit are given by

$$p_r^d(a) = \frac{[2(1-\theta)b + \beta M + \alpha_2 N\beta + (\beta S + \alpha_2 T) - 2\alpha_2 k_r \sqrt{a}]}{4\alpha_2} \tag{16}$$
\[ \Pi^d_d(a) = \frac{[(2(1-\theta) + \beta M - a_{\alpha_2} N )b +(\beta S - a_{\alpha_2} T + 2k_r )\sqrt{a} - 1]}{16a_{\alpha_2}} \]  

(17)

To find the optimal \( a \) to maximize \( \Pi^d_d(a) \), we differentiate \( \Pi^d_d(a) \) with respect to \( a \), which yields the first-order condition:

\[ \frac{d\Pi^d_d(a)}{da} = \frac{[2(1-\theta) + \beta M - a_{\alpha_2} N ]}{16a_{\alpha_2} \sqrt{a}} b \]

\[ + \frac{(\beta S - a_{\alpha_2} T + 2k_r )^2}{16a_{\alpha_2} \sqrt{a}} (a - 1) = 0 \]

We have:

\[ a_d = \left( \frac{[2(1-\theta) + \beta M - a_{\alpha_2} N ]}{16a_{\alpha_2} \sqrt{a}} b \right)^2 \]  

(18)

Substituting (18) into (14), (15) and (16), we obtain the optimal prices in the two channels.

\[ p^d_d(a) = \frac{1}{2} \left( M + \frac{N + T[2(1-\theta) + \beta M - a_{\alpha_2} N ]}{16a_{\alpha_2} \sqrt{a}} b \right) b \]

\[ w^d(a) = \frac{1}{2} \left( \frac{N + T[2(1-\theta) + \beta M - a_{\alpha_2} N ]}{16a_{\alpha_2} \sqrt{a}} b \right) b \]

\[ p^d_r(a) = \frac{4a_{\alpha_2}}{4a_{\alpha_2} + \beta S - a_{\alpha_2} T + 2k_r )^2} \]

\[ \sqrt{a} (\beta S + a_{\alpha_2} T + 2k_r )[2(1-\theta) + \beta M - a_{\alpha_2} N ](\beta S - a_{\alpha_2} T + 2k_r ) b \]

To examine the impacts of \( \theta \) and \( a \) on the manufacturer and retailer’s best pricing strategies, we take the first-order derivatives of \( p^d_d(a) \), \( p^d_r(a) \) and \( w^d(a) \) with respect to \( \theta \) and \( a \), respectively, and obtain the following corollary.

**Corollary 3:**

(i) The optimal direct channel price \( p^d_d(a) \) increases with increasing \( \theta \) while the retail channel price \( p^d_r(a) \) and wholesale price \( w \) decreases with increasing \( \theta \) for any given local advertising \( a \);

(ii) The optimal retail channel price \( p^d_r(a) \) and wholesale price \( w \) increases with increasing \( a \); When \( \lambda > 0 \), \( p^d_d(a) \) increases with increasing \( a \); when \( \lambda = 0 \), \( p^d_d(a) \) is a constant independent of \( a \); when \( \lambda < 0 \), \( p^d_d(a) \) decreases with increasing \( a \).

Corollary 3(i) shows that the optimal direct channel price \( p^d_d(a) \) increases with increasing \( \theta \) for any given local advertising, which is reasonable, because \( \theta \) reflects customers’ preference to the direct channel when the goods are free of charge, if customers’ preference to the direct channel is becoming big, the price will increase. It also shows that the retail channel price \( p^d_r(a) \) and wholesale price \( w \) decreases with increasing \( \theta \), intuitively, in order to attract customers form direct channel to retail channel, if \( \theta \) becomes big, the wholesale price and the retail price will decrease.

Corollary 3(ii) shows that the retail price and the wholesale price increase with increasing local advertising, which is similar to Corollary 1(iii).

### 6. Numerical analysis

In this section, we use numerical analysis to further examine the local advertising decisions and the pricing behaviors in the centralized and decentralized supply chain, respectively. The results of numerical analysis are summarized in Figs. 2 – 4, where \( b = 400 \), \( a_1 = 6.5 \), \( a_2 = 6 \), \( k_d = 5 \), \( k_r = 6 \), \( \beta = [4, 5, 5.5] \). The chosen parameters must make the models meaningful.

![Fig.2. Impact of \( \theta \) on the local advertising and prices (\( \lambda < 0 \))](image-url)
6.1. Centralized supply chain

From Fig. 2, when $\lambda < 0$, the local advertising $a$ decreases with increasing customers’ preference to the direct channel $\theta$ when $\theta$ less than a threshold and increases with increasing $\theta$ when $\theta$ more than the threshold. While from Figs. 3–4, when $\lambda = 0$ or $\lambda > 0$, the local advertising $a$ decreases with increasing $\theta$. From Fig. 2, when $\lambda < 0$, the retail price $p_r$ increases with increasing $\theta$ when $\theta$ less than a threshold and increases with increasing $\theta$ when $\theta$ more than the threshold. While from Figs. 3–4, when $\lambda = 0$ or $\lambda > 0$, the retail price $p_r$ decreases with increasing $\theta$.

From Figs. 2–3, when $\lambda < 0$ or $\lambda = 0$, the direct channel price $p_d$ increases with increasing $\theta$, which means that the more the base level of demand or demand rate in the direct channel is, the higher the direct channel price should be. However, from Fig. 4, when $\lambda > 0$, the direct channel price $p_d$ decreases with increasing $\theta$, it is different from our common knowledge: if the degree of customers’ preference to the direct channel is relatively high, the direct channel price should be set higher. From Figs. 2–3, when $\theta$ is below a threshold, the retail price is higher than the direct channel price. From Fig. 4, the retail price is higher than the direct channel price for any $\theta$. We can conclude that customers’ preference to the direct channel has a great impact on a vertically integrated supply chain’s pricing behavior.

6.2. Decentralized supply chain

From Figs. 2–4, the local advertising $a$ is identical and decreases with increasing $\theta$ for any $\lambda$. From Figs. 2–4, the retail price $p_r$ and the wholesale price $w$ decrease with increasing $\theta$ for any $\lambda$. The retail price $p_r$ decreases with increasing $\theta$ and great than or equal to the wholesale price $w$ for any $\lambda$. It is surprising that from Fig. 4, when $\lambda > 0$, the direct channel price decreases with increasing $\theta$ as similar as centralized setting, it is different from previous literature. From Figs. 2–4, when $\theta$ is relatively low, the wholesale price $w$ should be set higher than the direct channel price $p_d$. However, when $\theta$ is relatively high (e.g., higher than a threshold), the direct channel price $p_d$ should be set higher than the wholesale price $w$. In order to make the pricing feasible and meaningful, the degree of customers’ preference to the direct channel cannot below a threshold. From Fig. 2 and Fig. 4, when
\( \theta \) is below a threshold, the retail price \( p_r \) is higher than the direct channel price \( p_d \) for \( \lambda < 0 \) or \( \lambda > 0 \). From Fig. 3, for any \( \theta \), the retail price is higher than the direct channel price.

The above results indeed show that this observation also holds for the dual-channel supply chain, i.e., generally speaking, the higher (lower) the base level of demand in one channel is, the higher (lower) the price in that channel is, and if the base level of demand or demand rate in one channel is relatively high (e.g., higher than a threshold), the sale price in that channel should be set higher than that in the other channel.

6.3. Comparing the two settings

In this subsection, we compare the optimal local advertising and the optimal prices in the two settings. From Figs. 2–4, we can see that the local advertising \( a \) in the centralized supply chain should set higher than that in the decentralized one for any \( \theta \) and \( \lambda \), which conforms the decisions deviating in decentralized decisions.

Chiang et al [5] found that adding a direct channel and setting a proper wholesale price and direct channel price for the manufacturer can mitigate double marginalization when the degree of customers' preference to the direct channel is high enough. However, from Figs. 2–4, the retail price \( p_r \) decreases with increasing \( \theta \), when \( \lambda < 0 \) and \( \theta \) below a threshold or \( \lambda > 0 \), the retail price \( p_r \) in the centralized supply chain should be set higher than that in the decentralized one, and for most case, the direct sale prices \( p_d \) in the two settings are not equal, which shows that manufacturer and retailer in the decentralized supply chain maximizing their own profit leads to decisions deviating from the optimal decisions from the overall system's perspective, i.e., double marginalization always exists in the decentralized dual-channel supply chain.

We find that the wholesale price \( w \) and the retail price \( p_r \) in the decentralized supply chain become closer with increasing \( \theta \), in other words, \( p_r \to w \) and even \( p_r = w \) with increasing \( \theta \), which indicate that the retailer's margin is becoming small with increasing \( \theta \).

We observe that customers' preference to the direct channel has great effects on the local advertising and pricing decisions from above numerical analysis.

7. Conclusion

In this paper, we develop a framework to study the strategic roles of the local advertising in a dual-channel supply chain. Our results indicate that local advertising and price decisions are very important for the retailer and the manufacturer. We examined the optimal decisions for the local advertising and prices in a centralized and a decentralized dual-channel supply chain using the two-stage optimization technique and Stackelberg game. We provide a decision making aid for the manufacturer and the retailer, and analyze the impacts of the local advertising on the manufacturer and retailer's pricing decisions and profits.

We obtained some new results differ from those in the literature. We find that local advertising strongly influences the manufacturer and the retailer's pricing strategies and profits. Our numerical analysis shows that the parameter \( \lambda \) and customers' preference to the direct channel have great effects on the local advertising and pricing decisions. For example, when \( \lambda > 0 \), the direct channel price \( p_d \) (denoted by red and magenta in the below graphs of Fig. 4) decreases with increasing \( \theta \), may it is caused by fierce competition between direct and retail channel, the proper decrease of direct channel price will lead to sharp decrease of demand in retail channel, as a result, the demand of direct channel sharp increases.

In our work, we assume that the cross-price effect is symmetric and the manufacturer is more powerful than the retailer in the Stackelberg game. If the cross-price effect is unequal and if they are equally powerful or the retailer is more powerful, then the wholesale price or even the direct channel price may be set through negotiation. What are corresponding results in these setting? Therefore, these different settings should be explored in the future to determine whether the result remain exist validly. In another, we use static model to solve pricing decision and not consider other factors of product, such as deterioration of agricultural product and fresh food, so dynamic pricing and deteriorating items with local advertising in a dual-channel supply chain for future research.

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Appendix

Proof of Proposition 1
Taking the second-order partial derivatives of \( \Pi_c \)
with respect to \( p_r, p_d \) and \( a \), we have the Hessian matrix
\[
H = \begin{pmatrix}
\partial^2 \Pi / \partial p_r^2 & \partial^2 \Pi / \partial p_r \partial p_d & \partial^2 \Pi / \partial p_r \partial a \\
\partial^2 \Pi / \partial p_d \partial p_r & \partial^2 \Pi / \partial p_d^2 & \partial^2 \Pi / \partial p_d \partial a \\
\partial^2 \Pi / \partial a \partial p_r & \partial^2 \Pi / \partial a \partial p_d & \partial^2 \Pi / \partial a^2
\end{pmatrix}
\]
\[
= \begin{pmatrix}
-2\alpha_2 & 2\beta & \frac{1}{2}k_r a^{-\frac{1}{2}} \\
2\beta & -2\alpha_1 & -\frac{1}{2}k_r a^{-\frac{1}{2}} \\
\frac{1}{2}k_r a^{-\frac{1}{2}} & \frac{1}{2}k_r a^{-\frac{1}{2}} & \frac{1}{4}(p_d k_r - p_r k_r)a^{-\frac{1}{2}}
\end{pmatrix}
\]

Since \( \Delta_1 = -2\alpha_2 < 0 \), and \( \Delta_2 = 4(\alpha_1 \alpha_2 - \beta^2) > 0 \)
\( \Pi_1 \) is strictly jointly concave in \( p_r, p_d, \) and \( a \).
However, due to \( \partial^2 \Pi / \partial p_r^2 = -2\alpha_1 < 0 \), and
\[
\begin{align*}
\partial^2 \Pi / \partial p_d a &= -\frac{1}{2}a \left( p_d k_r - p_r k_r \right) a^{-\frac{3}{2}} - \frac{1}{4} k_r^2 a^{-1} \\

\end{align*}
\]
The above equation is negative, so \( \Pi_c \) is not jointly concave in \( p_r, p_d, \) and \( a \).

**Proof of Proposition 2**
Taking the first-order partial derivatives of \( \Pi_c \) with respect to \( p_r, p_d, \) and letting the derivatives be zero, we have
\[
\begin{align*}
\frac{\partial \Pi_c}{\partial p_r} &= (1 - \theta)b - 2\alpha_2 p_r + 2\beta p_d + k_r \sqrt{a} = 0 \\
\frac{\partial \Pi_c}{\partial p_d} &= \theta b + 2\beta p_r - 2\alpha_1 p_d - k_r \sqrt{a} = 0 \\
\end{align*}
\]
Then
\[
\begin{align*}
p_c^*(a) &= \frac{1}{2} \left( Nb + T \sqrt{a} \right) \\
p_d^*(a) &= \frac{1}{2} \left( Mb + S \sqrt{a} \right)
\end{align*}
\]

**Proof of Corollary 1**
(i) From (7) and (8), we have
\[
\frac{\partial p_c^*}{\partial a} = \frac{b(\beta - \alpha_1)}{2(\alpha_2 - \beta^2)} < 0, \quad \frac{\partial^2 p_c^*}{\partial a^2} = \frac{b(\beta - \alpha_2)}{2(\alpha_1^2 - \beta^2)} > 0
\]
(ii) From (7) and (8), we have
\[
\begin{align*}
\frac{dp_c^*}{da} &= \frac{\alpha_1 k_r - \beta k_d}{4(\alpha_2 - \beta^2) \sqrt{a}}, \\
\frac{dp_d^*}{da} &= \frac{\beta k_r - \alpha_2 k_d}{4(\alpha_1^2 - \beta^2) \sqrt{a}}
\end{align*}
\]
Since \( \alpha_1, \alpha_2, \beta, k_r > k_d, a > 0 \), so \( \frac{dp_c^*}{da} > \frac{dp_d^*}{da} \).
(iii) When \( \lambda > 0 \), \( \frac{dp_c^*}{da} > \frac{dp_d^*}{da} > 0 \); when \( \lambda < 0 \), \( \frac{dp_c^*}{da} < \frac{dp_d^*}{da} < 0 \);
and when \( \lambda = 0 \), \( \frac{dp_c^*}{da} = \frac{dp_d^*}{da} = 0 \), where \( \lambda = \beta k_r - \alpha_2 k_d \).

Therefore, we have corollary 1.

**Proof of Proposition 3**
Substituting (2) into (4), we have
\[
\Pi_r = (p_r - w)[(1 - \theta)b - 2\alpha_2 p_r + \beta p_d + k_r \sqrt{a}] - a
\]
Obviously, \( \Pi_r \) is a concave quadratic function of \( p_r \), using the first-order condition, we get
\[
p_r^d(p_d, w, a) = \frac{(1 - \theta)b + \beta p_d + \alpha_2 w + k_r \sqrt{a}}{2a}
\]
Substituting (11) into (4) and simplifying, we get (12).

**Proof of Proposition 4**
Taking the second-order partial derivatives of \( \Pi_m \) with respect to \( p_d, p_r \) and \( w \), respectively, we have the Hessian matrix
\[
H = \begin{pmatrix}
\partial^2 \Pi_m / \partial p_r^2 & \partial^2 \Pi_m / \partial p_r \partial w & \partial^2 \Pi_m / \partial p_r \partial a \\
\partial^2 \Pi_m / \partial p_d \partial p_r & \partial^2 \Pi_m / \partial p_d^2 & \partial^2 \Pi_m / \partial p_d \partial w \\
\partial^2 \Pi_m / \partial a \partial p_r & \partial^2 \Pi_m / \partial a \partial p_d & \partial^2 \Pi_m / \partial a^2
\end{pmatrix}
\]
Since \( \partial^2 \Pi_m / \partial p_r^2 < 0 \) and \( |H| = 2(a_1 \alpha_2 - \beta^2) > 0 \), \( \Pi_m \) is jointly concave in \( p_d, p_r, w \).

We use the two-stage optimization method to maximize the manufacturer’s profit \( \Pi_m \). Taking the first-order partial derivatives of \( \Pi_m \) with respect to \( p_d, p_r, w \), and letting the derivatives be zero as same as Proposition 2, we have
\[
\begin{align*}
p_d^*(a) &= \frac{1}{2} \left( Mb + S \sqrt{a} \right) \\
w^d(a) &= \frac{1}{2} \left( Nb + T \sqrt{a} \right)
\end{align*}
\]

**Proof of Proposition 5**
Substituting (14), (15) into (11) and (12) and simplifying, respectively, we get (16) and (17).

**Proof of Corollary 3**
From (14), (15) and (16), we have
\[
\begin{align*}
\frac{dp_d^*}{da} &= \frac{(\alpha_2 - \beta)}{2(\alpha_1 \alpha_2 - \beta^2)} > 0, \\
\frac{dw^d}{da} &= \frac{(\beta - \alpha_1)}{2(\alpha_1 \alpha_2 - \beta^2)} < 0 \ \\
\frac{dp_d^*}{da} &= \frac{([\beta_1^2 - \alpha_1^2] + 2\alpha_2 (\beta - \alpha_1))b}{4a_2 (\alpha_1 \alpha_2 - \beta^2)} < 0 \ \\
\frac{dp_d^*}{da} &= \frac{\beta k_r - \alpha_2 k_d}{4(\alpha_1 \alpha_2 - \beta^2) \sqrt{a}}
\end{align*}
\]
So, when $\lambda = \beta k_r - \alpha_3 k_d > 0$, \( \frac{dp_d}{da}(a) > 0 \); when $\lambda < 0$, \( \frac{dp_d}{da}(a) < 0 \), and when $\lambda = 0$, \( \frac{dp_d}{da}(a) = 0 \);
\[
\frac{dp_d}{da}(a) = \left(\frac{a_3(a_2 - \beta^2)k_r}{8a_5(a_5(a_2 - \beta^2))}\right) > 0
\]
\[
\frac{dw_d}{da}(a) = \left(\frac{a_1 k_r - \beta k_d}{4(a_1 a_2 - \beta^2)}\right) > 0
\]

So we have Corollary 2.

References